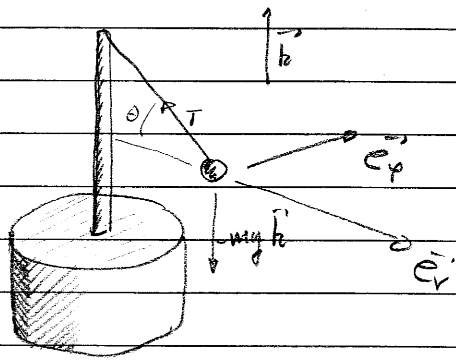
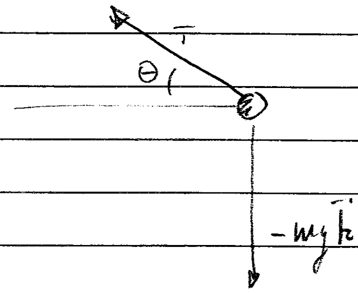
① ⓐ



$\vec{v} = d\dot{\varphi}\,\vec{e}_\varphi \qquad (d\dot{\varphi} = v)$

$\vec{a} = -d\dot{\varphi}^2\,\vec{e}_r$

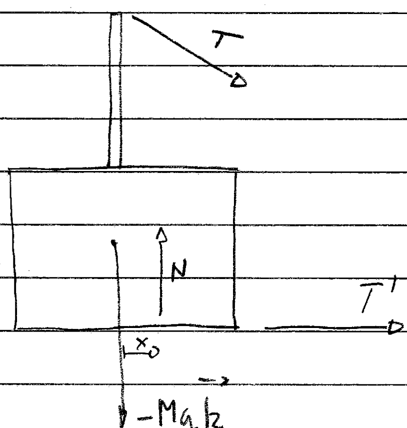
$-md\dot{\varphi}^2\vec{e}_r = -T\cos\theta\,\vec{e}_r + T\,\mathrm{sen}\,\theta\,\vec{k} - mg\,\vec{k}$



$$\begin{cases} \dfrac{mv^2}{d} = T\cos\theta \\ T\,\mathrm{sen}\,\theta = mg \end{cases} \qquad \frac{mv^2}{d} = mg\,\mathrm{ctg}\,\theta$$

$$\boxed{\vec{T} = \underbrace{-\frac{mv^2}{d}\vec{e}_r}_{T_r} + \underbrace{mg\,\vec{k}}_{T_z}} \qquad T$$

ⓑ



$T_r + T' = 0$

$-T_z + N - Mg = 0$

$T' = -\dfrac{mv^2}{d}$

$N = Mg + mg$

No desliz. $\Rightarrow \dfrac{mv^2}{d} \leq (M+m)gf$

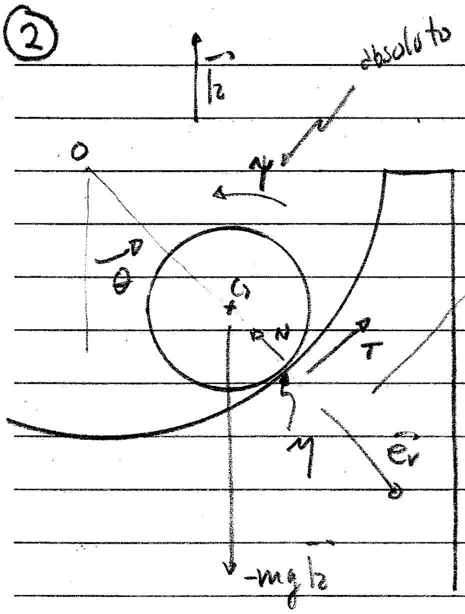
$$\boxed{v^2 \leq \left(\frac{M+m}{m}\right)gdf}$$

$0 = Nx - T_r H$

$(M+m)gx = \dfrac{mv^2}{d}H$

$|x| = \left|\dfrac{mv^2H}{gd(M+m)}\right| \leq a \qquad \boxed{v^2 \leq \left(\frac{M+m}{m}\right)gd\left(\frac{a}{H}\right)}$

(2)



(a)

$\vec{g}$

$$\vec{a_G} = (R-r)\ddot{\theta}\,\vec{e_\theta} - (R-r)\dot{\theta}^2\,\vec{e_r}$$

$$\vec{a_G}(t=0) = (R-r)\ddot{\theta}\,\vec{e_\theta}$$

$$(\vec{e_\theta}) \quad m(R-r)\ddot{\theta} = T - mg\,\text{sen}\,\theta$$

$$(\vec{e_r}) \quad -m(R-r)\dot{\theta}^2 = -N + mg\cos\theta$$

$$I_G = \frac{mr^2}{2} \Rightarrow \frac{mr^2}{2}\ddot{\psi} = Tr$$

$$\text{R.S.D} \Rightarrow \vec{v_C} = \vec{v_G} + r\dot{\psi} = 0 \Rightarrow (R-r)\dot{\theta} + r\dot{\psi} = 0.$$

(C: pto de contacto)

$$\dot{\psi} = \frac{-(R-r)}{r}\dot{\theta}$$

$$\frac{mr^2}{2}\left(-\frac{(R-r)}{r}\ddot{\theta}\right) = Tr \Rightarrow T = -\frac{m}{2}(R-r)\ddot{\theta}$$

$$m(R-r)\ddot{\theta} = -\frac{m}{2}(R-r)\ddot{\theta} - mg\,\text{sen}\,\theta$$

$$\ddot{\theta} = -\frac{2}{3}\frac{g}{R-r}\,\text{sen}\,\theta$$

En $t=0$

$$N = mg\cos\theta_0$$

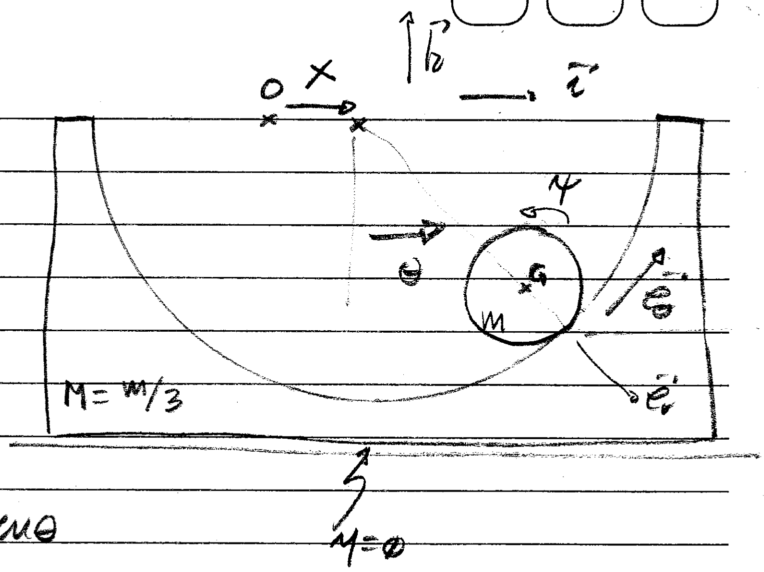
$$T = +\frac{mg}{3}\,\text{sen}\,\theta_0$$

$$|T| \leq \mu N \Rightarrow \frac{\text{sen}\,\theta_0}{3} \leq \mu\cos\theta_0$$

$$\boxed{\theta_{0_{max}} = \text{Arctg}(3\mu)}$$

(b)

$\vec{a}_G = (R-r)\ddot{\theta}\,\vec{e}_\theta - (R-r)\dot{\theta}^2\,\vec{e}_r + \ddot{x}\,\vec{i}$



$(\vec{e}_\theta)\quad m\left((R-r)\ddot{\theta} + \ddot{x}\cos\theta\right) = T - mg\,\mathrm{sen}\,\theta$

$(\vec{e}_r)\quad -m\left((R-r)\dot{\theta}^2 - \ddot{x}\,\mathrm{sen}\,\theta\right) = -N + mg\,\mathrm{sen}\,\theta$

$(-\vec{j})\quad \dfrac{mr^2}{2}\ddot{\psi} = Tr \qquad ; \qquad \ddot{\psi} = -\dfrac{(R-r)}{r}\ddot{\theta}$

El sistema global conserva $\vec{P}\cdot\vec{i} \Rightarrow \dfrac{m}{3}\dot{x} + m\left(\dot{x} + (R-r)\dot{\theta}\cos\theta\right) = c^{te}$

$\dot{\vec{P}}\cdot\vec{i} = 0 \Rightarrow \dfrac{m}{3}\ddot{x} + m\ddot{x} + m(R-r)\ddot{\theta}\cos\theta - m(R-r)\dot{\theta}^2\,\mathrm{sen}\,\theta = 0$

$\dfrac{4}{3}m\ddot{x} = -m(R-r)\ddot{\theta}\cos\theta + m(R-r)\dot{\theta}^2\,\mathrm{sen}\,\theta$

En $t=0$ $\quad T = -\dfrac{m}{2}(R-r)\ddot{\theta}$

$\cancel{m}(R-r)\ddot{\theta} + \dfrac{\cancel{m}}{2}(R-r)\ddot{\theta} - \dfrac{3}{4}\cancel{m}(R-r)\ddot{\theta}\cos^2\theta_0 = -\cancel{m}g\,\mathrm{sen}\,\theta_0$

$\dfrac{3}{4}\ddot{\theta}\left(2 - \cos^2\theta_0\right) = -\dfrac{g}{(R-r)}\mathrm{sen}\,\theta_0$

$\ddot{\theta} = \dfrac{-4g\,\mathrm{sen}\,\theta_0}{(R-r)(2-\cos^2\theta_0)} = \dfrac{-4g\,\mathrm{sen}\,\theta_0}{(R-r)(1+\mathrm{sen}^2\theta_0)}$

$T = \dfrac{2mg\,\mathrm{sen}\,\theta_0}{1+\mathrm{sen}^2\theta_0}$

$N = mg\,\mathrm{sen}\,\theta_0 - m\ddot{x}\,\mathrm{sen}\,\theta = mg\,\mathrm{sen}\,\theta_0 - m\,\mathrm{sen}\,\theta_0\left(-\dfrac{3}{4}(R-r)\cos\theta_0\right)\ddot{\theta} =$

$= mg\,\mathrm{sen}\,\theta_0 + 3mg\dfrac{\mathrm{sen}^2\theta_0\cos\theta_0}{1+\mathrm{sen}^2\theta_0} = mg\left(\mathrm{sen}\,\theta_0 + \dfrac{3\,\mathrm{sen}^2\theta_0\cos\theta_0}{1+\mathrm{sen}^2\theta_0}\right)$