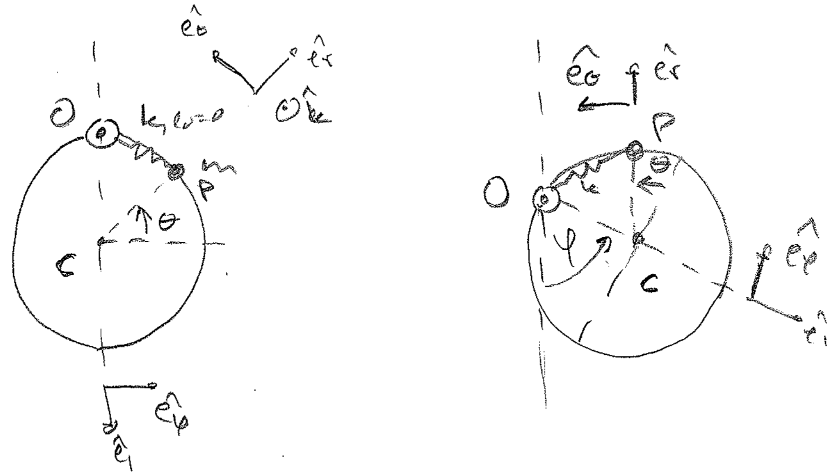
1



$\dot{\varphi} = \omega$ cte

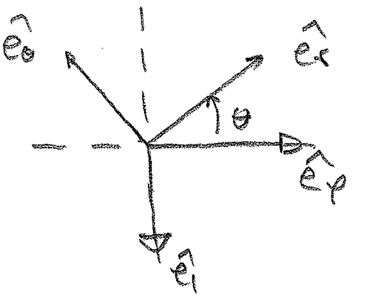
$$\vec{r} = R(\hat{e}_1 + \hat{e}_r)$$
$$\vec{v} = R(\dot{\hat{e}}_1 + \dot{\hat{e}}_r)$$

$$\dot{\hat{e}}_1 = \dot{\varphi}\hat{k} \wedge \hat{e}_1 = \dot{\varphi}\hat{e}_\varphi$$
$$\dot{\hat{e}}_\varphi = \dot{\varphi}\hat{k} \wedge \hat{e}_\varphi = -\dot{\varphi}\hat{e}_1$$
$$\dot{\hat{e}}_r = (\dot{\varphi}\hat{k} + \dot{\theta}\hat{k}) \wedge \hat{e}_r = (\dot{\varphi} + \dot{\theta})\hat{e}_\theta$$
$$\dot{\hat{e}}_\theta = (\dot{\varphi} + \dot{\theta})\hat{k} \wedge \hat{e}_\theta = -(\dot{\varphi} + \dot{\theta})\hat{e}_r$$

$$\vec{v} = R(\dot{\varphi}\hat{e}_\varphi + (\dot{\varphi} + \dot{\theta})\hat{e}_\theta)$$

$$\vec{a} = R(\dot{\varphi}\dot{\hat{e}}_\varphi + \ddot{\theta}\hat{e}_\theta + (\dot{\varphi} + \dot{\theta})\dot{\hat{e}}_\theta)$$
$$= R(-\dot{\varphi}^2\hat{e}_1 + \ddot{\theta}\hat{e}_\theta - (\dot{\varphi} + \dot{\theta})^2\hat{e}_r)$$

$$\vec{F}_N = -N\hat{e}_r + \vec{F}_{el} \qquad \vec{F}_{el} = -k(\vec{r}_P - \vec{r}_O)$$
$$= -kR(\hat{e}_1 + \hat{e}_r)$$



$$\hat{e}_\varphi = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$$
$$\hat{e}_1 = -\cos\theta\,\hat{e}_\theta - \sin\theta\,\hat{e}_r$$

$$\vec{F}_N = -N\hat{e}_r - kR(-\cos\theta\,\hat{e}_\theta + (1 - \sin\theta)\hat{e}_r)$$
$$\vec{a} = R(\ddot{\theta}\hat{e}_\theta - \dot{\varphi}^2\hat{e}_1 - (\dot{\varphi} + \dot{\theta})^2\hat{e}_r)$$

$\vec{F}_N = m\vec{a}$ según $\hat{e}_\theta$

$$kR\cos\theta = mR[\ddot{\theta} + \dot{\varphi}^2\cos\theta] \rightarrow$$

$$\boxed{\ddot{\theta} + \left(\omega^2 - \frac{k}{m}\right)\cos\theta = 0}$$ Ec. de mov.

b) Puntos de eq.

ceros de $f(\theta)=\left(\omega^2-\frac{k}{m}\right)\cos\theta$

$\rightarrow \cos\theta=0 \rightarrow \theta_1=\frac{\pi}{2}$ pues $\omega^2\neq\frac{k}{m}$

$\theta_2=-\frac{\pi}{2}$

$\left.\frac{df}{d\theta}\right|_{\theta_{eq}}>0$ estables.

$$\frac{df}{d\theta}=-\left(\omega^2-\frac{k}{m}\right)\sin\theta$$

$\theta_1=\frac{\pi}{2}$ $\left.\frac{df}{d\theta}\right|_{\theta_1}=-\left(\omega^2-\frac{k}{m}\right)>0$ si $\omega^2<\frac{k}{m}\rightarrow$ es estable.

<0 si $\omega^2>\frac{k}{m}$ es inestable.

$\theta_2=-\frac{\pi}{2}$ $\left.\frac{df}{d\theta}\right|_{\theta_2}=-\left(\omega^2-\frac{k}{m}\right)(-1)<0$ si $\omega^2<\frac{k}{m}$ es inestable.

>0 si $\omega^2>\frac{k}{m}$ es estable.

c) en $t=0$ $\theta_0=0$ y $\dot\theta(0)=\dot\theta_0>0$ y $\omega^2>\frac{k}{m}$

preintegro $\frac{\dot\theta^2}{2}-\frac{\dot\theta_0^2}{2}=-\left(\omega^2-\frac{k}{m}\right)\sin\theta\Big|_{\theta_0}^{\theta}$

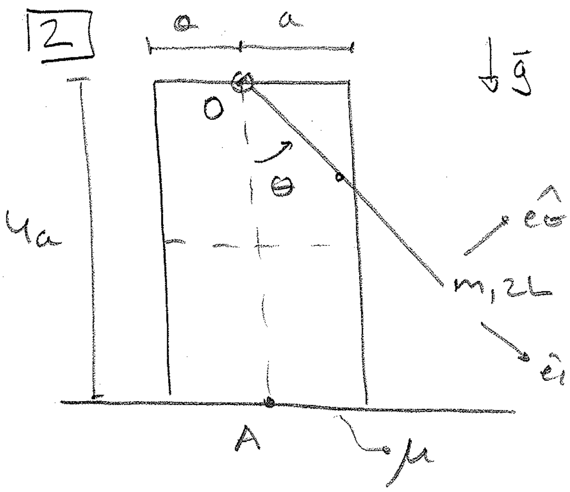
$$\frac{\dot\theta^2}{2}=\frac{\dot\theta_0^2}{2}-\left(\omega^2-\frac{k}{m}\right)\sin\theta$$

para que no llegue a $\theta=\frac{\pi}{2}$ θ_d t.q $\dot\theta(\theta_d)=0$ debe ocurrir con $\theta_d<\frac{\pi}{2}$

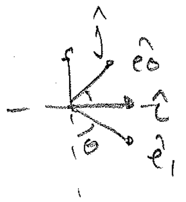
$0=\frac{\dot\theta_0^2}{2}-\left(\omega^2-\frac{k}{m}\right)\sin\theta_d \rightarrow \sin\theta_d=\frac{\dot\theta_0^2}{2\left(\omega^2-\frac{k}{m}\right)}<1 \rightarrow \boxed{\dot\theta_0^2<2\left(\omega^2-\frac{k}{m}\right)}$

pues $\sin(\pi/2)=1$

2



$\downarrow \vec{g}$ $\uparrow \hat{j}$

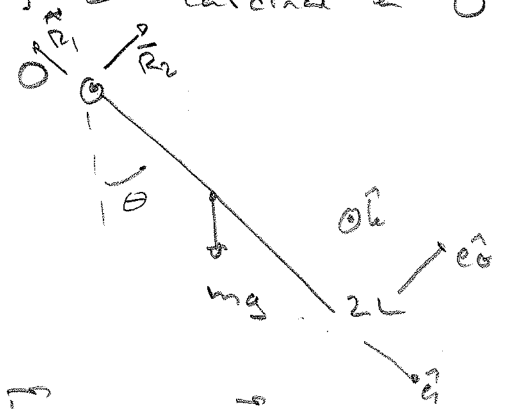


$\hat{j} = -\cos\theta\, \hat{e}_1 + \sin\theta\, \hat{e}_\theta$

$\hat{e}_1 = \sin\theta\, \hat{i} - \cos\theta\, \hat{j}$

$\hat{e}_\theta = \cos\theta\, \hat{i} + \sin\theta\, \hat{j}$

a) 2ª cardinal en O para la barra. En O articulación cilíndrica lisa



$$\vec{M}_O = -mgL \sin\theta\, \hat{k}$$

$$\mathbb{I}_O^{\{\hat{e}_1, \hat{e}_\theta, \hat{k}\}} = \frac{4}{3} mL^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vec{\Gamma}_O = \mathbb{I}_O \vec{\omega}$$

$$\vec{\omega} = \dot{\theta}\, \hat{k} \qquad \vec{\Gamma}_O = \frac{4}{3} mL^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$\vec{\Gamma}_O = \frac{4}{3} mL^2 \dot{\theta}\, \hat{k}$$

$$\Rightarrow \frac{4}{3} mL^2 \ddot{\theta} = -mgL \sin\theta \rightarrow \ddot{\theta} = -\frac{3}{4} \frac{g}{L} \sin\theta$$

en $t=0$ $\theta(0) = \frac{\pi}{2}$ y $\dot{\theta}(0) = 0$ → preintegro

$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^2}{2} = \frac{3}{4} \frac{g}{L} \cos\theta \Big|_{\theta_0}^{\theta} \rightarrow \dot{\theta}^2 = \frac{3}{2} \frac{g}{L} \cos\theta$$

b) 1ra cardinal barra.

$\vec{r}_G = L\hat{e}_1$, $\vec{v}_G = L\dot{\theta}\, \hat{e}_\theta$ $\vec{a}_G = L\ddot{\theta}\, \hat{e}_\theta - L\dot{\theta}^2\, \hat{e}_1$

$$\vec{F}_N = -R_1 \hat{e}_1 + R_2 \hat{e}_\theta - mg\hat{j} = (mg\cos\theta - R_1)\hat{e}_1 + (R_2 - mg\sin\theta)\hat{e}_\theta$$

$$mL\ddot{\theta} = R_2 - mg\sin\theta$$

$$-mL\dot{\theta}^2 = mg\cos\theta - R_1$$

Sustituyo $\ddot{\theta}$ y $\dot{\theta}$

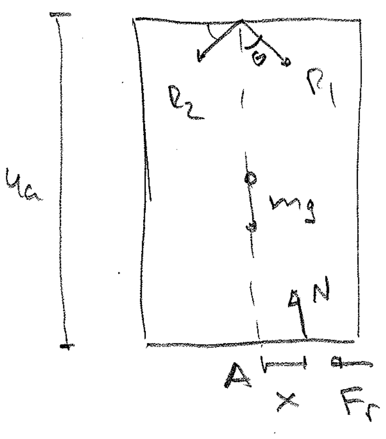
$$R_2 = mL\ddot{\theta} + mg\sin\theta = -\frac{3}{4}mg\sin\theta + mg\sin\theta$$

$$R_2 = \frac{1}{4}mg\sin\theta$$

$$R_1 = mg\cos\theta + mL\dot{\theta}^2 = mg\cos\theta\left(1 + \frac{3}{2}\right)$$

$$R_1 = \frac{5}{2}mg\cos\theta$$

c) Placa (Mientras no desliza)



1ra cardinal

$$\vec{F}_N = 0 = (N - mg)\hat{j} + R_1\hat{e} - R_2\hat{e}_\theta - F_r\hat{\imath} = 0$$

$$(N - mg)\hat{j} + R_1(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{j}) - R_2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{j}) - F_r\hat{\imath} = 0$$

$$\to N - mg - R_1\cos\theta - R_2\sin\theta = 0$$

$$R_1\sin\theta - R_2\cos\theta - F_r = 0$$

$$N = mg + R_1\cos\theta + R_2\sin\theta = mg + \frac{5}{2}mg\cos^2\theta + \frac{1}{4}mg\sin^2\theta$$

$$N = mg\left(1 + \frac{1}{4} + \frac{9}{4}\cos^2\theta\right) \to N = \frac{mg}{4}\left(5 + 9\cos^2\theta\right)$$

2^{da} cardinal en A:

$$N x \hat{k} + 4a(R_2\cos\theta - R_1\sin\theta) = 0$$

$$x = \frac{4a(R_1\sin\theta - R_2\cos\theta)}{\frac{mg}{4}(5+9\cos^2\theta)}$$

$$x = \frac{4a\left(\frac{5}{2}mg\sin\theta\cos\theta - \frac{1}{4}mg\sin\theta\cos\theta\right)}{\frac{mg}{4}(5+9\cos^2\theta)}$$

$$x = \frac{4a.9\sin\theta\cos\theta}{5+9\cos^2\theta} \rightarrow x = \frac{36a\sin\theta\cos\theta}{5+9\cos^2\theta}$$

No vuelco $\quad -a \leq x \leq a$

para $\theta \in [0, \frac{\pi}{2}]$ $\quad \begin{matrix}\sin\theta > 0\\ \cos\theta > 0\end{matrix} \rightarrow \frac{36a\sin\theta\cos\theta}{5+9\cos^2\theta} < a$

para $\theta \in [-\frac{\pi}{2}, 0]$ $\quad \begin{matrix}\sin\theta < 0\\ \cos\theta > 0\end{matrix} \rightarrow -a < \frac{36a\sin\theta\cos\theta}{5+9\cos^2\theta}$

Otra forma $|x| \leq a \rightarrow \frac{36a|\sin\theta|\cos\theta}{5+9\cos^2\theta} < a$

d) No desliza.

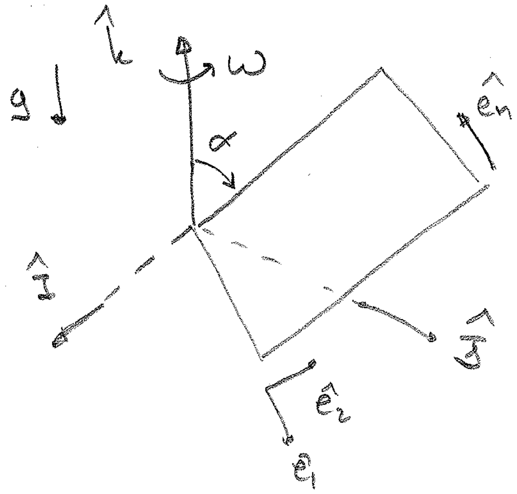
$$F_r = R_1\sin\theta - R_2\cos\theta = \frac{5}{2}mg\cos\theta\sin\theta - \frac{1}{4}mg\sin\theta\cos\theta$$

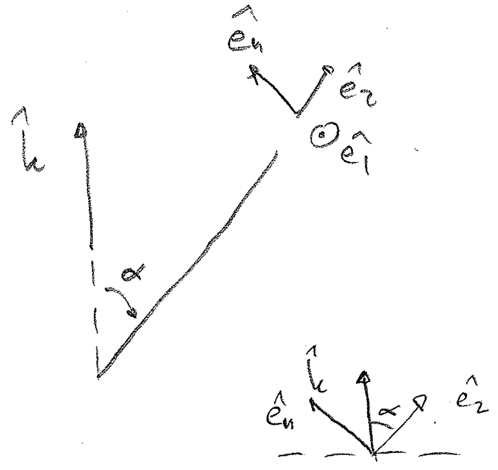
$$F_r = mg\frac{9}{4}\sin\theta\cos\theta$$

$$|F_r| \leq \mu|N| \rightarrow \frac{9}{4}mg|\sin\theta\cos\theta| \leq \mu\frac{mg}{4}(5+9\cos^2\theta)$$

$$\frac{9|\sin\theta|\cos\theta}{5+9\cos^2\theta} \leq \mu$$

$\boxed{3}$





$\hat{k} = \cos\alpha\, \hat{e}_2 + \sin\alpha\, \hat{e}_n$

a) $\vec{L}_O = \mathbb{I}_O \vec{\omega}$ $\quad \vec{\omega} = \omega \hat{k} = \omega\cos\alpha\, \hat{e}_2 + \omega\sin\alpha\, \hat{e}_n$

$\mathbb{I}_O = \mathbb{I}_G + \mathbb{J}_O^{M,G}$ $\quad \mathbb{I}_G^{\{\hat{e}_1, \hat{e}_2, \hat{e}_n\}} = \frac{m}{12}\begin{pmatrix} h^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & h^2+b^2 \end{pmatrix}$

$\mathbb{I}_G = \frac{m}{12}\begin{pmatrix} 16a^2 & 0 & 0 \\ 0 & 4a^2 & 0 \\ 0 & 0 & 20a^2 \end{pmatrix}$ $\quad b = 2a$, $h = 4a$

$\vec{r}_G - \vec{r}_O = a\hat{e}_1 + 2a\hat{e}_2$

$\mathbb{J}_O^{M,G} = m a^2 \begin{pmatrix} 4 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

$\mathbb{I}_O = m a^2 \begin{pmatrix} \frac{4}{3}+4 & -2 & 0 \\ -2 & \frac{1}{3}+1 & 0 \\ 0 & 0 & \frac{5}{3}+5 \end{pmatrix} = m a^2 \begin{pmatrix} \frac{16}{3} & -2 & 0 \\ -2 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{20}{3} \end{pmatrix}$

$\vec{L}_O = m a^2 \begin{pmatrix} \frac{16}{3} & -2 & 0 \\ -2 & 4/3 & 0 \\ 0 & 0 & \frac{20}{3} \end{pmatrix} \begin{pmatrix} 0 \\ \omega\cos\alpha \\ \omega\sin\alpha \end{pmatrix}$

$$\vec{L}_O = ma^2\left(-2\omega\cos\alpha\,\hat{e}_1 + \frac{4}{3}\omega\cos\alpha\,\hat{e}_2 + \frac{20}{3}\omega\sin\alpha\,\hat{e}_n\right)$$

b) $\mathring{\vec{L}}_O = \vec{M}_O^{ext}$ $\qquad \vec{M}_O^{(ext)} = \vec{r}_G \wedge (-mg\hat{k}) + \vec{M}_O^{react}$

$$\vec{r}_G = a\hat{e}_1 + 2a\hat{e}_2 \to \vec{M}_O^{(ext)} = a(\hat{e}_1 + 2\hat{e}_2)\wedge(-mg(\cos\alpha\,\hat{e}_2 + \sin\alpha\,\hat{e}_n)) + \vec{M}_O^{reac}$$

$$\vec{M}_O^{(ext)} = mga\left(-\cos\alpha\,\hat{e}_n + \sin\alpha\,\hat{e}_2 - 2\sin\alpha\,\hat{e}_1\right) + M_1\hat{e}_1 + M_2\hat{e}_2 + M_3\hat{e}_n$$

tomo $\vec{M}_O^{reac} = M_1\hat{e}_1 + M_2\hat{e}_2 + M_3\hat{e}_3$

$$\mathring{\vec{L}}_O = ma^2\left(-2\omega\cos\alpha\,\mathring{\hat{e}}_1 + \frac{4}{3}\omega\cos\alpha\,\mathring{\hat{e}}_2 + \frac{20}{3}\omega\sin\alpha\,\mathring{\hat{e}}_n\right)$$

$$\mathring{\hat{e}}_1 = \vec{\omega}\wedge\hat{e}_1 = \omega(\cos\alpha\,\hat{e}_2 + \sin\alpha\,\hat{e}_n)\wedge\hat{e}_1 = -\omega\cos\alpha\,\hat{e}_n + \omega\sin\alpha\,\hat{e}_2$$

$$\mathring{\hat{e}}_2 = \vec{\omega}\wedge\hat{e}_2 = \omega(\cos\alpha\,\hat{e}_2 + \sin\alpha\,\hat{e}_n)\wedge\hat{e}_2 = -\omega\sin\alpha\,\hat{e}_1$$

$$\mathring{\hat{e}}_n = \vec{\omega}\wedge\hat{e}_n = \omega(\cos\alpha\,\hat{e}_2 + \sin\alpha\,\hat{e}_n)\wedge\hat{e}_n = \omega\cos\alpha\,\hat{e}_1$$

$$\mathring{\vec{L}}_O = ma^2\left(-2\omega\cos\alpha(-\omega\cos\alpha\,\hat{e}_n + \omega\sin\alpha\,\hat{e}_2) - \frac{4}{3}\omega^2\cos\alpha\sin\alpha\,\hat{e}_1 + \frac{20}{3}\omega^2\sin\alpha\cos\alpha\,\hat{e}_1\right)$$

$$\to ma^2\omega^2\frac{16}{3}\cos\alpha\sin\alpha = -2mga\sin\alpha + M_1$$

$$\to -ma^2\omega^2\, 2\sin\alpha\cos\alpha = mga\sin\alpha + M_2$$

$$\to 2ma^2\omega^2\cos^2\alpha = -mga\cos\alpha + M_3$$