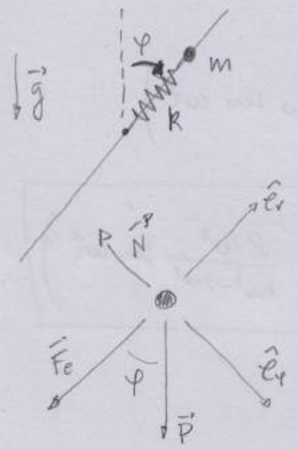
Mecánica Newtoniana

Examen 15 de febrero 2018

Problema 1 : ⓐ



$\vec{v} = \dot{r}\hat{e}_r + r\dot{\varphi}\hat{e}_\varphi$ $\qquad$ $\vec{F}_e = -kr\hat{e}_r$

$\vec{a} = (\ddot{r} - r\dot{\varphi}^2)\hat{e}_r + 2\dot{r}\dot{\varphi}\hat{e}_\varphi$ $\qquad$ $\vec{P} = -mg\,\hat{j}$

$$\begin{cases} (\hat{e}_r) & m(\ddot{r} - r\dot{\varphi}^2) = -kr - mg\cos\varphi \\ (\hat{e}_\varphi) & m\,2\dot{r}\dot{\varphi} = mg\,\text{sen}\,\varphi - N \end{cases}$$

$$\ddot{r} + \left(\frac{k}{m} - \omega^2\right) r = -g\cos(\omega t)$$

$$\boxed{\ddot{r} + (\omega_0^2 - \omega^2) r = -g\cos(\omega t)}$$

Solución homogénea

$$r_H = A e^{i\sqrt{\omega_0^2-\omega^2}\,t} + B e^{-i\sqrt{\omega_0^2-\omega^2}\,t}$$

Solución particular

$r_p = C\cos(\omega t)$

$\ddot{r}_p = -\omega^2 C\cos(\omega t) \Rightarrow (-\omega^2 + (\omega_0^2 - \omega^2))C = -g$

$C = \dfrac{-g}{\omega_0^2 - 2\omega^2}$

$$r(t) = \frac{-g}{\omega_0^2 - 2\omega^2}\cos\omega t + A e^{i\sqrt{\ }t} + B e^{-i\sqrt{\ }t}$$

$$\left(r(0) = 0 \;;\; \dot{r}(0) = 0\right) \Rightarrow \left(\frac{-g}{\omega_0^2 - 2\omega^2} + A + B = 0 \;;\; A = B\right) \Rightarrow$$

$$2A = \frac{g}{\omega_0^2 - 2\omega^2}$$

$$r(t) = \frac{g}{\omega_0^2 - 2\omega^2}\left(-\cos\omega t + \frac{1}{2}\left(e^{i\sqrt{\ }t} + e^{-i\sqrt{\ }t}\right)\right)$$

ⓑ Si $\omega < \omega_0 \Rightarrow$ $\quad r_H = \dfrac{g}{\omega_0^2 - 2\omega^2}\cos\left(\sqrt{\omega_0^2 - \omega^2}\,t\right)$ acotada,

en caso contrario $r_H(t \to \infty) \to \infty$

Además existe una resonancia para $\omega = \frac{\omega_0}{\sqrt{2}}$.

Condiciones para movimiento acotado: $\boxed{\omega < \omega_0 \text{ y } \omega \neq \frac{\omega_0}{\sqrt{2}}}$

$$\boxed{r(t) = \frac{g}{\omega_0^2 - 2\omega^2}\left(\cos\sqrt{\omega_0^2 - \omega^2}\, t - \cos \omega t\right)}$$

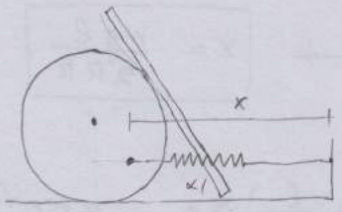
c)

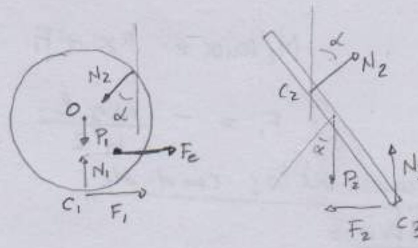
$$N = mg \operatorname{sen} \omega t - 2m\dot{r}\omega$$

$$\dot{r} = \frac{g}{\omega_0^2 - 2\omega^2}\left(-\sqrt{\omega_0^2 - \omega^2}\operatorname{sen}\sqrt{\omega_0^2 - \omega^2}\, t + \omega \operatorname{sen} \omega t\right)$$

$$\boxed{N = mg\left(\operatorname{sen}\omega t + \frac{2\omega\sqrt{\omega_0^2 - \omega^2}}{\omega_0^2 - 2\omega^2}\operatorname{sen}\sqrt{\omega_0^2 - \omega^2}\, t - \frac{2\omega^2}{\omega_0^2 - 2\omega^2}\operatorname{sen}\omega t\right)}$$

Problema 2





Equilibrio Disco

$$\sum \vec{F} \begin{cases} 0 = N_1 - mg - N_2 \cos\alpha \\ 0 = N_2 \operatorname{sen}\alpha - kx - F_1 \end{cases}$$

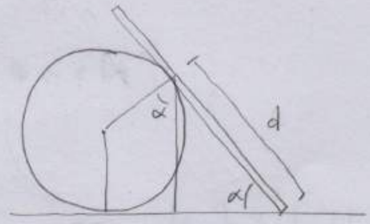
$$\vec{M}_0 \quad \{ \; 0 = kx \frac{R}{2} + F_1 R$$

Equilibrio Barra

$$\sum \vec{F} \begin{cases} 0 = N_3 - P_2 + N_2 \cos\alpha \\ 0 = N_2 \operatorname{sen}\alpha - F_2 \end{cases}$$

$$\vec{M}_{C_3} \quad \{ \; 0 = -N_2 \sqrt{3} R + l \cos\alpha \, mg$$

Distancia C_2, C_3



$$d \operatorname{sen}\alpha = R + R\cos\alpha$$

$$d = \frac{R}{\operatorname{sen}\alpha} + R \frac{\cos\alpha}{\operatorname{sen}\alpha}$$

$\operatorname{sen}\alpha = \sqrt{3}/2$

$\cos\alpha = 1/2$

$$d = \frac{2R}{\sqrt{3}} + \frac{R}{\sqrt{3}} = \sqrt{3} R$$

Condiciones de equilibrio:

$|F_1| \leq f_E N_1$ $\qquad$ $|F_2| \leq f_E N_3$

$N_2 \geq 0$ $\qquad$ $N_3 \geq 0$

$N_1 \geq 0$

Operando:

$$\left. \begin{aligned} N_2 &= \frac{mg\, l}{2\sqrt{3} R} \\ N_1 &= mg + \frac{N_2}{2} \end{aligned} \right\} \Rightarrow \quad N_1 = mg + \frac{mg\, l}{4\sqrt{3} R} = mg\left(1 + \frac{l}{4\sqrt{3}R}\right)$$

$$F_2 = N_2 \operatorname{sen}\alpha \quad \Rightarrow \quad F_2 = \frac{mg\, l}{2\sqrt{3} R} \cdot \frac{\sqrt{3}}{2} = \frac{mg\, l}{4R}$$

$$N_3 = mg - \frac{N_2}{2} \quad \Rightarrow \quad N_3 = mg - \frac{mg\, l}{4\sqrt{3} R} = mg\left(1 - \frac{l}{4\sqrt{3}R}\right)$$

$$\left.\begin{array}{l} F_1 = -\dfrac{kx}{2} \\ N_2 \operatorname{sen}\alpha = kx + F_1 \end{array}\right\} \Rightarrow \frac{mg\,\ell}{2\sqrt{3}R}\cdot\frac{\sqrt{3}}{2} = kx - \frac{kx}{2} = \frac{kx}{2}$$

$$F_1 = -\frac{mg\,\ell}{4R} \longleftarrow \boxed{x = \frac{mg\,\ell}{2Rk}}$$

Verificación de las cond. de equilibrio:

$$N_1 \geq 0 \Rightarrow mg\left(1+\frac{\ell}{4\sqrt{3}R}\right) \geq 0$$ se verifica siempre

$$N_2 \geq 0 \Rightarrow \frac{mg\,\ell}{2\sqrt{3}R} \geq 0$$ se verifica siempre.

$$N_3 \geq 0 \Rightarrow mg\left(1-\frac{\ell}{4\sqrt{3}R}\right) \geq 0 \Rightarrow \boxed{4\sqrt{3}R \geq \ell}$$

$$|F_1| \leq f_E N_1 \Rightarrow \frac{\cancel{mg}\,\ell}{4R} \leq f_E\, \cancel{mg}\left(1+\frac{\ell}{4\sqrt{3}R}\right)$$

Obs: si $f_E \geq \sqrt{3}$ se verifica siempre.

$$\boxed{f_E \geq \frac{\sqrt{3}\,\ell}{4\sqrt{3}R+\ell}} \quad \text{ó} \quad \boxed{4\sqrt{3}R\left(\frac{f_E}{\sqrt{3}-f_E}\right) \geq \ell}$$

$$|F_2| \leq f_E N_3 \Rightarrow \frac{\cancel{mg}\,\ell}{4R} \leq f_E\, \cancel{mg}\left(1-\frac{\ell}{4\sqrt{3}R}\right)$$

Esta condición es más restrictiva que esta

$$\boxed{f_E \geq \frac{\sqrt{3}\,\ell}{4\sqrt{3}R-\ell}} \quad \text{ó} \quad \boxed{4\sqrt{3}R\left(\frac{f_E}{\sqrt{3}+f_E}\right) \geq \ell}$$

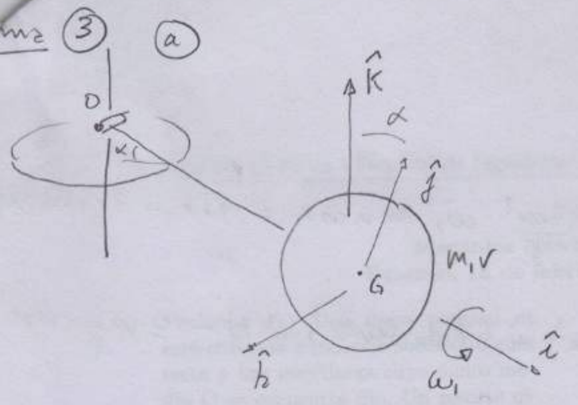
A su vez como $\dfrac{f_E}{\sqrt{3}+f_E} < 1$, la condición más restrictiva es

$$\boxed{4\sqrt{3}R\left(\frac{f_E}{\sqrt{3}+f_E}\right) \geq \ell}$$

El equilibrio se rompe porque la barra desliza.

$$\boxed{\text{Máxima longitud barra} = 8\sqrt{3}R\left(\frac{f_E}{\sqrt{3}+f_E}\right)}$$

ma (3) (a)



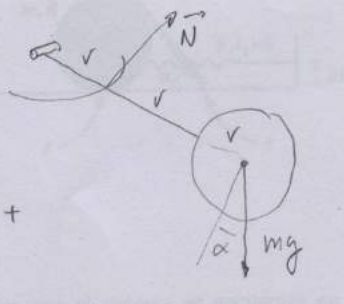
$$\mathbb{I}_0 = \frac{2}{5} m r^2 \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + m(9r^2) \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix} = \frac{mr^2}{5} \begin{pmatrix} 2 & & \\ & 47 & \\ & & 47 \end{pmatrix}$$

$$\vec{\omega} = \omega_1 \hat{\imath} + \omega_2 \hat{K} = \omega_1 \hat{\imath} + \omega_2 \left(\cos\alpha\, \hat{j} - \operatorname{sen}\alpha\, \hat{\imath} \right) =$$

$$= \left(\omega_1 - \omega_2 \operatorname{sen}\alpha \right) \hat{\imath} + \omega_2 \cos\alpha\, \hat{j}$$

$$\vec{L}_0 = \mathbb{I}_0 \vec{\omega} = \frac{mr^2}{5} \left(2 \left(\omega_1 - \omega_2 \operatorname{sen}\alpha \right) \hat{\imath} + 47 \omega_2 \cos\alpha\, \hat{j} \right)$$

(b)



$$\frac{d\vec{L}_0}{dt} = \frac{mr^2}{5} \Big(-2 \dot{\omega}_2 \operatorname{sen}\alpha\, \hat{\imath} +$$

$$+ 47 \dot{\omega}_2 \cos\alpha\, \hat{j} + 2 \left(\omega_1 - \omega_2 \operatorname{sen}\alpha \right) \dot{\hat{\imath}} +$$

$$+ 47 \omega_2 \cos\alpha\, \dot{\hat{j}} \Big) =$$

$$\dot{\hat{\imath}} = \vec{\omega}' \times \hat{\imath} = -\omega_2 \cos\alpha\, \hat{k} \qquad \vec{\omega}' = \omega_2 \hat{K}$$

$$\dot{\hat{j}} = \left(-\omega_2 \operatorname{sen}\alpha \right) \hat{k}$$

$$= \frac{mr^2}{5} \Big(-2 \dot{\omega}_2 \operatorname{sen}\alpha\, \hat{\imath} + 47 \dot{\omega}_2 \cos\alpha\, \hat{j} - 2 \left(\omega_1 - \omega_2 \operatorname{sen}\alpha \right) \omega_2 \cos\alpha\, \hat{k}$$

$$- 47 \omega_2 \cos\alpha\, \omega_2 \operatorname{sen}\alpha\, \hat{k} \Big)$$

$$\frac{d\vec{L}_0}{dt} \cdot \hat{k} = \vec{M}_0^{ext} \cdot \hat{k}$$

$$-\frac{2mr^2}{5} w_2 \cos\alpha \left(w_1 - w_2 \operatorname{sen}\alpha\right) - \frac{47mr^2}{5} w_2^2 \operatorname{sen}\alpha \cos\alpha = Nr - 3r\cos\alpha\, mg$$

$$-\frac{mr}{5} w_2 \cos\alpha \left(2\left(w_1 - w_2 \operatorname{sen}\alpha\right) + 47 w_2 \operatorname{sen}\alpha\right) = N - 3mg\cos\alpha$$

$$N = 3mg\cos\alpha - mr w_2 \cos\alpha \left(\tfrac{2}{5} w_1 + 9 w_2 \operatorname{sen}\alpha\right)$$

La varilla permanece apoyada siempre que $N \geq 0$

$$N \geq 0 \Rightarrow 3g - r w_2 \left(2/5\, w_1 + 9 w_2 \operatorname{sen}\alpha\right) \geq 0$$

$$\boxed{3g > r w_2 \left(2/5\, w_1 + 9 w_2 \operatorname{sen}\alpha\right)}$$