$a_n = a_{n-1} + a_{n-2} \rightarrow 1a_n - 1 \cdot a_{n-1} - 1a_{n-2} = 0$

$$\begin{cases} C_0 a_n + C_1 a_{n-1} = f(n) & \text{(Primer orden)} \\ C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} = f(n) & \text{(Segundo orden)} \end{cases}$$

$$\begin{cases} f(n) = 0 & \text{(Homogenea)} \\ f(n) \neq 0 & \text{(No Homogenea)} \end{cases}$$

Primer orden Homogenea

$(e)\quad C_0 a_n + C_1 a_{n-1} = 0 \quad \leftarrow \quad a_0 = A$

$$C_0 \lambda + C_1 = 0 \rightarrow \lambda = \frac{-C_1}{C_0}$$

$a_n = \lambda^n$ es una sol de $(e)$ $\quad \{\lambda^n\}$

$C_0 \lambda^n + C_1 \lambda^{n-1} = 0 \qquad C_0\left(\frac{-C_1}{C_0}\right)^n + C_1\left(\frac{-C_1}{C_0}\right)^{n-1} = 0$

$$-C_1\left(\frac{-C_1}{C_0}\right)^{(n-1)} + C_1\left(\frac{-C_1}{C_0}\right)^{(n-1)} = 0 \checkmark$$

En general $\underline{a_n = k \cdot \lambda^n}$ es solución $\forall k$

$$a_0 = k \cdot \lambda^0 \to k = A$$

<u>Ej:</u> $a_{n+1} - \frac{3}{2} a_n = 0$ $\qquad a_1 = 9/2$

$$\lambda - \frac{3}{2} = 0 \to \lambda = \frac{3}{2}$$

$\Rightarrow$ $a_n = k \cdot \left(\frac{3}{2}\right)^n$ $\qquad a_1 = k \cdot \left(\frac{3}{2}\right)^1$

$$\frac{9}{2} = k \cdot \frac{3}{2} \Rightarrow k = 3$$

$$\boxed{a_n = 3 \cdot \left(\frac{3}{2}\right)^n}$$

# Segundo orden Homogenea

$$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} = 0$$

$$C_0 \lambda^2 + C_1 \lambda + C_2 = 0 \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

Sol de $a_n$:

- $\lambda_1 \neq \lambda_2 \Rightarrow \underline{a_n = \alpha \lambda_1^n + \beta \lambda_2^n}$ $\quad \{\lambda_1^n, \lambda_2^n\}$
- $\lambda_1 = \lambda_2 = \lambda \Rightarrow a_n = \alpha \lambda^n + \beta n \lambda^n$ $\quad \{\lambda^n, n\lambda^n\}$

**Ejercicio 1**

Expresar explícitamente en $n$ las sucesiones:

(a) $a_n = 5a_{n-1} - 6a_{n-2}, \quad \forall n \geqslant 2$, con $a_0 = 1, a_1 = 3$.

(b) $3a_{n+1} = 2a_n + a_{n-1}, \quad \forall n \geqslant 1$, con $a_0 = 7, a_1 = 3$.

(c) $a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad \forall n \geqslant 2$, con $a_0 = 5, a_1 = 12$.

(d) $a_n = 2a_{n-1} + n2^n, \quad \forall n \geqslant 1$, con $a_0 = 1$.

a) $a_n - 5a_{n+1} + 6a_{n+2} = 0 \qquad a_0 = 1, a_1 = 3$

$$\lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3) = 0 \begin{cases} \lambda_1 = 2 \\ \lambda_2 = 3 \end{cases}$$

$$\rightarrow \begin{cases} a_n = \alpha \cdot 2^n + \beta \cdot 3^n \\ a_0 = 1 \\ a_1 = 3 \end{cases}$$

$$\begin{cases} a_0 = \alpha + \beta = 1 \rightarrow \alpha = 1 - \beta \rightarrow \boxed{\alpha = 0} \\ a_1 = \alpha \cdot 2 + \beta \cdot 3 = 3 \rightarrow 2 - 2\beta + 3\beta = 3 \rightarrow \boxed{\beta = 1} \end{cases}$$

$$\boxed{\text{Sol}: a_n = 3^n}$$

**Ejercicio 1**

Expresar explícitamente en $n$ las sucesiones:

(a) $a_n = 5a_{n-1} - 6a_{n-2}, \quad \forall n \geqslant 2$, con $a_0 = 1, a_1 = 3$.

(b) $3a_{n+1} = 2a_n + a_{n-1}, \quad \forall n \geqslant 1$, con $a_0 = 7, a_1 = 3$.

(c) $a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad \forall n \geqslant 2$, con $a_0 = 5, a_1 = 12$.

(d) $a_n = 2a_{n-1} + n2^n, \quad \forall n \geqslant 1$, con $a_0 = 1$.

c) $\begin{cases} a_n - 6a_{n-1} + 9a_{n-2} = 0 \\ a_0 = 5 \\ a_1 = 12 \end{cases}$

$\lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 = 0 \rightarrow \boxed{\lambda = 3}$ Doble

$\begin{cases} a_n = \alpha \cdot 3^n + \beta \cdot n \cdot 3^n \\ a_0 = 5 \\ a_1 = 12 \end{cases}$

$a_0 = \boxed{\alpha = 5}$

$a_1 = \alpha \cdot 3 + \beta \cdot 3 = 12 \rightarrow \boxed{\beta = \frac{-3}{3} = -1}$

Sol: $a_n = 5 \cdot 3^n - n \cdot 3^n$