Examen Mecánica Newtoniana (1122)

30/7/2021

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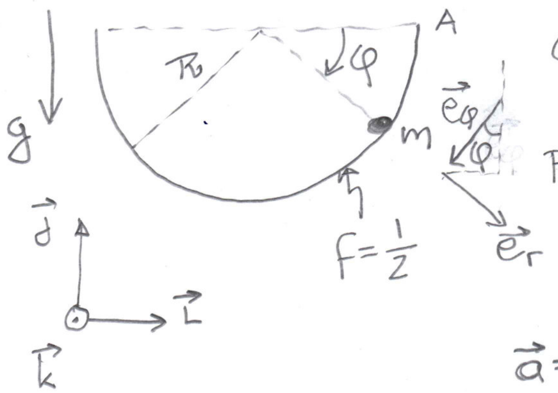
Ejercicio Nº 1



$\varphi(0)=0$

$\dot\varphi(0)=\dfrac{v_0}{R}$

parte: $m\vec a = -mg\vec J - T\vec e_\varphi - N\vec e_r$

$T>0$ (Se opone al mov.)

$N>0$ (Vínculo Unilateral)

$\vec a = R\ddot\varphi\,\vec e_\varphi - R\dot\varphi^2\vec e_r$

$\Rightarrow mR\ddot\varphi = -mg\underbrace{\vec J\cdot\vec e_\varphi}_{-\cos\varphi} - T = mg\cos\varphi - T$

$-mR\dot\varphi^2 = -mg\underbrace{\vec J\cdot\vec e_r}_{-\operatorname{sen}\varphi} - N \Rightarrow N = mR\dot\varphi^2 + mg\operatorname{sen}\varphi > 0$ en $0\le\varphi\le\pi$

$T = fN = \dfrac{N}{2}$

$\cancel{m}R\ddot\varphi = \cancel{m}g\cos\varphi - \dfrac{\cancel{m}R\dot\varphi^2}{2} - \dfrac{\cancel{m}g}{2}\operatorname{sen}\varphi$

$$\boxed{\ddot\varphi + \frac{\dot\varphi^2}{2} = \frac{g}{R}\cos\varphi - \frac{g}{2R}\operatorname{sen}\varphi}$$

parte b: $u=\dot\varphi^2 \Rightarrow u'\dot\varphi = 2\dot\varphi\ddot\varphi \Rightarrow \ddot\varphi = \dfrac{u'}{2}$

$u'+u = \dfrac{2g}{R}\cos\varphi - \dfrac{g}{R}\operatorname{sen}\varphi \Rightarrow u = u_h + u_p$ con $u_h = \mathbb{C}e^{-\varphi}$

$$\left.\begin{aligned} u_p &= A\cos\varphi + B\operatorname{sen}\varphi\\ u'_p &= -A\operatorname{sen}\varphi + B\cos\varphi\end{aligned}\right\} \Rightarrow \begin{aligned} B+A &= \frac{2g}{R}\\ -A+B &= -\frac{g}{R}\end{aligned}\quad\Bigg\}\quad \begin{aligned} B&=\frac{g}{2R}\\ A&=\frac{3g}{2R}\end{aligned}$$

$u(\varphi) = \mathbb{C}e^{-\varphi} + \dfrac{3g}{2R}\cos\varphi + \dfrac{g}{2R}\operatorname{sen}\varphi$

$u(0) = \dfrac{v_0^2}{R^2} = \mathbb{C} + \dfrac{3g}{2R} \Rightarrow \mathbb{C} = \dfrac{v_0^2}{R^2} - \dfrac{3g}{2R}$

$$\boxed{u(\varphi) = \left(\frac{v_0^2}{R^2} - \frac{3g}{2R}\right)e^{-\varphi} + \frac{3g}{2R}\cos\varphi + \frac{g}{2R}\operatorname{sen}\varphi}$$

parte c: $u\left(\frac{3\pi}{4}\right) = 0 = \left(\dfrac{v_0^2}{R^2} - \dfrac{3g}{2R}\right)e^{-\frac{3\pi}{4}} + \dfrac{3g}{2R}\underbrace{\cos\frac{3\pi}{4}}_{-\frac{\sqrt2}{2}} + \dfrac{g}{2R}\underbrace{\operatorname{sen}\frac{3\pi}{4}}_{\frac{\sqrt2}{2}}$

$\left(\dfrac{v_0^2}{R^2} - \dfrac{3g}{2R}\right)e^{-\frac{3\pi}{4}} = \dfrac{g}{R}\dfrac{\sqrt2}{2} \Rightarrow v_0^2 = \dfrac{gR}{2}\left(\sqrt2\,e^{+\frac{3\pi}{4}} + 3\right)$

$$\boxed{v_0 = \sqrt{\frac{gR}{2}\left(\sqrt2\,e^{+\frac{3\pi}{4}} + 3\right)}}$$

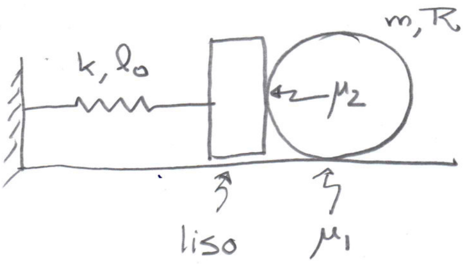
parte d:

$$W = \Delta(T+U) = \underbrace{T(B)}_{=0} + \underbrace{U(B)}_{= -mgR\frac{\sqrt{2}}{2}} - \underbrace{T(A)}_{=\frac{mv_0^2}{2}} - \underbrace{U(A)}_{=0} < 0$$

$$\boxed{W = -\frac{mv_0^2}{2} - mgR\frac{\sqrt{2}}{2} = -\frac{mgR}{4}\left[\sqrt{2}\left(e^{\frac{3\pi}{4}}+2\right)+3\right]} < 0$$

Ejercicio Nº 2: Examen Mecánica Newtoniana (1122)

30/7/2021



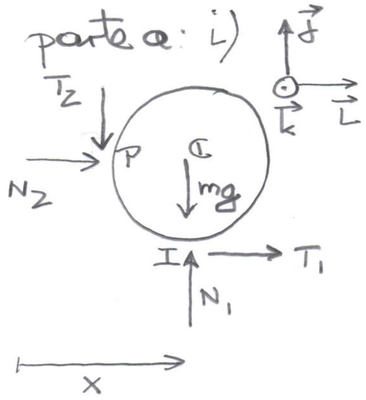
$\mu_2 = \frac{4}{5}$

$l(0) = l_0 - \Delta l$

$\dot l(0) = 0$

$I = \frac{2}{5} m R^2$

parte a: i)



$x = l + a + R$

↖ ancho del bloque

$\vec v_P = \underbrace{\vec v_C}_{\dot x \vec i} + \underbrace{\vec\omega}_{\omega \vec k} \wedge \underbrace{(P - C)}_{-R\vec i} = \dot x \vec i - R\omega \vec j$

$\vec v_{bloque} = \dot x \vec i \Rightarrow \vec v'_P = -R\omega \vec j = \dot x \vec j$

Por la rodadura: $\dot x + R\omega = 0$

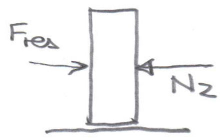
La fuerza del resorte impulsa la esfera hacia la derecha $\Rightarrow \dot x > 0 \Rightarrow \vec v'_P$ según $\vec j \Rightarrow \vec T_2 = -T_2 \vec j$ con $T_2 = \mu_2 |N_2| > 0$

ii) 1ª cardinal a la esfera: $m\ddot x = T_1 + N_2$

$N_1 - T_2 - mg = 0$

2ª " " " en $C = G$: $\frac{2}{5} m R^2 \dot\omega = R(T_1 + T_2)$

1ª " al bloque: $F_{res} - N_2 = 0 \Rightarrow N_2 = F_{res} = -k(l - l_0)$



Resorte comprimido $\Rightarrow l < l_0$ y $N_2 > 0$

$T_2 = -\frac{4}{5} k (l - l_0)$ $R\omega = -\dot x = -\dot l \Rightarrow R\dot\omega = -\ddot l$

$-\frac{2}{5} m \cancel{R} \ddot l = \cancel{R} T_1 - \frac{4}{5} k \cancel{R} (l - l_0) \Rightarrow T_1 = \frac{4}{5} k (l - l_0) - \frac{2}{5} m \ddot l$

$m\ddot l = \frac{4}{5} k (l - l_0) - \frac{2}{5} m \ddot l - k(l - l_0) \Rightarrow \frac{7m}{5} \ddot l = -\frac{k}{5}(l - l_0)$

$$\boxed{\ddot l = -\frac{k}{7m}(l - l_0)}$$

parte b: $N_2 \geq 0 \Rightarrow l \leq l_0$ $\ddot l + \frac{k}{7m} l = \frac{k}{7m} l_0$

$l = l_h + \underset{l_0}{\underset{\shortparallel}{l_p}}$ $l_h = A\cos\omega_0 t + B \operatorname{sen}\omega_0 t$ $\omega_0 = \sqrt{\frac{k}{7m}}$

$l = l_0 + A\cos\omega_0 t + B\operatorname{sen}\omega_0 t$

$l(0) = l_0 + A = l_0 - \Delta l_0 \Rightarrow A = -\Delta l$

$\dot l = -\omega_0 A \operatorname{sen}\omega_0 t + \omega_0 B \cos\omega_0 t \Rightarrow \dot l(0) = B\omega_0 = 0 \Rightarrow B = 0$

$l(t) = l_0 - \Delta l \cos\omega_0 t^* = l_0 \Rightarrow \cos\omega_0 t^* = 0 \Rightarrow \omega_0 t^* = \frac{\pi}{2}$

$$\boxed{t^* = \frac{\pi}{2}\sqrt{\frac{7m}{k}}}$$

parte c: $\Delta \ell = \frac{5mg}{4k}$

$|T_1| \leq \mu_1 |N_1|$

$$\frac{28+2}{7} = \frac{30}{7}$$

$$\left.\begin{aligned} &T_1(0) = \frac{4}{5}k\left[\ell(0)-\ell_0\right] - \frac{2}{5}m\ddot{\ell}(0) \\ &\ell(0) = \ell_0 - \Delta\ell \Rightarrow \ell(0)-\ell_0 = -\Delta\ell \\ &\ddot{\ell}(0) = -\frac{k}{7m}\left[\ell(0)-\ell_0\right] = +\frac{k\Delta\ell}{7m} \end{aligned}\right\} T_1(0) = -\frac{k\Delta\ell}{5}\left(4+\frac{2}{7}\right) = -\frac{6}{7}k\Delta\ell$$

$$N_1(0) = T_2(0) + mg = -\frac{4}{5}k\left[\ell(0)-\ell_0\right] + mg = \frac{4}{5}k\Delta\ell + mg$$

$$\frac{6}{7}k\Delta\ell \leq \mu_1\left(\frac{4}{5}k\Delta\ell + mg\right) = 2\mu_1 mg \Rightarrow \mu_1 \geq \frac{3}{7}\frac{5}{4} \Rightarrow \boxed{\mu_1 \geq \frac{15}{28}}$$

parte d:

$$P_{N_2} = N_2\vec{i}\cdot\vec{v}_P = -k(\ell-\ell_0)\dot{\ell}$$

$$W_{N_2} = \int_0^t -k(\ell-\ell_0)\dot{\ell}\,dt = -k\int_{\ell_0-\Delta\ell}^{\ell_0}(\ell-\ell_0)\,d\ell = -k\frac{(\ell-\ell_0)^2}{2}\Bigg|_{\ell_0-\Delta\ell}^{\ell_0} = \frac{k(\Delta\ell)^2}{2}$$

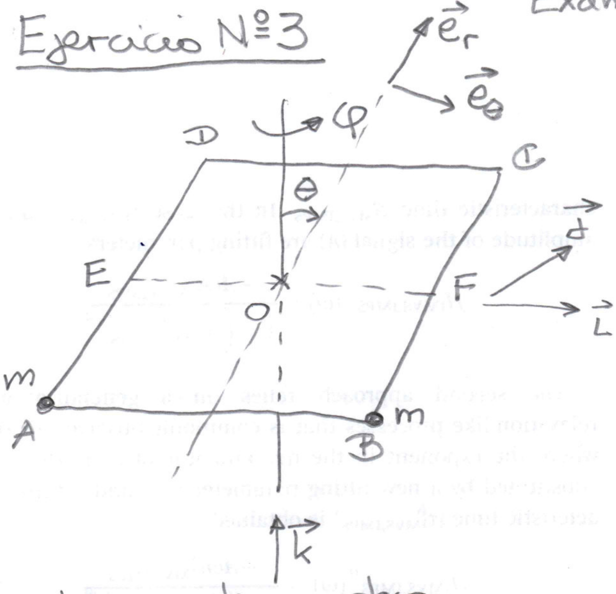
$$\boxed{W_{N_2} = \frac{k(\Delta\ell)^2}{2} > 0}$$

$$P_{T_2} = -T_2\vec{j}\cdot\vec{v}_P = \frac{4}{5}k(\ell-\ell_0)\overbrace{(-R\omega)}^{\dot{\ell}} = \frac{4}{5}k(\ell-\ell_0)\dot{\ell}$$

$$W_{T_2} = \int_0^t \frac{4}{5}k(\ell-\ell_0)\dot{\ell}\,dt = \frac{4k}{5}\int_{\ell_0-\Delta\ell}^{\ell_0}(\ell-\ell_0)\,d\ell = \frac{4k}{5}\frac{(\ell-\ell_0)^2}{2}\Bigg|_{\ell_0-\Delta\ell}^{\ell_0}$$

$$\boxed{W_{T_2} = -\frac{2k}{5}(\Delta\ell)^2 < 0}$$

Ejercicio Nº3



$$M = 6m,\ a$$

$$\theta(0) = \frac{\pi}{2},\quad \vec{\omega} = \omega_0 \vec{k}$$

$$I^{placa}_{O,\vec{e}_\theta} = \frac{Ma^2}{6} = ma^2$$

parte a: $\vec{L}_O = \mathbb{I}_O \vec{\omega}$

$$\vec{\omega} = \dot{\varphi}\vec{k} - \dot{\theta}\vec{l}$$

$$\mathbb{I}_O = \mathbb{I}_O^{(placa)} + \mathbb{I}_O^{(masas)}$$

$\vec{e}_\theta, \vec{e}_r, \vec{l}$: base ortonormal directa de ejes principales

$$I^{placa}_{O,\vec{e}_r} + I^{placa}_{O,\vec{l}} = I^{placa}_{O,\vec{e}_\theta} = ma^2$$

$$I^{placa}_{O,\vec{e}_r} = I^{placa}_{O,\vec{l}} = \frac{ma^2}{2}$$

$$\mathbb{I}_O(placa, \vec{e}_\theta, \vec{e}_r, \vec{l}) = \begin{pmatrix} ma^2 & 0 & 0 \\ 0 & \frac{ma^2}{2} & 0 \\ 0 & 0 & \frac{ma^2}{2} \end{pmatrix}$$

$$\mathbb{I}_O(masas, \vec{e}_\theta, \vec{e}_r, \vec{l}) = \begin{pmatrix} \sum_{i=1}^{2} m_i(y_i^2+z_i^2) & -\sum_{i=1}^{2} m_i x_i y_i & -\sum_{i=1}^{2} m_i x_i z_i \\ -\sum_{i=1}^{2} m_i x_i y_i & \sum_{i=1}^{2} m_i(x_i^2+z_i^2) & -\sum_{i=1}^{2} m_i y_i z_i \\ -\sum_{i=1}^{2} m_i x_i z_i & -\sum_{i=1}^{2} m_i y_i z_i & \sum_{i=1}^{2} m_i(x_i^2+y_i^2) \end{pmatrix} =$$

$$= \begin{pmatrix} m\left(\frac{a^2}{4}+\frac{a^2}{4}\right).2 & 0 & 0 \\ 0 & \frac{ma^2}{4}.2 & -\frac{ma^2}{4}+\frac{ma^2}{4} \\ 0 & -\frac{ma^2}{4}+\frac{ma^2}{4} & \frac{ma^2}{4}.2 \end{pmatrix} = \begin{pmatrix} ma^2 & 0 & 0 \\ 0 & \frac{ma^2}{2} & 0 \\ 0 & 0 & \frac{ma^2}{2} \end{pmatrix}$$

$$\mathbb{I}_O = \begin{pmatrix} 2ma^2 & 0 & 0 \\ 0 & ma^2 & 0 \\ 0 & 0 & ma^2 \end{pmatrix}$$

$$\vec{k} = \cos\theta\, \vec{e}_r - \operatorname{sen}\theta\, \vec{e}_\theta$$

$$\vec{\omega} = -\dot{\varphi}\operatorname{sen}\theta\, \vec{e}_\theta + \dot{\varphi}\cos\theta\, \vec{e}_r - \dot{\theta}\vec{l}$$

$$\boxed{\vec{L}_O = ma^2\left(-2\dot{\varphi}\operatorname{sen}\theta\, \vec{e}_\theta + \dot{\varphi}\cos\theta\, \vec{e}_r - \dot{\theta}\vec{l}\right)}$$

$$T = \frac{1}{2}\vec{\omega}.\mathbb{I}_O\vec{\omega} = \frac{1}{2}\vec{\omega}.\vec{L}_O = \frac{ma^2}{2}\left(2\dot{\varphi}^2\operatorname{sen}^2\theta + \dot{\varphi}^2\cos^2\theta + \dot{\theta}^2\right)$$

$$\boxed{T = \frac{ma^2}{2}\left[\dot{\varphi}^2\left(1+\operatorname{sen}^2\theta\right) + \dot{\theta}^2\right]}$$

parte b: $\dot{\vec{L}}_O = \vec{M}_O^{(ext)} = \vec{M}_O^{(peso)} + \vec{M}_O^{(reacción\ en\ O)}$

$$\dot{\vec{L}}_O.\vec{k} = \underbrace{\vec{M}_O^{(peso)}.\vec{k}}_{\text{"0 porque peso vertical"}} + \underbrace{\vec{M}_O^{(reacción\ en\ O)}.\vec{k}}_{\text{"0 porque sistema gira libremente según } \vec{k}}$$

$$(\vec{L}_o\cdot\vec{k})^{\circ} = \underbrace{\dot{\vec{L}}_o\cdot\vec{k}}_{0} + \vec{L}_o\cdot\underbrace{\dot{\vec{k}}}_{0 \text{ porque } \vec{k} \text{ es fijo}}$$

$$\Rightarrow \vec{L}_o\cdot\vec{k} = cte$$

¿Se conserva la energía mecánica del sistema?

1) Peso conservativo: $U_g = \underbrace{U_g^{placa}}_{0} + \underbrace{U_g^{masas}}_{-mg\frac{a}{2}\cos\theta\times 2} = -mga\cos\theta$

2) Fuerzas internas del rígido placa + masas de potencia nula

3) Reacciones en O: $P^{O} = \underbrace{\vec{R}^{O}\cdot\vec{v}_o}_{0 \text{ por ser O fijo}} + \vec{\omega}\cdot\vec{M}_o^{O}$

Momento externo sobre articulación de eje vertical:

$$P = \vec{\omega}^{art}\cdot\vec{M}_o = \dot{\varphi}\vec{k}\cdot\vec{M}_o = 0 \text{ porque articulación es lisa}$$

Momento entre articulación horizontal (EF) y placa cuadrada:

$$P = \underbrace{\vec{\omega}^{art}\cdot\vec{M}_o^{art} + \vec{\omega}\cdot(-\vec{M}_o^{art})}_{\text{momentos iguales y opuesto x acción y reacción}} = \vec{M}_o^{art}(\dot{\varphi}\vec{k} - \dot{\varphi}\vec{k} + \dot{\theta}\vec{i}) = \dot{\theta}\,\underbrace{\vec{M}_o^{(art)}\cdot\vec{i}}_{0 \text{ porque articulación eje EF es lisa}}$$

La reacción entre las articulaciones es de potencia nula porque ellas se comportan como un sistema rígido.

$$\Rightarrow \boxed{T + U = E = cte}$$

parte c: $\frac{ma^2}{2}\left[\dot{\varphi}^2(1+\operatorname{sen}^2\theta) + \dot{\theta}^2\right] - mga\cos\theta = E$

$\dot{\varphi}(0) = \omega_o$, $\dot{\theta}(0) = 0$, $\operatorname{sen}\theta(0) = 1$, $\cos\theta(0) = 0 \Rightarrow E = \frac{ma^2\omega_o^2 2}{2} = ma^2\omega_o^2$

$\vec{L}_o\cdot\vec{k} = ma^2(2\dot{\varphi}\operatorname{sen}^2\theta + \dot{\varphi}\cos^2\theta) = ma^2\dot{\varphi}(1+\operatorname{sen}^2\theta) = ma^2\omega_o^2$

$$\dot{\varphi} = \frac{2\omega_o}{1+\operatorname{sen}^2\theta}$$

$$\frac{\cancel{m}a^{\cancel{2}}}{2}\left[\frac{4\omega_o^2}{1+\operatorname{sen}^2\theta} + \dot{\theta}^2\right] - \cancel{m}g\cancel{a}\cos\theta = \cancel{m}a^{\cancel{2}}\omega_o^2$$

$\dot{\theta}(0)$ en $\theta = \frac{\pi}{3} = 60°$ $\operatorname{sen}60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$

$$\frac{a\,2\omega_o^2}{1+\frac{3}{4}} - \frac{g}{2} = a\omega_o^2 \Rightarrow \underbrace{\left(\frac{8}{7} - 1\right)}_{\frac{1}{7}} a\omega_o^2 = \frac{g}{2} \Rightarrow \boxed{\omega_o = \pm\sqrt{\frac{7g}{2a}}}$$