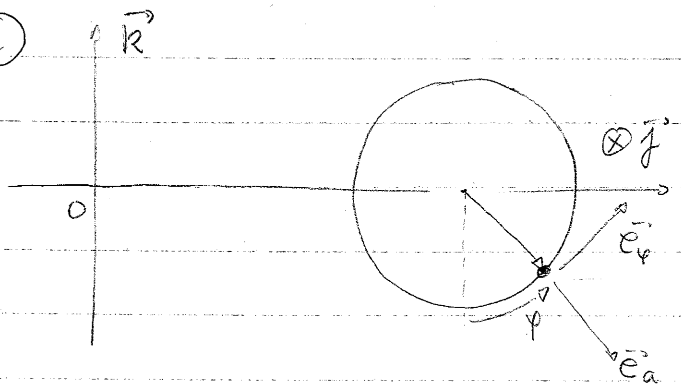
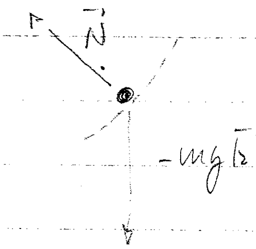
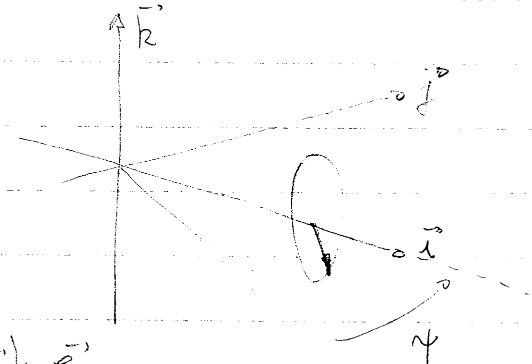
①





ⓐ



$$\vec{L}_o = \vec{r} \times \vec{p}$$

$$\vec{r} = b\vec{i} + a\vec{e}_a$$

$$\vec{v} = b\dot{\psi}\vec{j} + a\dot{\vec{e}}_a =$$

$$= (b + a\,\text{sen}\varphi)\dot{\psi}\vec{j} + a\dot{\varphi}\vec{e}_\varphi \qquad \dot{\vec{e}}_a = (\dot{\psi}\vec{k} - \dot{\varphi}\vec{j}) \times \vec{e}_a =$$

$$= \dot{\psi}\,\text{sen}\varphi\,\vec{j} + \dot{\varphi}\vec{e}_\varphi$$

$$\vec{L}_o = (b\vec{i} + a\vec{e}_a) \times m\left[(b + a\,\text{sen}\varphi)\dot{\psi}\vec{j} + a\dot{\varphi}\vec{e}_\varphi\right] = mb(b + a\,\text{sen}\varphi)\dot{\psi}\vec{k} +$$

$$- ab\dot{\varphi}\,\text{sen}\varphi\,\vec{j} + ma(b + a\,\text{sen}\varphi)\dot{\psi}\vec{e}_\varphi - ma^2\dot{\varphi}\vec{j}$$

$$\vec{L}_o \cdot \vec{k} = mb(b + a\,\text{sen}\varphi)\dot{\psi} + ma(b + a\,\text{sen}\varphi)\dot{\psi}\,\text{sen}\varphi =$$

$$= m\dot{\psi}\left(b^2 + 2ab\,\text{sen}\varphi + a^2\text{sen}^2\varphi\right) = \underline{m\dot{\psi}(b + a\,\text{sen}\varphi)^2}$$

$m\vec{a} \cdot \vec{j} = 0$ (∄ $\vec{F}$ según $\vec{j}$)

$$\vec{a} \cdot \vec{j} = (b + a\,\text{sen}\varphi)\ddot{\psi} + a\cos\varphi\,\dot{\varphi}\dot{\psi} + a\dot{\psi}\,\dot{\vec{e}}_\varphi \cdot \vec{j} = 0$$

$$\dot{\vec{e}}_\varphi = (\dot{\psi}\vec{k} - \dot{\varphi}\vec{j}) \times \vec{e}_\varphi = \dot{\psi}\cos\varphi\,\vec{j} - \dot{\varphi}\,\vec{e}_a$$

$$\dot{\vec{e}}_\varphi \cdot \vec{j} = \dot{\psi}\cos\varphi$$

$$(b + a\,\text{sen}\varphi)\ddot{\psi} + 2a\cos\varphi\,\dot{\varphi}\dot{\psi} = 0 \qquad ⊛$$

$$\frac{d\,\vec{L}_o\cdot\vec{k}}{dt} = m\ddot{\psi}(b+a\,\text{sen}\,\varphi)^2 + 2m\dot{\psi}(b+a\,\text{sen}\,\varphi)\,a\cos\varphi\,\dot{\varphi} =$$

$$= m(b+a\,\text{sen}\,\varphi)\left((b+a\,\text{sen}\,\varphi)\ddot{\psi} + 2a\dot{\varphi}\dot{\psi}\cos\varphi\right) \doteq 0$$

(*)

(b) $$\dot{\psi}(b+a\,\text{sen}\,\varphi)^2 = \ell \qquad \left((b+a\,\text{sen}\,\varphi_0)\,\dot{\psi}_0 = v_0\right)$$

$$\boxed{\ell = (b+a\,\text{sen}\,\varphi_0)\,v_0}$$

$E_{mec.} = c^{te}$

$$E_{mec.} = \frac{1}{2}m\left((b+a\,\text{sen}\,\varphi)\dot{\psi}\,\vec{j} + a\dot{\varphi}\,\vec{e}_\varphi\right)^2 - mga\cos\varphi$$

$$E = \frac{1}{2}m\left[(b+a\,\text{sen}\,\varphi)^2\dot{\psi}^2 + a^2\dot{\varphi}^2\right] - mga\cos\varphi \qquad \boxed{E = \frac{1}{2}mv_0^2 - mga\cos\varphi_0}$$

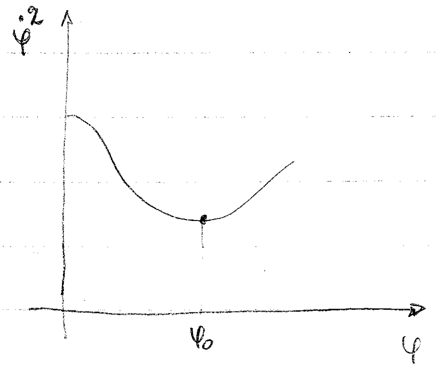
$$E = \frac{1}{2}m\left(\frac{\ell^2}{(b+a\,\text{sen}\,\varphi)^2} + a^2\dot{\varphi}^2\right) - mga\cos\varphi$$

$$\frac{2E}{m} + 2ga\cos\varphi - \frac{\ell^2}{(b+a\,\text{sen}\,\varphi)^2} = a^2\dot{\varphi}^2$$

$$a\dot{\varphi} = \sqrt{\frac{2E}{m} + 2ga\cos\varphi - \frac{\ell^2}{(b+a\,\text{sen}\,\varphi)^2}}$$

$$\boxed{\vec{v} = \frac{\ell}{(b+a\,\text{sen}\,\varphi)}\,\vec{j} + \sqrt{\frac{2E}{m} + 2ga\cos\varphi - \frac{\ell^2}{(b+a\,\text{sen}\,\varphi)^2}}\;\vec{e}_\varphi}$$

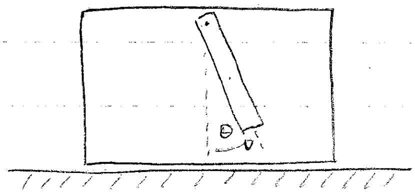
(c)



$$\left.\frac{\partial\dot{\varphi}^2}{\partial\varphi}\right|_{\varphi_0} = 0 \Rightarrow -2ga\,\text{sen}\,\varphi_0 + \frac{2\ell^2 a\cos\varphi_0}{(b+a\,\text{sen}\,\varphi_0)^3} = 0.$$

$$\boxed{\frac{v_0^2\cos\varphi_0}{(b+a\,\text{sen}\,\varphi_0)} = g\,\text{sen}\,\varphi_0}$$

② ⓐ



$E_{mec.} = E_0$

$$E_{mec.} = \overbrace{\frac{1}{2}m\left(d\dot{\theta}\right)^2 + \frac{1}{2}\,\frac{md^2}{3}\,\dot{\theta}^2}^{T} - mgd\cos\theta$$

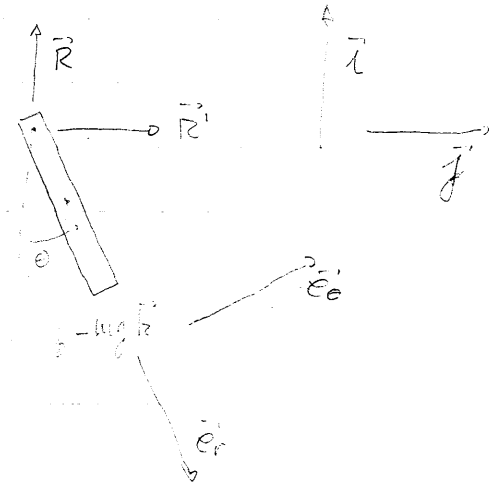
Ⓗ Bloque en reposo

$E_0 = 0$

$$\frac{2}{3}md^2\dot{\theta}^2 - mgd\cos\theta = 0$$

$\frac{\partial}{\partial t}$

$$\frac{4}{3}md^2\ddot{\theta} + mgd\,\mathrm{sen}\,\theta = 0.$$



$$m\vec{a}_G = \vec{R} + \vec{R}' \quad ; \quad \vec{a}_G = d\ddot{\theta}\,\vec{e}_\theta - d\dot{\theta}^2\vec{e}_r$$

$$m\left(d\ddot{\theta}\,\vec{e}_\theta - d\dot{\theta}^2\vec{e}_r\right) = R\,\vec{\imath} + R'\,\vec{\jmath} - mg\,\vec{\imath}$$

$(\vec{\imath})$ $-mg + R = m d\ddot{\theta}\,\mathrm{sen}\,\theta + m d\dot{\theta}^2\cos\theta$

$(\vec{\jmath})$ $R' = m d\ddot{\theta}\cos\theta - m d\dot{\theta}^2\,\mathrm{sen}\,\theta$

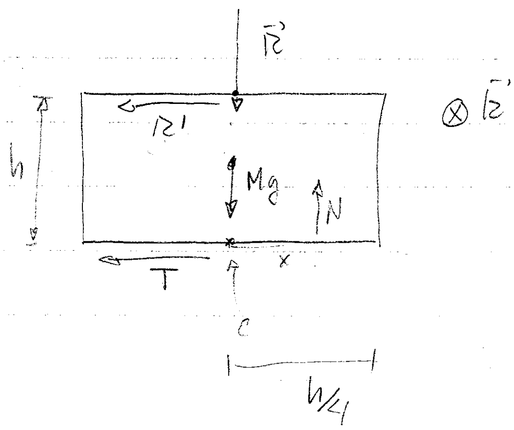
$$\ddot{\theta} = -\frac{3}{4}\,g\,\frac{\mathrm{sen}\,\theta}{d} \quad ; \quad \dot{\theta}^2 = \frac{3}{2}\,g\,\frac{\cos\theta}{d}$$

$$-mg + R = -\frac{3}{4}mg\,\mathrm{sen}^2\theta + \frac{3}{2}mg\cos^2\theta = \frac{3}{4}mg\left(2\cos^2\theta - \mathrm{sen}^2\theta\right) = \frac{3}{4}mg\left(3\cos^2\theta - 1\right)$$

$$R' = -\frac{3}{4}mg\,\mathrm{sen}\,\theta\cos\theta - \frac{3}{2}mg\,\mathrm{sen}\,\theta\cos\theta = -\frac{9}{4}mg\,\mathrm{sen}\,\theta\cos\theta$$

$$R = \frac{mg}{4}\left(9\cos^2\theta + 1\right)$$

$$\sum \vec{F}^{ext} = \emptyset$$



$$-R - Mg + N = 0 \Rightarrow N = Mg + R$$

$$-R' - T = 0 \Rightarrow T = -R'$$

$$\sum \vec{M}'^{ext}_{C} = \emptyset \Rightarrow -R'h\,\vec{k}' - Nx\,\vec{k}' = \emptyset \Rightarrow -R'h = Nx$$

$$N = Mg + \frac{1}{4} mg \left(9\cos^2\theta + 1\right) \overset{3m=M}{=} \frac{mg}{4}\left(9\cos^2\theta + 13\right)$$

$$T = \frac{9}{4} mg \operatorname{sen}\theta\cos\theta$$

No deslizamiento $\Rightarrow$ $|T| \leq \mu N \Rightarrow 9|\operatorname{sen}\theta\cos\theta| \leq \mu(9\cos^2\theta + 13)$

$$\frac{|\operatorname{sen}\theta|\cos\theta}{\frac{13}{9} + \cos^2\theta} \leq \mu$$

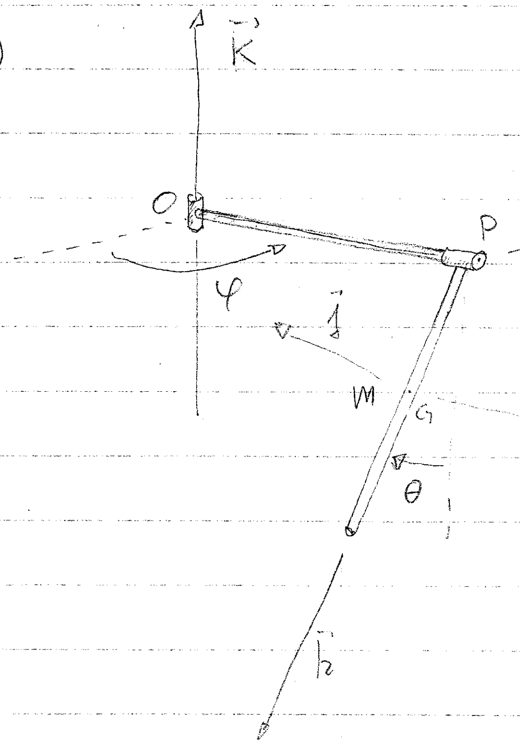
$$\boxed{\frac{|\operatorname{sen}\theta_d|\cos\theta_d}{13/9 + \cos^2\theta_d} = \mu}$$

c) No vuelco $\Rightarrow$ $x = \dfrac{9/4\, mg \operatorname{sen}\theta\cos\theta\, h}{\frac{mg}{4}(13 + 9\cos^2\theta)} = \dfrac{\operatorname{sen}\theta\cos\theta\, h}{\left(\frac{13}{9} + \cos^2\theta\right)}$

$$|x| \leq \frac{h}{4} \Rightarrow \frac{|\operatorname{sen}\theta|\cos\theta}{\frac{13}{9} + \cos^2\theta} \leq \frac{1}{4}$$

$$\boxed{\frac{|\operatorname{sen}\theta_v|\cos\theta_v}{13/9 + \cos^2\theta_v} = \frac{1}{4}}$$

③



$$\begin{cases} E_{mec} = c^{te} \\ \vec{L}_O \cdot \vec{K} = c^{te} \quad (\vec{M}^{ext} \cdot \vec{K} = 0) \end{cases}$$

$$\mathbb{I}_{G\,(i,j,k)} = \frac{md^2}{3}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$$

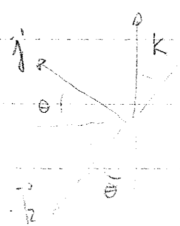
$$\vec{\Omega} = \dot{\varphi}\vec{K} - \dot{\theta}\vec{i}$$

$$\vec{v}_G = \vec{v}_P + \vec{\Omega} \times d\vec{k} =$$
$$= 2d\dot{\varphi}\,\vec{e}_\varphi + (\dot{\varphi}\vec{K} - \dot{\theta}\vec{i}) \times d\vec{k} =$$
$$= 2d\dot{\varphi}\,\vec{e}_\varphi + d\dot{\varphi}\,\mathrm{sen}\,\theta\,\vec{i} + d\dot{\theta}\,\vec{j}$$

ⓐ $$\mathbb{I}_O = \mathbb{I}_G + \begin{bmatrix} md^2 & 0 & -2md^2 \\ 0 & 5md^2 & 0 \\ -2md^2 & 0 & 4md^2 \end{bmatrix} = \begin{bmatrix} \frac{4}{3}md^2 & 0 & -2md^2 \\ 0 & \frac{16md^2}{3} & 0 \\ -2md^2 & 0 & 4md^2 \end{bmatrix}$$

ⓑ $$\vec{L}_O = \mathbb{I}_O\,\vec{\Omega} = \left(-\frac{4}{3}md^2\dot{\theta} + 2md^2\dot{\varphi}\cos\theta\right)\vec{i} + \frac{16md^2}{3}\dot{\varphi}\,\mathrm{sen}\,\theta\,\vec{j} + \left(-2md^2\dot{\theta} - 4md^2\dot{\varphi}\cos\theta\right)\vec{k}$$

$$\vec{\Omega} = \dot{\varphi}\vec{K} - \dot{\theta}\vec{i} = -\dot{\theta}\vec{i} + \dot{\varphi}\,\mathrm{sen}\,\theta\,\vec{j} - \dot{\varphi}\cos\theta\,\vec{k}$$



$$T = \frac{1}{2}\vec{\Omega}\,\mathbb{I}_O\,\vec{\Omega} = \frac{1}{2}\left(+\frac{4}{3}md^2\dot{\theta}^2 - 2md^2\dot{\varphi}\dot{\theta}\cos\theta + \frac{16}{3}md^2\dot{\varphi}^2\mathrm{sen}^2\theta - 2md^2\dot{\varphi}\dot{\theta}\cos\theta + 4md^2\dot{\varphi}^2\cos^2\theta\right)$$

$$T = \frac{md^2}{2}\left(\frac{4}{3}\dot{\theta}^2 - 4\dot{\theta}\dot{\varphi}\cos\theta + 4\dot{\varphi}^2 + \frac{4}{3}\dot{\varphi}^2\mathrm{sen}^2\theta\right)$$

(c) $U = -mgd\cos\theta$

(i) $E_{mec} = T + U = c^{te}$; $\vec{L}_0 \cdot \vec{K} = c^{te}$ $\quad (\vec{K} = \operatorname{sen}\theta\, \vec{j} - \cos\theta\, \vec{k})$

(ii) $\vec{L}_0 \cdot \vec{K} = \frac{16}{3} md^2 \dot{\varphi} \operatorname{sen}^2\theta - 2md^2\dot{\theta}\cos\theta + 4md^2\dot{\varphi}\cos^2\theta =$

$$= md^2\left(4\dot{\varphi} + \frac{4}{3}\dot{\varphi}\operatorname{sen}^2\theta - 2\dot{\theta}\cos\theta\right) = c^{te}$$

(d) $\left(t=0 \Rightarrow \theta = \pi/2,\ \dot{\theta} = 0,\ \dot{\varphi} = 0\right)$

(i) $md^2\left(\frac{2}{3}\dot{\theta}^2 - 2\dot{\theta}\dot{\varphi}\cos\theta + 2\dot{\varphi}^2 + \frac{2}{3}\dot{\varphi}^2\operatorname{sen}^2\theta\right) - mgd\cos\theta = 0$

(ii) $md^2\left(4\dot{\varphi} + \frac{4}{3}\dot{\varphi}\operatorname{sen}^2\theta - 2\dot{\theta}\cos\theta\right) = 0$

$$\theta = 0 \Rightarrow \begin{cases} 2\dot{\varphi} = \dot{\theta} \\ \frac{2}{3}\dot{\theta}^2 - 2\dot{\theta}\dot{\varphi} + 2\dot{\varphi}^2 = g/d \end{cases}$$

$$\frac{8}{3}\dot{\varphi}^2 - 4\dot{\varphi}^2 + 2\dot{\varphi}^2 = g/d$$

$$\frac{2\dot{\varphi}^2}{3} = g/d \longrightarrow \dot{\varphi}^2 = \frac{3}{2} g/d$$

$$\boxed{\vec{\Omega}_0 = \sqrt{\frac{3}{2} g/d}\,\left(\vec{K} - 2\vec{\imath}\right)}$$