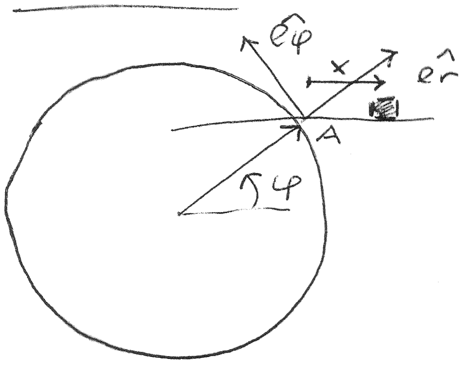
Ejercicio 1



a) $\vec v_R = \dot x \,\hat\imath'$, $\vec a_R = \ddot x \,\hat\imath'$

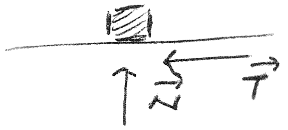
b) $\vec v = \vec v_R + \vec v_A = \dot x\,\hat\imath' + \omega R\,\hat e_\varphi$

$$\boxed{\vec v = (\dot x - \omega R \operatorname{sen}\varphi)\,\hat\imath' + \omega R\cos\varphi\,\hat\jmath'}$$

$$\vec a = \vec a_R + \vec a_A = \ddot x\,\hat\imath' - \omega^2 R\,\hat e_r$$

$$\boxed{\vec a = (\ddot x - \omega^2 R\cos\varphi)\,\hat\imath' - \omega^2 R \operatorname{sen}\varphi\,\hat\jmath'}$$

c)



En reposo relativo, $\dot x = 0$, $\ddot x = 0$

$$\Rightarrow \quad N - mg = m(-\omega^2 R \operatorname{sen}\varphi)$$
$$-T = m(-\omega^2 R\cos\varphi)$$

$t \simeq 0$: $\quad N - mg = 0$

$$T = m\omega^2 R$$

$|T| \le f_E |N|$: $\quad m\omega^2 R \le f_E\, mg$: $\quad \boxed{f_E \ge \dfrac{\omega^2 R}{g}}$

d) Suponiendo $\dot x(t) > 0$ en un entorno del instante inicial: $\vec T = -f_D N\,\hat\imath'$

$$\Rightarrow \begin{cases} N - mg = -m\omega^2 R \operatorname{sen}\varphi \\ -f_D N = m(\ddot x - \omega^2 R\cos\varphi)\end{cases}$$; eliminando N entre las anteriores:

$$-f_D(mg - m\omega^2 R \operatorname{sen}\varphi) = m(\ddot x - \omega^2 R\cos\varphi)$$

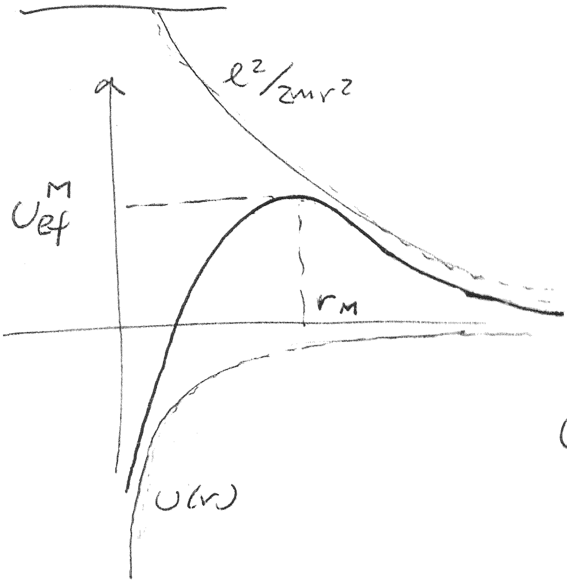
$$\Rightarrow \boxed{\ddot x(t) = -f_D g + \omega^2 R(\cos\varphi + f_D \operatorname{sen}\varphi)} \underset{t\simeq 0}{\simeq} \omega^2 R - f_D g > 0$$

(siendo $\varphi = \omega t$) $\qquad$ $\left(f_D < f_E < \dfrac{\omega^2 R}{g}\right)$

luego: $\boxed{\dot x(t) = -f_D g t + \omega R(\operatorname{sen}\varphi - f_D(\cos\varphi - 1))} \underset{t\simeq 0}{\simeq} (\omega^2 R - f_D g)t > 0$ ✓

$$\underline{\underline{\vec v\cdot\hat\imath'}} = \dot x - \omega R\operatorname{sen}\varphi = \boxed{-f_D\left[gt + \omega R(\cos\varphi - 1)\right]} \underset{t\simeq 0}{\simeq} -f_D g t$$

Ejercicio 2



a) $U_{ef}(r) = \dfrac{\ell^2}{2mr^2} + U(r) = \dfrac{\ell^2}{2mr^2} - \dfrac{C}{r^4}$

b) Para que la partícula alcance el origen su energía E debe verificar: $E > U_{ef}^M$

U_{ef}^M : $\dfrac{dU_{ef}}{dr} = 0$: $-\dfrac{\ell^2}{mr^3} + \dfrac{4C}{r^5} = 0$:

$$r_M^2 = \frac{4mC}{\ell^2}$$

$$\Rightarrow U_{ef}^M = U_{ef}(r_M) = \boxed{\frac{\ell^4}{16m^2C} < E}$$

c) $\ell = mbv_0$

$E_\infty = \frac{1}{2}mv_0^2$

$\dfrac{(mbv_0)^4}{16m^2C} < \dfrac{1}{2}mv_0^2$: $\boxed{b < \left(\dfrac{8C}{mv_0^2}\right)^{1/4}}$

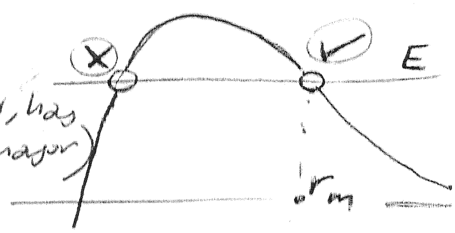
d) $E = \frac{1}{2}m\dot{r}^2 + U_{ef}(r)$; para la mínima distancia: $\dot{r} = 0$

$\Rightarrow E = U_{ef}(r) = \dfrac{\ell^2}{2mr^2} - \dfrac{C}{r^4}$: $r^4 - \dfrac{\ell^2}{2mE}r^2 + \dfrac{C}{E} = 0$

sustituyendo $\ell = mbv_0$, $E = \frac{1}{2}mv_0^2$: $r^4 - b^2r^2 + \dfrac{2C}{mv_0^2} = 0 \Rightarrow$

$$\Rightarrow \boxed{r_m^2 = \frac{1}{2}\left(b^2 \pm \sqrt{b^4 - \frac{8C}{mv_0^2}}\right)}$$

(de las raíces, hay que elegir la mayor)



La velocidad máxima corresponde al mínimo valor de U, que se obtiene para $r = r_m$:

$E = \frac{1}{2}mv^2 + U(r)$: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_M^2 + U(r_m)$

$$\boxed{v_M = \sqrt{v_0^2 + \frac{2C}{m}r_m^{-4}}}$$