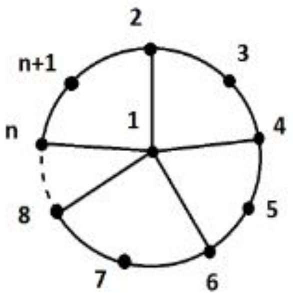
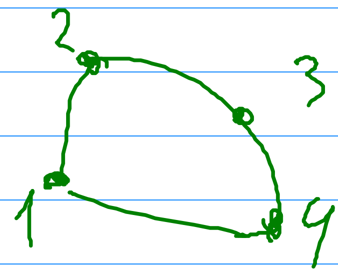
5. En el siguiente grafo de $n+1$ vértices (asumir n par), el vértice 1 está unido por una arista con cada vértice par.

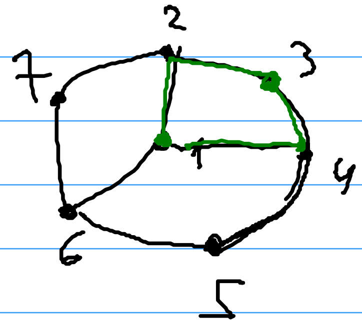


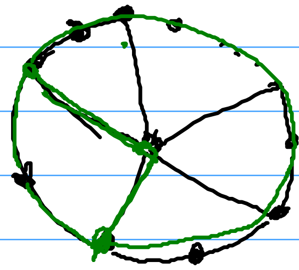


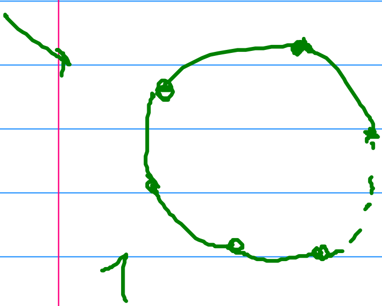
Halle la cantidad de ciclos (tamaño ≥ 3) en el grafo.

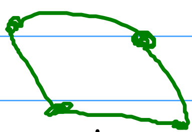
$\boxed{\text{(A) } \frac{n}{2}(\frac{n}{2}-1)+1}$ (B) $\frac{n}{2}(\frac{n}{2}-2)+1$
(C) $n(\frac{n}{2}-1)+1$ (D) $n(\frac{n}{2}-2)+1$
(E) $n(\frac{n}{2})+1$

1-2-3-4-1
2-3-4-1-2









$$\frac{(n/2)\cdot(n/2-1)}{2} \cdot \frac{\cancel{(n/2-2)}!}{\cancel{(n/2-2)}!}$$

$$\binom{n/2}{2} = \frac{n/2!}{2\cdot(n/2-2)!} = \boxed{\frac{(n/2-1)(n/2)}{2}} \ast$$

$$\cancel{2}\frac{(\frac{n}{2})(n/2-1)}{\cancel{2}} + 1 = \frac{n}{2}(\frac{n}{2}-1)+1$$

Múltiple Opción 3

Hallar la cantidad de relaciones de equivalencia en $A = \{1, \ldots, 10\}$ tales que $\#[1] = 5$ y $\#[2] = 4$.
Opciones: A) 410; B) 350; C) 280; D) 115; E) 125.

$[1] = \{1, n_1, n_2, n_3, n_4\}$ $\binom{8}{4}$ $\frac{8!}{4!\cdot(8-4)!}$

$[2] = \{2, m_1, m_2, m_3\}$ $\binom{4}{3}$

$[x] = \{x\}$ $\cdot 1$

$$\frac{8!}{4!\,\cancel{4!}} \cdot \frac{\cancel{4!}}{3!} = \frac{5\cdot\cancel{6}\cdot 7\cdot 8}{\cancel{6}} = 280$$

- $[1] = [2]$ $\ldots$

$+$

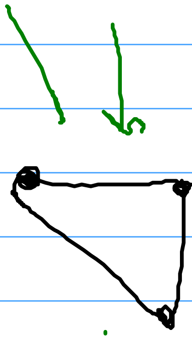
- $[1] \neq [2]$ $\ldots$

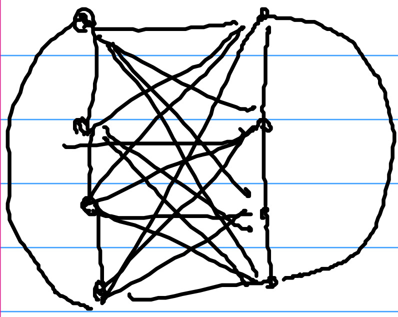
Múltiple Opción 3

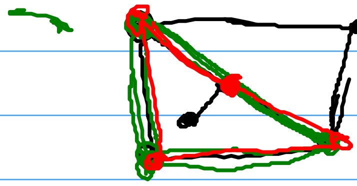
Sea $G = (V, E)$ un grafo simple 6–regular con 8 vértices (un ejemplo de tal grafo es el Hiperoctahedro). Entonces:

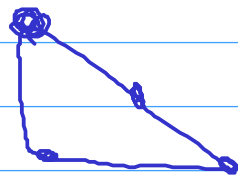
A) G no es plano y contiene un subgrafo isomorfo a K_5;
B) G no es plano y contiene un subgrafo isomorfo a $K_{3,3}$;
C) G es plano y tiene un ciclo hamiltoniano;
D) G es plano y no tiene un ciclo hamiltoniano.

$$2e = 6 \cdot 8 \to \boxed{e = 24}$$

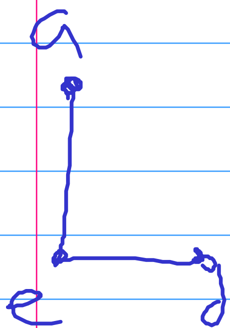


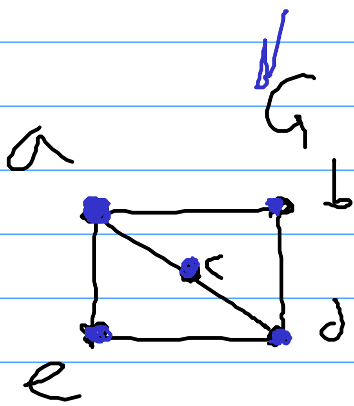


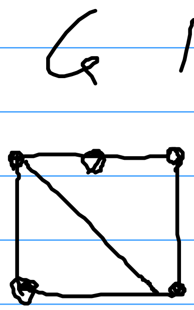




$$\begin{tikzcd} G \arrow[r] & G' \\ H \arrow[u] \arrow[ur] & \end{tikzcd}$$

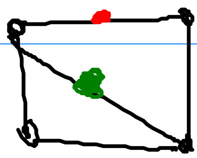






$G - e - e'$

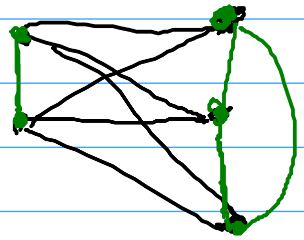
H



$G = (V, E)$. $S = (V', E')$

$V' \subseteq V$, $E' \subseteq E$

$K_{2,3}$ $\overline{K_{2,3}}$



$K_2 \cup K_3$