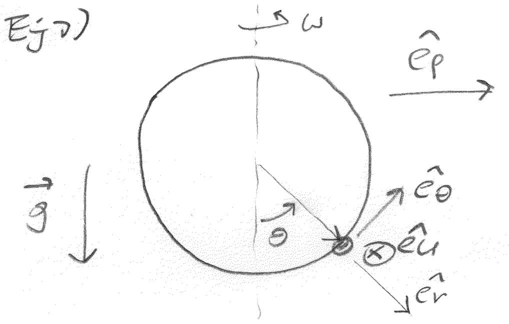
Ej 1)



a) $-mg\,\text{sen}\theta = m\left(\vec{a}_T \cdot \hat{e}_\theta + R\ddot{\theta}\right)$

$\vec{a}_T = -\omega^2 R\,\text{sen}\theta\,\hat{e}_\rho$

$\Rightarrow mR\ddot{\theta} - m\omega^2 R\,\text{sen}\theta\cos\theta + mg\,\text{sen}\theta = 0$

b) mult por R la ec. anterior:

$$mR^2\ddot{\theta} + \underbrace{\left[-m\omega^2R^2\,\text{sen}\theta\cos\theta + mgR\,\text{sen}\theta\right]}_{\frac{dU_{ef}}{d\theta}} = 0$$

siendo $U_{ef} = -\left[\frac{1}{2}m\omega^2R^2\,\text{sen}^2\theta + mgR\cos\theta\right]$

c) equilibrio: $\frac{dU_{ef}}{d\theta} = 0$: $\left(\frac{g}{R} - \omega^2\cos\theta\right)\text{sen}\theta = 0$ $\begin{cases} \theta = 0, \pi \\ \cos\theta_{eq} = g/R\omega^2 \end{cases}$

$(\exists$ si $\omega^2 \geq g/R)$

estabilidad: $\frac{d^2U_{ef}}{d\theta^2} = mR^2\left[\omega^2\,\text{sen}^2\theta + \left(\frac{g}{R} - \omega^2\cos\theta\right)\cos\theta\right]$

$\left.\frac{d^2U_{ef}}{d\theta^2}\right|_{\theta=\pi} = -mR^2\left(g/R + \omega^2\right) < 0 \Rightarrow \theta = \pi$ inestable

$\left.\frac{d^2U_{ef}}{d\theta^2}\right|_{\theta=0} = mR^2\left(\frac{g}{R} - \omega^2\right) > 0$ si $\omega^2 < g/R$

$\Rightarrow \theta = 0$ estable para $\omega^2 < g/R$

$\left.\frac{d^2U_{ef}}{d\theta^2}\right|_{\cos\theta_{eq} = g/R\omega^2} = mR^2\left[\omega^2(1-\cos^2\theta) + \left(\frac{g}{R} - \omega^2\cos\theta\right)\cos\theta\right]_{\cos\theta_{eq} = g/R\omega^2}$

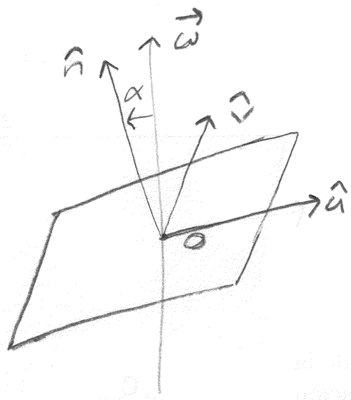
$= mR^2\left[\omega^2\left(1 - \left(g/R\omega^2\right)^2\right) + \right] > 0$ si $\omega^2 > g/R$: si $\exists\, \theta_{eq}$, es estable

$\omega^2 = \frac{g}{R} \Rightarrow \frac{dU_{ef}}{d\theta} = mgR\left(1 - \cos\theta\right)\text{sen}\theta$ $\begin{cases} > 0 \text{ para } \theta \to 0^+ \\ < 0 \text{ para } \theta \to 0^- \end{cases}$

$\Rightarrow$ la derivada de U_{ef} crece en un entorno de $\theta = 0$ por lo que tenemos un mínimo de U_{ef}

y la posición de equilibrio es estable

Ej 2)



a) Sean I_{11}, I_{22}, I_{33} los momentos de inercia alrededor de los ejes $\hat{u}, \hat{v}, \hat{n}$ respectivamente.

Se cumple $I_{33} = I_{11} + I_{22}$, $I_{11} = I_{22}$ $\rightarrow$ $I_{22} = I_{11}$, $I_{33} = 2I_{11}$

$$I_{11} = \int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{a}{2}}^{\frac{a}{2}} dy \left(\frac{M}{a^2}\right) y^2 = \frac{M}{a^2} a \left.\frac{y^3}{3}\right|_{-\frac{a}{2}}^{\frac{a}{2}} = \boxed{\frac{Ma^2}{12}}$$

b) $\vec{L}_o = \mathbb{I}_o \vec{\omega}$, $\vec{\omega} = \omega\cos\alpha\,\hat{n} + \omega\operatorname{sen}\alpha\,\hat{u}$

$$I_o^{\{\hat{u},\hat{v},\hat{n}\}} = \frac{Ma^2}{12}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow \boxed{\vec{L}_o = \frac{Ma^2}{12}\omega\left(\operatorname{sen}\alpha\,\hat{u} + 2\cos\alpha\,\hat{n}\right)}$$

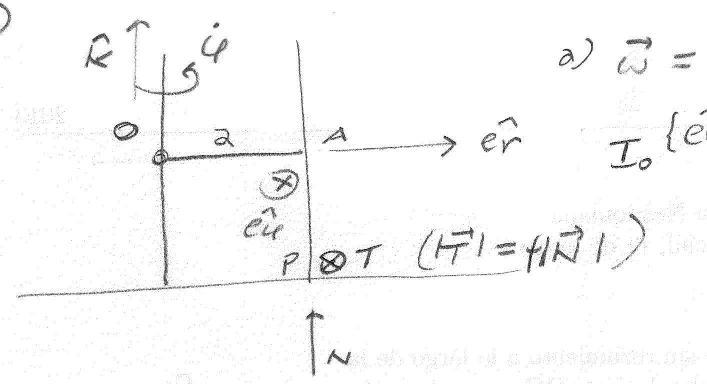
$$\vec{\omega}\cdot\vec{L}_o = |\vec{\omega}||\vec{L}_o|\cos\theta \longleftrightarrow \theta = \operatorname{Arcos}\left(\frac{\vec{\omega}\cdot\vec{L}_o}{\omega|\vec{L}_o|}\right)$$

$$\boxed{\theta = \operatorname{Arcos}\left(\frac{1+\cos^2\alpha}{\sqrt{1+3\cos^2\alpha}}\right)}$$

c) $\boxed{\vec{M}_o^{(ext)} = \dot{\vec{L}}_o = \vec{\omega}\times\vec{L}_o = -\frac{Ma^2}{12}\omega^2\operatorname{sen}\alpha\cos\alpha\,\hat{v}}$ es el momento reactivo

d) $P^{(react)}_{(M_o)} = \vec{M}_o^{(ext)}\cdot\vec{\omega} = (\vec{\omega}\times\vec{L}_o)\cdot\vec{\omega} = 0 \Rightarrow$ La energía se conserva

Ej 3)



a) $\vec{\omega} = \dot\varphi \hat{k} + \dot\psi \hat{e_r}$

$$I_O\{\hat{e_r},\hat{e_\varphi},\hat{k}\} = \begin{pmatrix} \frac{ma^2}{2} & 0 & 0 \\ 0 & \frac{ma^2}{4}+ma^2 & 0 \\ 0 & 0 & \frac{ma^2}{4}+ma^2 \end{pmatrix} = \frac{ma^2}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 5/2 & 0 \\ 0 & 0 & 5/2 \end{pmatrix}$$

$$\vec{L_O} = \mathbb{I}_O\vec{\omega} = \frac{ma^2}{2}\left(\frac{5}{2}\dot\varphi\hat{k} + \dot\psi\hat{e_r}\right)$$

b) $$\dot{\vec{L_O}} = \vec{M}_O^{(ext)} = a(\hat{e_r}-\hat{k})\times(fN\hat{e_\varphi}+N\hat{k}) + amg\,\hat{e_\varphi}$$

$$= afN\hat{k} - aN\hat{e_\varphi} + afN\hat{e_r} + amg\,\hat{e_\varphi}$$

$$\dot{\vec{L_O}} = afN\hat{e_r} + a(mg-N)\hat{e_\varphi} + afN\hat{k}$$

$$\dot{\vec{L_O}} = \frac{ma^2}{2}\left(\frac{5}{2}\ddot\varphi\hat{k} + \ddot\psi\hat{e_r} + \dot\varphi\dot\psi\hat{e_\varphi}\right)$$

$$\rightarrow \begin{cases} afN = \frac{ma^2}{2}\ddot\psi & \text{(i)} \\ a(mg-N) = \frac{ma^2}{2}\dot\varphi\dot\psi & \text{(ii)} \\ afN = \frac{ma^2}{2}\left(\frac{5}{2}\ddot\varphi\right) & \text{(iii)} \end{cases}$$

de (i) y (iii) $\ddot\psi = \frac{5}{2}\ddot\varphi \xrightarrow{\text{integro en } t} \boxed{\dot\psi = \frac{5}{2}\dot\varphi}$ (iv) ; sust. (iv) y (iii) en (ii):

$$amg - \frac{1}{f}\frac{ma^2}{2}\left(\frac{5}{2}\ddot\varphi\right) = \frac{ma^2}{2}\dot\varphi\left(\frac{5}{2}\dot\varphi\right)$$

$$\boxed{\ddot\varphi + f\dot\varphi^2 = \frac{4}{5}f\,g/a}$$

c) $u(\varphi) = \dot\varphi^2 \Rightarrow \frac{du}{dt} = \frac{du}{d\varphi}\dot\varphi = 2\dot\varphi\ddot\varphi \;\therefore\; \ddot\varphi = \frac{1}{2}\frac{du}{d\varphi}$

$\Rightarrow \frac{1}{2}\frac{du}{d\varphi} + fu = \frac{4}{5}f\,g/a$ homog. $u_H(\varphi) = Ae^{-2f\varphi}$; $u_P = \frac{4}{5}g/a$

$u(\varphi) = Ae^{-2f\varphi} + \frac{4}{5}g/a$; $u(0) = 0 \;\therefore\; A = -\frac{4}{5}g/a$

$$\Rightarrow \boxed{\dot\varphi^2 = \frac{4}{5}g/a\left(1 - e^{-2f\varphi}\right)}$$

d) $\vec{v_P}^{(\text{rígido})} = a(\dot\varphi + \dot\psi)\hat{e_\varphi} \overset{(iv)}{=} a\frac{7}{2}\dot\varphi\hat{e_\varphi}$

$\vec{v_P}^{(\text{plano})} = a\Omega\hat{e_\varphi}$

para que siempre haya deslizamiento:

$$\Omega > \frac{7}{2}\dot\varphi \;\; \forall\varphi \iff \Omega > \max\left(\frac{7}{2}\dot\varphi\right)$$

$$\boxed{\Omega > \frac{7}{2}\sqrt{\frac{4}{5}g/a}}\,.$$