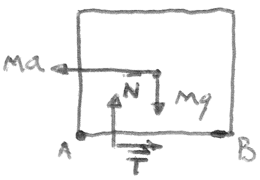
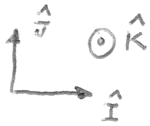
1) I) $\vec{F}_i = m_i \vec{a} \Longrightarrow \vec{M}_O = \sum_i \cancel{m}(P_i - O)\wedge \vec{F}_i = \underbrace{\sum_i m_i (P_i - O)}_{M(G-O)} \wedge \vec{a} = (G-O)\wedge(M\vec{a})$

$$\vec{R} = \sum_i \vec{F}_i = \underbrace{\sum_i m_i}_{M} \cdot \vec{a} = M \cdot \vec{a}$$

Entonces el sistema de fuerzas $\vec{F}_i$ es equivalente a una unica fuerza $M.\vec{a}$ aplicada en el centro de masas

II) a) Voy a trabajar en el sistema relativo (camión), donde aparece una fuerza ficticia $\vec{F} = -m.\vec{a}$ sobre el lavarropa.



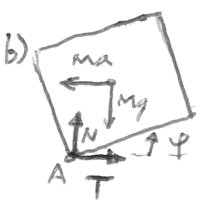


1^a cardinal placa $\Longrightarrow \begin{cases} T = M.a \\ N = M.g \end{cases}$

condición de no deslizamiento $\longrightarrow T \leq f.N \Longrightarrow M.a \leq f.M.g \Longrightarrow \boxed{f \geq \frac{a}{g}}$

condición de no vuelco $\longrightarrow \begin{cases} \vec{M}_A^{act} \cdot \hat{k} < 0 \\ \vec{M}_B^{act} \cdot \hat{k} > 0 \end{cases} \Longrightarrow \begin{cases} \vec{M}_A^{act} \cdot \hat{k} = \frac{Ml}{2}(a-g) < 0 \\ \vec{M}_B^{act} \cdot \hat{k} = \frac{Ml}{2}(a+g) > 0 \end{cases} \Longrightarrow \boxed{\frac{a}{g} < 1}$

b)



$d = \frac{l}{\sqrt{2}} \longrightarrow$ semidiagonal placa

Aplico primera cardinal en $t=0 \longrightarrow Md.\ddot{\varphi}.\hat{e}_\varphi|_{t=0} = (T - Ma)\hat{I} + (N - Mg)\hat{J}$

$\hat{e}_\varphi|_{t=0} = \frac{\sqrt{2}}{2}(\hat{J} - \hat{I}) \Longrightarrow Md\ddot{\varphi}(0) \cdot \frac{\sqrt{2}}{2}(\hat{J} - \hat{I}) = (T - Ma)\hat{I} + (N - Mg)\hat{J} \Longrightarrow \begin{cases} -M\frac{l}{2}\ddot{\varphi}(0) = T - Ma \\ M\frac{l}{2}\ddot{\varphi}(0) = N - Mg \end{cases}$

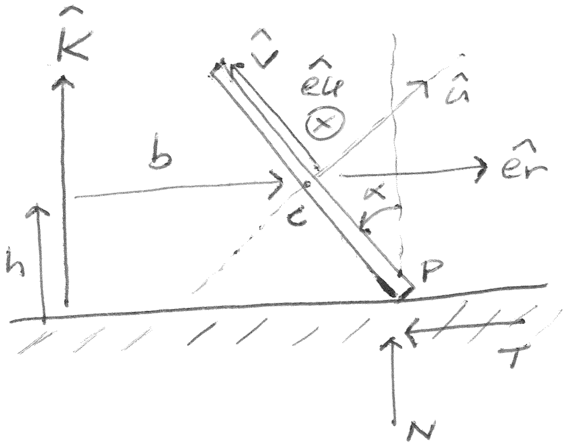
$I_A = I_G + M.d^2 = \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{4}{6}Ml^2 = \frac{2}{3}Ml^2$

Ahora aplico 2^a cardinal en A proyectada en la dirección de $\hat{k}$ $\longrightarrow \frac{2}{3}Ml^2\ddot{\varphi}(0) = Ma.\frac{l}{2} - Mg.l/2 \Longrightarrow \ddot{\varphi}(0) = \frac{3}{4l}.(a-g)$

$$\Longrightarrow \begin{cases} N = Mg + M\frac{l}{2}\ddot{\varphi}(0) = \frac{M}{8}(5g+3a) \\ T = Ma - M\frac{l}{2}\ddot{\varphi}(0) = \frac{M}{8}(3g+5a) \end{cases}$$

vuelco $\longrightarrow \ddot{\varphi}(0) > 0 \Longrightarrow \boxed{\frac{a}{g} > 1}$

no deslizamiento $\longrightarrow T \leq f.N \Longrightarrow f \geq \frac{3g+5a}{5g+3a} \Longrightarrow \boxed{f \geq \frac{3 + 5\,a/g}{5 + 3\,a/g}}$



$\cos\alpha = \frac{h}{a}$

$\hat{u} = \cos\alpha\, \hat{e}_r + \operatorname{sen}\alpha\, \hat{k}$

$\hat{v} = \cos\alpha\, \hat{k} - \operatorname{sen}\alpha\, \hat{e}_r$

$(\hat{k} = \cos\alpha\, \hat{v} + \operatorname{sen}\alpha\, \hat{u})$

a) $\dot{\varphi} = v/b$; $\vec{\omega} = \dot{\varphi}\hat{k} + \dot{\psi}\hat{u}$

Como la moneda rueda sin deslizar: $\vec{v}_P = 0 \Rightarrow \vec{v}_C = \vec{v}_P + \vec{\omega} \times (C-P)$

$= \vec{\omega} \times (C-P)$:

$$v\,\hat{e}_\varphi = (\dot{\varphi}\hat{k} + \dot{\psi}\hat{u}) \times a\hat{v}$$

$$v\,\hat{e}_\varphi = \left[\dot{\varphi}(\cos\alpha\,\hat{v} + \operatorname{sen}\alpha\,\hat{u}) + \dot{\psi}\hat{u}\right] \times a\hat{v}$$

$$= \left[\frac{v}{b}\cos\alpha\,\hat{v} + \left(\frac{v}{b}\operatorname{sen}\alpha + \dot{\psi}\right)\hat{u}\right] \times a\hat{v} = -a\left(\frac{v}{b}\operatorname{sen}\alpha + \dot{\psi}\right)\hat{e}_\varphi$$

$$\Rightarrow \boxed{\dot{\psi} = -\frac{v}{a}\left(\frac{a}{b}\operatorname{sen}\alpha + 1\right)} \; ; \; \boxed{\vec{\omega} = \frac{v}{b}\cos\alpha\,\hat{v} - \frac{v}{a}\hat{u}}$$

b) $\vec{a}_G = -b\dot{\varphi}^2\hat{e}_r \Rightarrow -m\frac{v^2}{b}\hat{e}_r = -T\hat{e}_r + (N - mg)\hat{k}$

$$\Rightarrow \boxed{\begin{aligned} T &= \frac{mv^2}{b} \\ N &= mg \end{aligned}}$$

c) $\dot{\vec{L}}_G = \vec{M}_G^{(ext)} = \vec{M}_C^{(ext)} = (hT - a\operatorname{sen}\alpha\, N)\,\hat{e}_\varphi = \left(\frac{h m v^2}{b} - a\operatorname{sen}\alpha\, mg\right)\hat{e}_\varphi$

$$\vec{L}_C = \mathbb{I}_C\vec{\omega} = \frac{ma^2}{4}\frac{v}{b}\cos\alpha\,\hat{v} - \frac{ma^2}{2}\frac{v}{a}\hat{u}$$

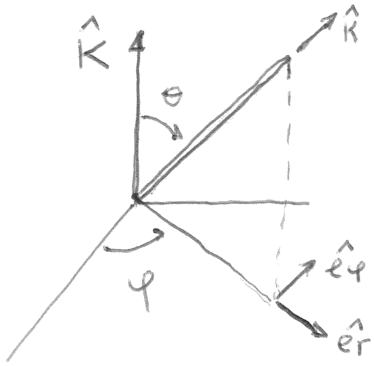
$$\dot{\vec{L}}_C = \dot{\varphi}\hat{k} \times \vec{L}_C = \left(\frac{v}{b}\right)^2\frac{ma^2}{4}\cos\alpha\,\hat{k}\times\hat{v} - \frac{v^2 a}{b}\frac{m}{2}\hat{k}\times\hat{u}$$

$$= -\frac{mv^2 a}{2b}\left(\frac{1}{2}\cos\alpha\,\frac{a}{b}\operatorname{sen}\alpha + \cos\alpha\right)\hat{e}_\varphi$$

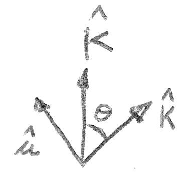
$$\Rightarrow \frac{h m v^2}{b} - mga\operatorname{sen}\alpha = -\frac{mv^2}{2}\frac{a}{b}\left(\frac{a}{2b}\operatorname{sen}\alpha + 1\right)\cos\alpha \quad \frac{h}{a}$$

$$\Rightarrow \boxed{v^2 = \frac{gab\operatorname{sen}\alpha}{h\left[(1 + a/4b)\operatorname{sen}\alpha + \frac{1}{2}\right]}}$$

3)



$\hat{u} = \hat{k} \wedge \hat{e}_\varphi \Longrightarrow \{\hat{e}_\varphi, \hat{u}, \hat{k}\}$ base de ejes principales



$\hat{K} = \cos\theta \cdot \hat{k} + \operatorname{sen}\theta \cdot \hat{u}$

a)

$$\vec{w} = \dot{\varphi}\hat{K} + \dot{\theta}\cdot\hat{e}_\varphi = \dot{\varphi}\cdot\operatorname{sen}\theta\cdot\hat{u} + \dot{\theta}\cdot\hat{e}_\varphi + \dot{\varphi}\cdot\cos\theta\cdot\hat{k}$$

$$\mathbb{I}_O^{\{\hat{e}_\varphi, \hat{u}, \hat{k}\}} = \frac{Ml^2}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Longrightarrow \boxed{\vec{L}_O = \frac{Ml^2}{3}\left[\dot{\varphi}\cdot\operatorname{sen}\theta\cdot\hat{u} + \dot{\theta}\,\hat{e}_\varphi\right]}$$

$$T = \frac{1}{2}\vec{w}\,\mathbb{I}_O\,\vec{w} = \frac{Ml^2}{6}\left[\dot{\varphi}^2\cdot\operatorname{sen}^2\theta + \dot{\theta}^2\right]$$

b) * se conserva la energía porque la única fuerza ~~no conservativa~~ que trabaja es el peso y es conservativa, ya que la reacción en O, es de potencia nula ~~porque~~ porque $\vec{V}_O = 0$

* se conserva $\vec{L}_O \cdot \hat{K}$ porque la única fuerza que hace momento en O es el peso y dicho momento no tiene componente vertical.

c) $L_z = \vec{L}_O\cdot\hat{K} = cte \Longrightarrow L_z = \frac{Ml^2}{3}\Big[\dot{\varphi}\cdot\operatorname{sen}\theta\cdot\underbrace{\hat{u}\cdot\hat{K}}_{\operatorname{sen}\theta} + \dot{\theta}\,\underbrace{\hat{e}_\varphi\cdot\hat{K}}_{0}\Big] = \dot{\varphi}\cdot\operatorname{sen}^2\theta = \underbrace{\dot{\varphi}(0)\cdot\operatorname{sen}^2(\pi/2)}_{w}$

$\Longrightarrow \dot{\varphi} = \frac{w}{\operatorname{sen}^2\theta}$

$E = T + U = cte \Longrightarrow E = \frac{Ml^2}{6}\left[\dot{\varphi}^2\cdot\operatorname{sen}^2\theta + \dot{\theta}^2\right] + Mg\frac{l}{2}\cdot\cos\theta = \frac{Ml^2}{6}\cdot w^2$

$\Longrightarrow \frac{Ml^2}{6}\left[\dot{\theta}^2 + \frac{w^2}{\operatorname{sen}^2\theta}\right] + Mg\frac{l}{2}\cdot\cos\theta = \frac{Ml^2}{6}w^2 \Longrightarrow \dot{\theta}^2 = w^2\left[1 - \frac{1}{\operatorname{sen}^2\theta}\right] - 3(g/l)\cdot\cos\theta$

$\Longrightarrow \boxed{\dot{\theta}^2 = w^2\cdot\frac{\cos^2\theta}{\cos^2\theta - 1} - 3(g/l)\cdot\cos\theta}$

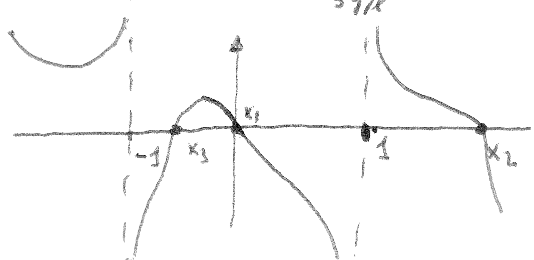
d) $\dot{\theta}^2 > 0 \longrightarrow w^2\cdot\frac{\cos^2\theta}{\cos^2\theta - 1} - 3(g/l)\cdot\cos\theta > 0$ hago el cambio de variable $x = \cos\theta$

$F(x) =$
$\longrightarrow \frac{w^2\cdot x^2}{x^2 - 1} - 3(g/l)\,x > 0$, ahora busco los ceros de esta función para analizar su signo

$x\left(\frac{w^2}{x^2-1} - 3g/l\right) = 0 \Longrightarrow x_1 = 0$, $x^2 - \frac{w^2}{3g/l}\cdot x - 1 = 0 \Longrightarrow \begin{cases} x_2 = \left(\frac{w^2}{3g/l} + \sqrt{\left(\frac{w^2}{3g/l}\right)^2 + 4}\right)\frac{1}{2} \\ x_3 = \left(\frac{w^2}{3g/l} - \sqrt{\left(\frac{w^2}{3g/l}\right)^2 + 4}\right)1/2 \end{cases}$

$\Longrightarrow x_2 = \frac{\sqrt{5}+3}{2} = 2{,}62$

$x_3 = \frac{\sqrt{5}-3}{2} = -0{,}38$



$F(x) > 0$ para $x_3 \le x \le 0$

$\Longrightarrow 90° \le \theta \le \underbrace{\arccos(-0{,}38)}_{112°}$