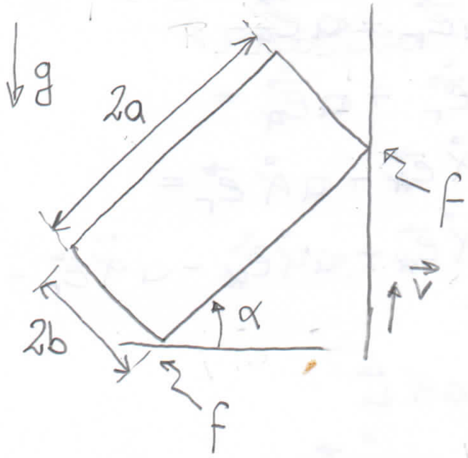
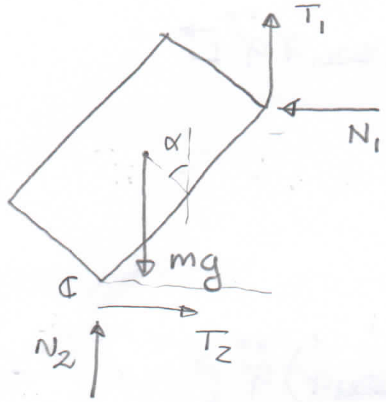
Ejercicio Nº 3





parte a:

$$T_1 = f N_1$$

$$N_1 = T_2$$

$$T_1 + N_2 = mg$$

$$-mg(a\cos\alpha - b\,\mathrm{sen}\,\alpha) + N_1 2a\,\mathrm{sen}\,\alpha + T_1 2a\cos\alpha = 0$$

$$N_1(2a\,\mathrm{sen}\,\alpha + f2a\cos\alpha) = mg(a\cos\alpha - b\,\mathrm{sen}\,\alpha)$$

$$N_1 = \frac{mg(a\cos\alpha - b\,\mathrm{sen}\,\alpha)}{2a(\mathrm{sen}\,\alpha + f\cos\alpha)}$$

$$N_1 \geq 0 \qquad a\cos\alpha \geq b\,\mathrm{sen}\,\alpha$$

$$\boxed{\mathrm{tg}\,\alpha \leq \frac{a}{b}}$$

$$N_2 = mg - f\frac{mg(a\cos\alpha - b\,\mathrm{sen}\,\alpha)}{2a(\mathrm{sen}\,\alpha + f\cos\alpha)} > 0$$

$$\cancel{mg}\,2a(\mathrm{sen}\,\alpha + f\cos\alpha) > f\cancel{mg}(a\cos\alpha - b\,\mathrm{sen}\,\alpha)$$

$$2a\,\mathrm{sen}\,\alpha + fa\cos\alpha > -fb\,\mathrm{sen}\,\alpha \checkmark$$

$$|T_2| \leq f N_2$$

$$\frac{\cancel{mg}(a\cos\alpha - b\,\mathrm{sen}\,\alpha)}{2a(\mathrm{sen}\,\alpha + f\cos\alpha)} \leq f\left[\cancel{mg} - f\frac{\cancel{mg}(a\cos\alpha - b\,\mathrm{sen}\,\alpha)}{2a(\mathrm{sen}\,\alpha + f\cos\alpha)}\right]$$

$$a\cos\alpha - b\,\mathrm{sen}\,\alpha \leq f\left[2a(\mathrm{sen}\,\alpha + f\cos\alpha) - f(a\cos\alpha - b\,\mathrm{sen}\,\alpha)\right]$$

$$(a - 2af^2 + af^2)\cos\alpha \leq [b + f(2a + fb)]\,\mathrm{sen}\,\alpha$$

$$\boxed{\mathrm{tg}\,\alpha \geq \frac{a(1-f^2)}{b(1+f^2) + 2fa}}$$

parte b:

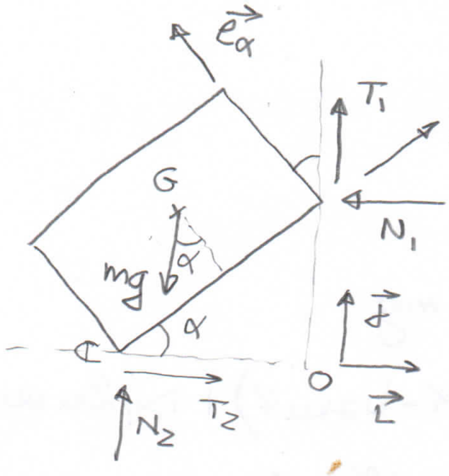
$b = a$

$$\frac{1-f^2}{1+f^2+2f} \leq \mathrm{tg}\,\alpha \leq 1$$

$$\frac{1-f}{1+f} \leq \mathrm{tg}\,\alpha \leq 1$$

Se rompe deslizamiento

$$\mathrm{tg}\,\alpha < \frac{1-f}{1+f} \leq 1$$



$$G = C + a\vec{e}_r + a\vec{e}_\alpha$$

$$\dot{G} = \dot{C} + a\dot{\vec{e}}_r + a\dot{\vec{e}}_\alpha =$$

$$= \dot{C} + a\dot{\alpha}\vec{e}_\alpha - a\dot{\alpha}\vec{e}_r =$$

$$= \ddot{C} + a\ddot{\alpha}\vec{e}_\alpha + a\dot{\alpha}\dot{\vec{e}}_\alpha - a\ddot{\alpha}\vec{e}_r - a\dot{\alpha}\dot{\vec{e}}_r$$

$$\dot{\alpha}(0) = 0$$

$$C = O - 2a\cos\alpha\,\vec{i}$$

$$\dot{C} = \dot{O} + 2a\,\mathrm{sen}\,\alpha\,\dot{\alpha}\,\vec{i}$$

$$\ddot{C} = \ddot{O} + 2a\cos\alpha\,\dot{\alpha}^2 + 2a\,\mathrm{sen}\,\alpha\,\ddot{\alpha}\,\vec{i} = 2a\,\mathrm{sen}\,\alpha\,\ddot{\alpha}\,\vec{i}$$

$$\ddot{G} = 2a\,\mathrm{sen}\,\alpha\,\ddot{\alpha}\,\vec{i} + a\ddot{\alpha}\vec{e}_\alpha - a\ddot{\alpha}\vec{e}_r$$

$$\vec{e}_r = \cos\alpha\,\vec{i} + \mathrm{sen}\,\alpha\,\vec{j}$$

$$\vec{e}_\alpha = -\mathrm{sen}\,\alpha\,\vec{i} + \cos\alpha\,\vec{j}$$

$$\ddot{G} = a(\mathrm{sen}\,\alpha - \cos\alpha)\ddot{\alpha}\,\vec{i} + a(\cos\alpha - \mathrm{sen}\,\alpha)\ddot{\alpha}\,\vec{j}$$

$$ma(\mathrm{sen}\,\alpha - \cos\alpha)\ddot{\alpha} = T_2 - N_1 \qquad T_2 = fN_2$$

$$ma(\cos\alpha - \mathrm{sen}\,\alpha)\ddot{\alpha} = T_1 + N_2 - mg \qquad T_1 = fN_1$$

$$I_G\ddot{\alpha} = N_1(a\,\mathrm{sen}\,\alpha - a\cos\alpha) + T_1(a\,\mathrm{sen}\,\alpha + a\cos\alpha) +$$

$$+ T_2(a\,\mathrm{sen}\,\alpha + a\cos\alpha) - N_2(a\cos\alpha - a\,\mathrm{sen}\,\alpha)$$

$$I_G = \frac{m}{12}(4a^2 + 4b^2) = \frac{8ma^2}{12} = \frac{2ma^2}{3}$$

$$\frac{2ma}{3}\ddot{\alpha} = N_1(\mathrm{sen}\,\alpha - \cos\alpha) + T_1(\mathrm{sen}\,\alpha + \cos\alpha) +$$

$$+ T_2(\mathrm{sen}\,\alpha + \cos\alpha) - N_2(\cos\alpha - \mathrm{sen}\,\alpha) =$$

$$= N_1[(1+f)\mathrm{sen}\,\alpha - (1+f)\cos\alpha] +$$

$$+ N_2[(1+f)\mathrm{sen}\,\alpha - (1-f)\cos\alpha]$$

$$\frac{2ma}{3}\ddot{\alpha} = (N_1 + N_2)[(1+f)\mathrm{sen}\,\alpha - (1-f)\cos\alpha]$$

$$\frac{\mathrm{sen}\,\alpha}{\cos\alpha} < \frac{1-f}{1+f} \Rightarrow (1+f)\mathrm{sen}\,\alpha - (1-f)\cos\alpha < 0 \Rightarrow \ddot{\alpha} < 0$$

xg $N_1 N_2 > 0$

$$ma(\operatorname{sen}\alpha-\cos\alpha)\ddot{\alpha} = fN_2 - N_1$$

$$ma(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} = fN_1 + N_2 - mg$$

$$ma(1-f)(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} = (1+f^2)N_2 - mg$$

$$N_2 = \frac{ma(1-f)(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} + mg}{1+f^2}$$

$$-ma(1+f)(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} = -(1+f^2)N_1 + fmg$$

$$N_1 = \frac{ma(1+f)(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} + fmg}{1+f^2}$$

$$N_1 + N_2 = \frac{2ma(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} + (1+f)mg}{1+f^2}$$

$$\frac{2\cancel{m}a}{3}\ddot{\alpha} = \frac{[2\cancel{m}a(\cos\alpha-\operatorname{sen}\alpha)\ddot{\alpha} + (1+f)\cancel{m}g][(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha]}{1+f^2}$$

$$\left\{\frac{2}{3}(1+f^2) - 2(\cos\alpha-\operatorname{sen}\alpha)[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha]\right\} a\ddot{\alpha} =$$

$$= (1+f)g[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha]$$

$$\boxed{\ddot{\alpha} = (1+f)\frac{g}{a}\,\frac{(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha}{\frac{2}{3}(1+f^2) - 2(\cos\alpha-\operatorname{sen}\alpha)[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha]}}$$

$\ddot{\alpha} < 0$

¿$N_1 > 0$? $\quad (1+f)^2(\cos\alpha-\operatorname{sen}\alpha)[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha]$

$$+ f\frac{2}{3}(1+f^2) - 2f(\cos\alpha-\operatorname{sen}\alpha)[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha] > 0$$

$$\underbrace{[(1+f)^2-2f]}_{1+\cancel{f^2}}(\cos\alpha-\operatorname{sen}\alpha)[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha] + f\frac{2}{3}\cancel{(1+f^2)} > 0$$

$$\frac{2}{3}f > (\cos\alpha-\operatorname{sen}\alpha)[(1-f)\cos\alpha-(1+f)\operatorname{sen}\alpha] =$$

$$= (\cos\alpha-\operatorname{sen}\alpha)(\cos\alpha-\operatorname{sen}\alpha) - f(\cos^2\alpha-\operatorname{sen}^2\alpha)$$

$$f\left[\frac{2}{3}+\underbrace{\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}_{\cos 2\alpha}\right]>\left(\cos\alpha-\operatorname{sen}\alpha\right)^2$$

$$\boxed{f>\frac{\left(\cos\alpha-\operatorname{sen}\alpha\right)^2}{\frac{2}{3}+\cos 2\alpha}=\frac{1-\operatorname{sen}2\alpha}{\frac{2}{3}+\cos 2\alpha}}$$

¿$N_2>0$?

$$(1+f)^2\left(\cos\alpha-\operatorname{sen}\alpha\right)\left[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha\right]$$
$$+\frac{2}{3}\left(1+f^2\right)-2\left(\cos\alpha-\operatorname{sen}\alpha\right)\left[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha\right]>0$$

$$\underbrace{\left[(1+f)^2-2\right]}_{f^2+2f-1}\left(\cos\alpha-\operatorname{sen}\alpha\right)\left[(1+f)\operatorname{sen}\alpha-(1-f)\cos\alpha\right]+\frac{2}{3}\left(1+f^2\right)>0$$

Pero si $f<1$ $\quad N_2>0$ es más estricta que $N_1>0$, así que si se cumple esta se verifica la primera

$\frac{1}{3\sqrt{3}} >$

$= \frac{6-3\sqrt{3}}{7}$

$0.192 > 0.1148$