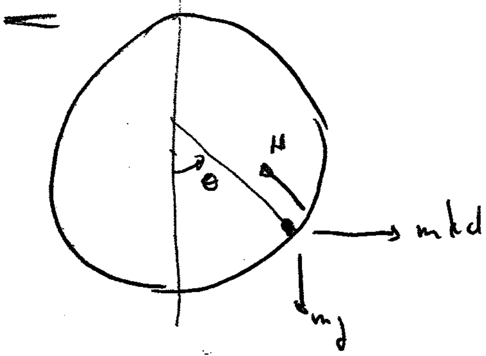
1.-



$$ma\ddot{\theta} = -mg\sin\theta + mka\sin\theta\cos\theta \qquad (1)$$

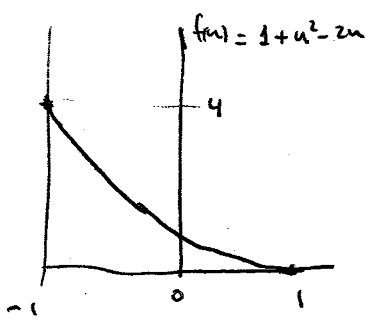
$$ma\dot{\theta}^2 = N - mg\cos\theta - mka\sin^2\theta \qquad (2)$$

Multiplicando (1) por $\dot{\theta}$ e integrando:

$$\Rightarrow \dot{\theta}^2 = \frac{2g}{a}\cos\theta + k\sin^2\theta + \frac{v_0^2}{a^2} - \frac{2g}{a}$$

$u = \cos\theta$ ‖ $$= \frac{g}{a}\left(\frac{v_0^2}{ag} - 1 - u^2 + 2u\right) \geq 0$$

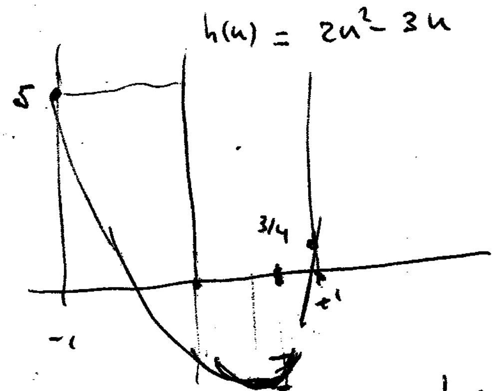
en $-1 \leq u \leq 1$



$$\Rightarrow \underline{\frac{v_0^2}{ag} \geq 4} \qquad (I)$$

$$N = mg\left(2\sin^2\theta + 3\cos\theta + \frac{v_0^2}{ag} - 2\right) = mg\left(\frac{v_0^2}{ag} - 2u^2 + 3u\right) \geq 0$$

en $-1 \leq u \leq 1$



$$\Rightarrow \underline{\frac{v_0^2}{ag} \geq 5} \qquad (II)$$

La más restrictiva es (II) $\Rightarrow$ $\boxed{v_0^2 \geq 5ag}$

2a.- Pos. de eq.: $\underline{\sin\theta\,(ka\cos\theta - g) = 0}$ $\qquad \underline{N \geq 0}$

$N = mg\cos\theta + mka\sin^2\theta$

$\boxed{\theta_1 = 0}$ $\qquad N(0) = mg > 0$

$\theta = \pi$ no es pos. de eq. pues $N(\pi) = -mg < 0$

$\boxed{\cos\theta_2 = \frac{g}{ka}}$ existe y es $\neq \theta_1$ si $\underline{\frac{g}{ka} < 1}$

$N(\theta_2) > 0$ pues $\cos\theta_2 > 0$

$f(\theta) = \operatorname{sen}\theta\,(ka\cos\theta - g)$

$f'(\theta) = \cos\theta\,(ka\cos\theta - g) - ka\operatorname{sen}^2\theta$

$f'(\theta_1) = ka - g$

$\theta_1 = 0$ es estable si $ka \leq g$

es inestable si $ka > g$

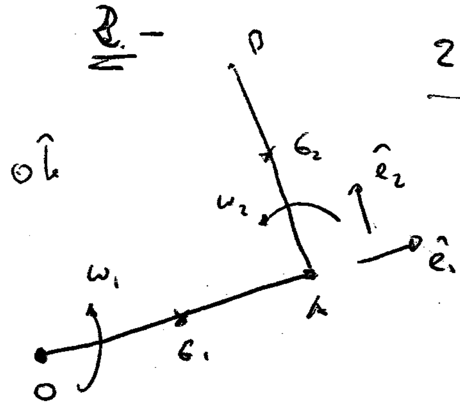
El caso $ka = g$ se decide observando que

$f''(0) = 0$ y $f'''(0) = -3ka < 0$. —

θ_2 es estable pues $f'(\theta_2) = -ka\operatorname{sen}^2\theta_2 < 0$

También si $f(\theta) = ka\operatorname{sen}\theta(\cos\theta - 1)$, una primitiva es $F(\theta) = -2ka\operatorname{sen}^4\frac{\theta}{2}$, que presenta un máximo en $\theta = 0$. —

1.-

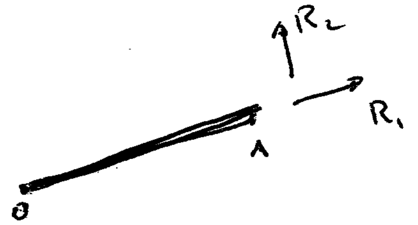


2^{da} card. en A a la barra AB:

$$\frac{4}{3} m l^2 \overset{\circ}{\omega}_2 + m\left[(G_2 - A) \wedge \vec{a}_A\right] \cdot \hat{k} = 0$$

$$\vec{a}_A = 2l\,\overset{\circ}{\omega}_1 \hat{e}_2 - 2l\,\underbrace{\omega_1^2}_{=\omega} \hat{e}_1 \quad ; \quad G_2 - A = l\,\hat{e}_2$$

$$\Rightarrow \frac{4}{3}\overset{\circ}{\omega}_2 + 2\omega^2 = 0 \Rightarrow \boxed{\overset{\circ}{\omega}_2 = -\frac{3}{2}\omega^2}$$



2^{da} card. en O a la barra OA

$$\Rightarrow \frac{4}{3} m l^2 \overset{\circ}{\omega}_1 = 2l R_2 \quad (1)$$

1^{ra} card. a la barra AB: $\quad m\,\vec{a}_{G_2} = -R_1\,\hat{e}_1 - R_2\,\hat{e}_2$

$$\vec{a}_{G_2} = \vec{a}_A + \vec{\overset{\circ}{\omega}}_2 \wedge (G_2 - A) + \underbrace{\vec{\omega}_2 \cdot (G_2 - A)}_{=0}\,\vec{\omega}_2 - \underbrace{\omega_2^2}_{=\omega}(G_2 - A)$$

$$= 2l\,\overset{\circ}{\omega}_1 \hat{e}_2 - 2l\omega^2 \hat{e}_1 + l\,\overset{\circ}{\omega}_2 (\hat{k} \wedge \hat{e}_2) - l\omega^2 \hat{e}_2$$

$$\Rightarrow -ml\frac{\omega^2}{2} = -R_1 \quad (2)$$

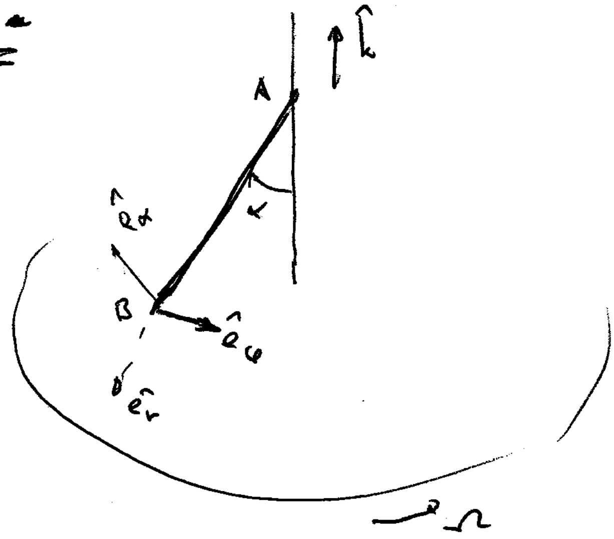
$$ml(2\overset{\circ}{\omega}_1 - \omega^2) = -R_2 \quad (3)$$

Eliminando R_2 entre (1) y (3) $\rightarrow$ $\boxed{\overset{\circ}{\omega}_1 = \frac{3}{8}\omega^2}$

2.- $\boxed{R_1 = ml\frac{\omega^2}{2}}$

$$\boxed{R_2 = +ml\frac{\omega^2}{4}}$$

3.a



$$\vec{\omega} = \dot{\varphi}\,\hat{k} = -\dot{\varphi}\cos\alpha\,\hat{e}_r + \dot{\varphi}\,\mathrm{sen}\,\alpha\,\hat{e}_\alpha$$

$$\mathbb{II}_A\,\vec{\omega} = \frac{4}{3} m l^2\,\mathrm{sen}\,\alpha\,\dot{\varphi}\,\hat{e}_\alpha$$

$$\frac{d(\mathbb{II}_A\vec{\omega})}{dt} = \frac{4}{3} m l^2\,\mathrm{sen}\,\alpha\,(\ddot{\varphi}\,\hat{e}_\alpha + \dot{\varphi}\,\mathring{\hat{e}}_\alpha)$$

$$= \vec{M}_A^{(ext)} = (B-A)\wedge\left[\left(N - \tfrac{1}{2}mg\right)\hat{k} + T\hat{e}_\varphi\right]$$

$$= 2l\left[\left(\tfrac{1}{2}mg - N\right)\mathrm{sen}\,\alpha\,\hat{e}_\varphi + T\hat{e}_\alpha\right]$$

$$\mathring{\hat{e}}_\alpha = \vec{\omega}\wedge\hat{e}_\alpha = \dot{\varphi}\,\hat{k}\wedge\hat{e}_\alpha = \dot{\varphi}\cos\alpha\,\hat{e}_\varphi$$

$$\frac{4}{3} m l^2\,\mathrm{sen}\,\alpha\,\ddot{\varphi} = 2lT$$

$$\frac{4}{3} m l^2\,\cancel{\mathrm{sen}\,\alpha}\cos\alpha\,\dot{\varphi}^2 = (mg\,l - 2lN)\,\cancel{\mathrm{sen}\,\alpha}$$

$$\boxed{T = \frac{2}{3} m l\,\mathrm{sen}\,\alpha\,\ddot{\varphi}} \quad ; \quad \boxed{N = \frac{mg}{2} - \frac{2}{3} m l\cos\alpha\,\dot{\varphi}^2}$$

1b.- $T = f\,|N| = f\,N \Rightarrow \frac{2}{3} m l\,\mathrm{sen}\,\alpha\,\ddot{\varphi} = f\frac{mg}{2} - \frac{2}{3} m l f\cos\alpha$

ó $\boxed{\ddot{\varphi} = \frac{3}{4}\frac{f g}{l\,\mathrm{sen}\,\alpha} - f\,\mathrm{cotg}\,\alpha\,\dot{\varphi}^2}$

1c.- $u(\varphi) = \dot{\varphi}^2 \Rightarrow u'(\varphi) = 2\dot{\varphi}\ddot{\varphi}\frac{1}{\dot{\varphi}} = 2\ddot{\varphi}$

$\Rightarrow u' = \frac{3}{2}\frac{fg}{l\,\mathrm{sen}\,\alpha} - 2f\,\mathrm{cotg}\,\alpha\,u \;\therefore\; u(\varphi) = A\,e^{-2f\,\mathrm{cotg}\,\alpha\,\varphi} + \frac{3}{4}\frac{g}{l\cos\alpha}$

$u(0) = 0 \Rightarrow \boxed{u(\varphi) = \frac{3}{4}\frac{g}{l\cos\alpha}\left(1 - e^{-2f\,\mathrm{cotg}\,\alpha\,\varphi}\right)}$

2a.- $\Omega^2 > u \quad \forall\varphi \Rightarrow \boxed{\Omega > \frac{1}{2}\sqrt{\frac{3g}{l\cos\alpha}}}$

2b.- $N = \frac{mg}{2} - \frac{2}{3} m l u\cos\alpha = \frac{mg}{2}\,e^{-2f\,\mathrm{cotg}\,\alpha\,\varphi} > 0$ ✓