Quiero probar: $\forall n \in \mathbb{N}, n \geq n_0 \quad P(n)$

- $\sum_{i=0}^{n} i^2 = n(n+1)(n+2)$
- $2^n > n^2+1$
- $7^n - 2^n = \dot{5}$
- $n = 2i+3$;
- $f^{(n)}(x) = e^x(x+n)$, $f(x) = xe^x$

Paso base:

$P(n_0)$ /

Paso Inductivo:

$P(n) \Rightarrow P(n+1)$ /

Por PIC se cumple

$P(n) \ \forall n > n_0$

Queremos probar $\forall n \geq 0 \quad \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = P(n)$

Paso base: $n=0$

$$\sum_{i=0}^{0} i^2 = 0 = \frac{0 \cdot 1 \cdot 1}{6} \quad \checkmark$$

Paso inductivo

HI $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

TI $\sum_{i=0}^{n+1} i^2 = \frac{(n+1)(n+1+1)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$

$$= \frac{2n^3 + 6n^2 + 4n + 3n^2 + 9n + 6}{6}$$

$$= \frac{2n^3 + 9n^2 + 13n + 6}{6}$$

Dem

$$\sum_{i=0}^{n+1} i^2 = \overbrace{0^2 + 1^2 + 2^2 + \dots + n^2}^{\sum_{i=0}^{n} i^2} + (n+1)^2$$

$$= \sum_{i=0}^{n} i^2 + (n+1)^2 \overset{HI}{=} \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{2n^3 + n^2 + 2n^2 + n}{6} + \frac{6 \cdot n^2 + 12n + 6}{6}$$

$$= \frac{2n^3 + 9n^2 + 13n + 6}{6} \qquad \boxtimes$$

<u>Por PIC se cumple $P(n)$ $\forall n \geq 0$</u>

$$\sum_{i=0}^{n} i = \frac{(n+1)n}{2} = \frac{n^2+n}{2}$$

$$= \overbrace{n+1}^{n+1} + \overbrace{(n-1)+2}^{n+1} + \overbrace{(n-2)+2}^{n+1} + \dots + \overbrace{\quad}^{n+1}$$