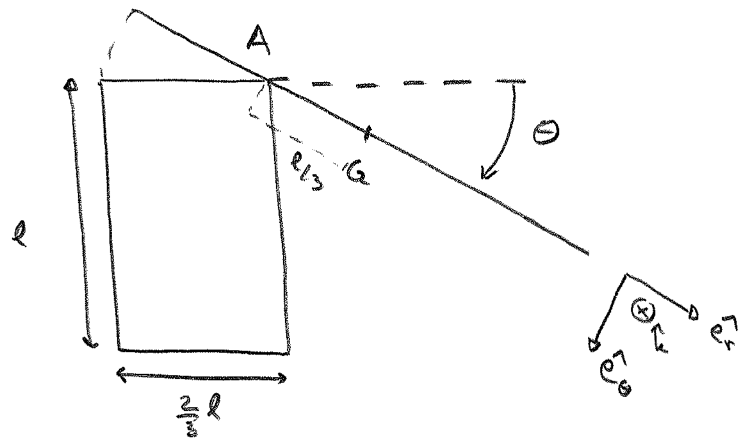
Ejercicio 1:

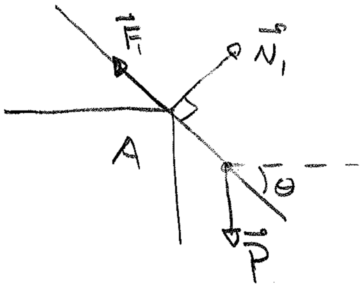


a) Mientras no desliza:

$\vec{r}_G = \frac{\ell}{3}\hat{e}_r$

$\vec{v}_G = \frac{\ell}{3}\dot{\theta}\,\hat{e}_\theta$

$\vec{a}_G = -\frac{\ell}{3}\dot{\theta}^2\hat{e}_r + \frac{\ell}{3}\ddot{\theta}\,\hat{e}_\theta$



1ra cardinal

$\hat{e}_r)\ mg\sin\theta - F_1 = -m\frac{\ell}{3}\dot{\theta}^2$

$\hat{e}_\theta)\ mg\cos\theta - N_1 = m\frac{\ell}{3}\ddot{\theta}$

2da cardinal

$\mathbb{I}_A\ddot{\theta}\hat{k} = mg\frac{\ell}{3}\cos\theta\,\hat{k}$

$\frac{4}{9}m\ell^2\ddot{\theta} = mg\frac{\ell}{3}\cos\theta$

$$\boxed{\ddot{\theta} = \frac{3}{4}\frac{g}{\ell}\cos\theta}$$

Otro camino: mientras no desliza se conserva la energía

$T = \frac{1}{2}\vec{\omega}\cdot\mathbb{I}_A\vec{\omega} = \frac{1}{2}\frac{4}{9}m\ell^2\dot{\theta}^2 \to T = \frac{2}{9}m\ell^2\dot{\theta}^2$

$U_g = -mg\frac{\ell}{3}\sin\theta \Rightarrow E = \frac{2}{9}m\ell^2\dot{\theta}^2 - mg\frac{\ell}{3}\sin\theta$

$\frac{dE}{dt} = 0 \to \boxed{\ddot{\theta} = \frac{3}{4}\frac{g}{\ell}\cos\theta}$

b) Las C.I. $\theta(0)=0 \quad \dot{\theta}(0)=0 \Rightarrow E_0=0$

$$\Rightarrow \quad \dot{\theta}^2 = \frac{3}{2}\frac{g}{l}\sin\theta$$

de la 1ra cardinal: $\hat{e}_r)$ $F_1 = mg\sin\theta + m\frac{l}{3}\dot{\theta}^2$

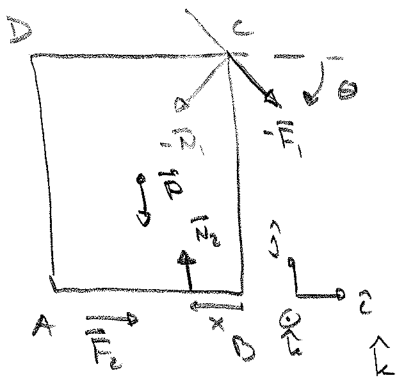
$$\boxed{F_1 = \frac{3}{2} mg\sin\theta}$$

$\hat{e}_\theta)$ $N_1 = mg\cos\theta - m\frac{l}{3}\ddot{\theta}$

$$\boxed{N_1 = \frac{3}{4} mg\cos\theta}$$

c) La barra comienza a deslizar cuando $|\vec{F}_1| = f|\vec{N}_1|$

$$\frac{3}{2}mg\sin\theta = f\frac{3}{4}mg\cos\theta \rightarrow \boxed{tg(\theta_d) = \frac{f}{2}}$$

d) 

1ra cardinal:

$\hat{i})$ $F_2 - N_1\sin\theta + F_1\cos\theta = 0$

$\hat{j})$ $N_2 - mg - N_1\cos\theta - F_1\sin\theta = 0$

$\hat{k}.$ $M_B = mg\frac{l}{3} + N_1 l\sin\theta - F_1 l\cos\theta - N_2 x = 0$

$$N_2 = mg + N_1\cos\theta + F_1\sin\theta = mg\left(1 + \frac{3}{4}\cos^2\theta + \frac{3}{2}\sin^2\theta\right)$$

$$N_2 = \frac{mg}{4}\left(7 + 3\cos^2\theta\right)$$

$$N_2 x = mgl\left(\frac{1}{3} - \frac{3}{2}\sin\theta\cos\theta + \frac{3}{4}\sin\theta\cos\theta\right)$$

$$x = \frac{4l\left(\frac{1}{3} - \frac{3}{4}\sin\theta\cos\theta\right)}{7 + 3\sin^2\theta} \qquad 0 \leq x \leq \frac{2}{3}l$$

$$0 \leq x \Rightarrow \quad x=0 \rightarrow \frac{1}{3} - \frac{3}{4}\sin\theta\cos\theta = 0 \Rightarrow \boxed{\sin(2\theta_v) = \frac{8}{9}}$$

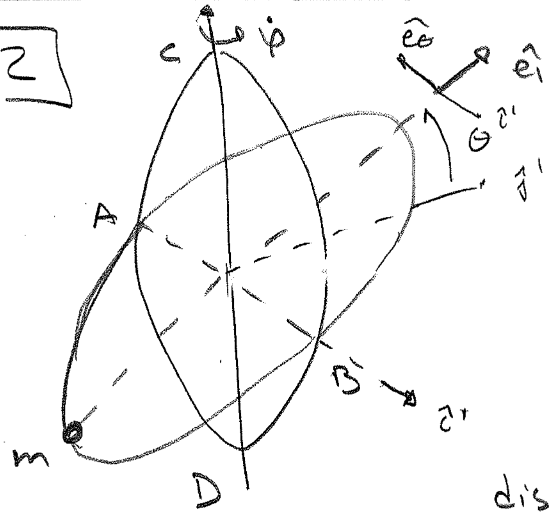
$$\sin(2\theta_v) = \frac{2\,tg\theta_v}{1+tg^2\theta_v} = \frac{8}{9} \Rightarrow tg^2\theta_v - \frac{9}{4}tg\theta_v + 1 = 0$$

$$\Rightarrow \quad tg\theta_v = \frac{9 \pm \sqrt{17}}{8}$$ dada la C.I. $\theta(0)=0 \Rightarrow tg\theta_v = \frac{9-\sqrt{17}}{8}$

para que no vuelque mientras desliza $\theta_v > \theta_d \Rightarrow$

$$tg\theta_v \geq tg\theta_d \Rightarrow \frac{f}{2} \leq \frac{9-\sqrt{17}}{8}$$

$\boxed{2}$



$$\bar r_i - \bar r_0 = -R\hat e_1$$

a) $$\mathbb{I}_0^{\{\hat\imath', \hat e_1, \hat e_\theta\}} = \underbrace{\frac{MR^2}{2}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{disco}} + \underbrace{MR^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{partícula}}$$

$$\mathbb{I}_0 = MR^2\begin{pmatrix} 5/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 3/2 \end{pmatrix}$$

b) O es pto fijo

$$\vec\omega = \dot\varphi\hat k + \dot\theta\hat\imath'$$

$$\hat k = \sin\theta\,\hat e_1 + \cos\theta\,\hat e_2$$

$$\vec\omega = \dot\varphi\sin\theta\,\hat e_1 + \dot\varphi\cos\theta\,\hat e_2 + \dot\theta\,\hat\imath'$$

$$\mathbb{I}_0\vec\omega = \frac{MR^2}{2}\begin{pmatrix} 5/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} \dot\theta \\ \dot\varphi\sin\theta \\ \dot\varphi\cos\theta \end{pmatrix} \Rightarrow \bar L_0 = \mathbb{I}_0\vec\omega = \frac{MR^2}{2}\left(\frac{5}{2}\dot\theta\,\hat\imath' + \frac{1}{2}\dot\varphi\sin\theta\,\hat e_1 + 3\dot\varphi\cos\theta\,\hat e_\theta\right)$$

$$T = \frac{1}{2}\vec\omega\cdot\mathbb{I}_0\vec\omega \Rightarrow T = \frac{MR^2}{4}\left(\frac{5}{2}\dot\theta^2 + \frac{1}{2}\dot\varphi^2\sin^2\theta + 3\dot\varphi^2\cos^2\theta\right)$$

$$T = \frac{MR^2}{8}\left(5\dot\theta^2 + \dot\varphi^2(1 + 5\cos^2\theta)\right)$$

c) $$\frac{d}{dt}(\bar L_0\cdot\hat k) = 0\ ?$$

$$\frac{d}{dt}(\bar L_0\cdot\hat k) \underset{\hat k \text{ es fijo}}{=} \hat k\cdot\frac{d\bar L_0}{dt} = \hat k\cdot\vec M_0^{(ext)}$$

$$\Rightarrow \text{si } \bar M_0^{ext}\cdot\hat k = 0 \Rightarrow \frac{d}{dt}(\bar L_0\cdot\hat k) = 0 \text{ y } \bar L_0\cdot\hat k = \text{cte.}$$

· Sobre el sistema total actúa el Peso $\bar{P} = -mg\hat{k}$ sobre la partícula

· Supongamos que tenemos fuerzas reactivas $\bar{F}_C$ y $\bar{F}_D$ en C y D respectivamente.

· Las fuerzas reactivas en A y B son internas al sistema: aro + disco + partícula.

$$\bar{M}_O^{ext} = (\bar{r}_p - \bar{r}_O) \wedge (-mg\hat{k}) + (\bar{r}_C - \bar{r}_O) \wedge \bar{F}_C + (\bar{r}_D - \bar{r}_O) \wedge \bar{F}_D$$

$$\left.\begin{array}{l} \bar{r}_p - \bar{r}_O = -R\hat{e}_1 \\ \bar{r}_O - \bar{r}_D = R\hat{k} \\ \bar{r}_O - \bar{r}_C = -R\hat{k} \end{array}\right\}$$ Los tres productos vectoriales dan vectores $\perp$ a $\hat{k}$

$$\Rightarrow \bar{M}_O^{ext} \cdot \hat{k} = 0 \quad \Rightarrow \bar{L}_O \cdot \hat{k} = cte \quad \square$$

d) $$\bar{L}_O \cdot \hat{k} = \frac{MR^2}{2}\left(\frac{5}{2}\dot{\theta}\hat{i}' + \frac{1}{2}\dot{\varphi}\sin\theta\,\hat{e}_1 + 3\dot{\varphi}\cos\theta\,\hat{e}_2\right)\cdot\left(\sin\theta\hat{e}_1 + \cos\theta\hat{e}_2\right)$$

$$= \frac{MR^2}{2}\left(\frac{1}{2}\dot{\varphi}\sin^2\theta + 3\dot{\varphi}\cos^2\theta\right) = \frac{MR^2}{4}\dot{\varphi}\left(1 + 5\cos^2\theta\right) = l_0$$

$$U_g = -mgR\sin\theta$$

$$E = \frac{MR^2}{8}\left(5\dot{\theta}^2 + \dot{\varphi}^2(1 + 5\cos^2\theta)\right) - MgR\sin\theta$$

La energía se conserva $\quad$ C.I. $\rightarrow \theta(0) = 0;\ \dot{\theta}(0) = 0;\ \bar{\omega}(0) = \dot{\varphi}_0\hat{k}$

$$l_0 = \frac{3}{2}MR^2\dot{\varphi}_0; \quad E_0 = \frac{3}{4}MR^2\dot{\varphi}_0^2$$

$$\dot{\varphi}(1 + 5\cos^2\theta) = 6\dot{\varphi}_0 \rightarrow \dot{\varphi} = \frac{6\dot{\varphi}_0}{1 + 5\cos^2\theta}$$

$$\frac{MR^2}{8}\left(5\dot{\theta}^2 + \dot{\varphi}^2(1 + 5\cos^2\theta)\right) - MgR\sin\theta = \frac{3}{4}MR^2\dot{\varphi}_0^2$$

$$\frac{MR^2}{8}\left(5\dot{\theta}^2 + \frac{(6\dot{\varphi}_0)^2}{1 + 5\cos^2\theta}\right) - MgR\sin\theta = \frac{3}{4}MR^2\dot{\varphi}_0^2$$

$$5\dot{\theta}^2 = 6\dot{\varphi}_0^2\left(1 - \frac{6}{1 + 5\cos^2\theta}\right) + 8\frac{g}{R}\sin\theta$$

$$\boxed{\dot{\theta}^2 = 6\dot{\varphi}_0^2 \frac{(\cos^2\theta - 1)}{1 + 5\cos^2\theta} + \frac{8}{5}\frac{g}{R}\sin\theta}$$

e) para llegar a $\theta = \frac{\pi}{2}$ $\dot{\theta}^2\left(\frac{\pi}{2}\right) \geq 0$

$$\left. 6\dot{\varphi}_0^2 \frac{(\cos^2\theta - 1)}{1+5\cos^2\theta} + \frac{8}{5}\frac{g}{R}\sin\theta \right|_{\theta = \pi/2} \geq 0$$

$$-6\dot{\varphi}_0^2 + \frac{8}{5}\frac{g}{R} \geq 0 \Rightarrow \boxed{\frac{4}{15}\frac{g}{R}\frac{1}{\dot{\varphi}_0^2} \geq 1}$$