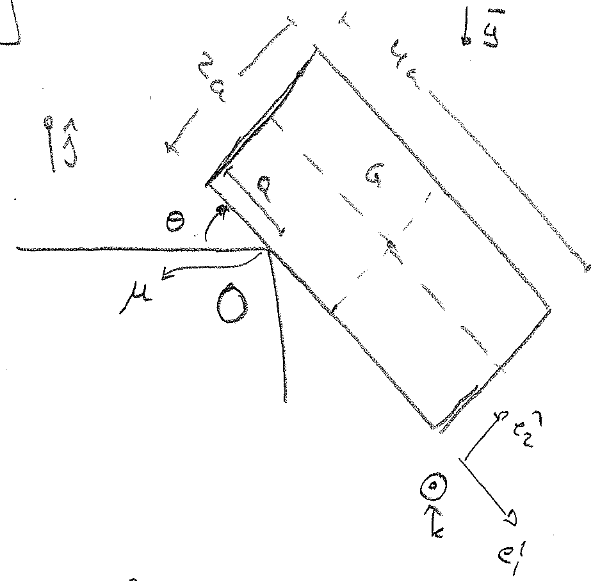
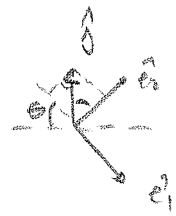
1





$\hat{j} = \cos\theta\,\hat{e}_2 - \sin\theta\,\hat{e}_1$

a) $I_G^{\{\hat{e}_1,\hat{e}_2,\hat{k}\}} = \frac{4m}{3}a^2 \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{5}{4} \end{pmatrix}$

$I_O = I_G + \mathbb{J}_G^{M,O}$ $\qquad \mathbb{J}_G^{M,G} = ma^2 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$\vec{r}_G - \vec{r}_O = a\hat{e}_1 + a\hat{e}_2$

$\mathbb{I}_O = ma^2 \begin{pmatrix} \frac{4}{3} & -1 & 0 \\ -1 & \frac{7}{3} & 0 \\ 0 & 0 & \frac{11}{3} \end{pmatrix}$

b) Mientras no deslice. (También vale usar conservación de la Energía)

$\vec{\omega} = -\dot{\theta}\hat{k} \qquad \vec{L}_O = -\frac{11}{3}ma^2\dot{\theta}\hat{k} \rightarrow \frac{d\vec{L}_O}{dt} = -\frac{11}{3}ma^2\ddot{\theta}\hat{k}$

$\vec{a}_O = 0 \rightarrow \frac{d\vec{L}_O}{dt} = \vec{M}_O^{(ext)} \qquad \vec{M}_O^{(ext)} = (a\hat{e}_1 + a\hat{e}_2) \wedge -mg\hat{j}$

$\vec{M}_O^{(ext)} = mga(-\cos\theta\hat{k} - \sin\theta\hat{k})$

$\rightarrow -\frac{11}{3}ma^2\ddot{\theta} = -mga(\cos\theta + \sin\theta) \rightarrow \boxed{\ddot{\theta} = \frac{3}{11}\frac{g}{a}(\cos\theta + \sin\theta)}$

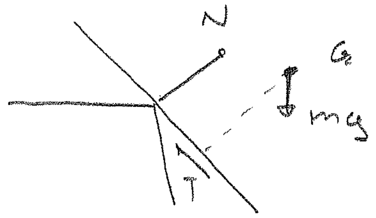
c) $\vec{r}_G = a(\hat{e}_1 + \hat{e}_2)$

$$\vec{v}_G = a(\dot{\hat{e}}_1 + \dot{\hat{e}}_2) = -a\dot{\theta}(\hat{e}_2 - \hat{e}_1)$$

$$\vec{a}_G = -a\ddot{\theta}(\hat{e}_2 - \hat{e}_1) - a\dot{\theta}(\dot{\hat{e}}_2 - \dot{\hat{e}}_1)$$

$$= -a\ddot{\theta}(\hat{e}_2 - \hat{e}_1) + a\dot{\theta}^2(-\hat{e}_1 - \hat{e}_2)$$

Fuerzas:



$$\vec{F} = N\hat{e}_2 - T\hat{e}_1 - mg\hat{j}$$
$$= (mg\sin\theta - T)\hat{e}_1 + (N - mg\cos\theta)\hat{e}_2$$

1ra cardinal:

$$ma(\ddot{\theta} - \dot{\theta}^2) = mg\sin\theta - T$$

$$-ma(\ddot{\theta} + \dot{\theta}^2) = N - mg\cos\theta$$

En el instante inicial $\dot{\theta}(0) = 0$ $\theta = 0 \rightarrow \sin\theta = 0$ $\cos\theta = 1$

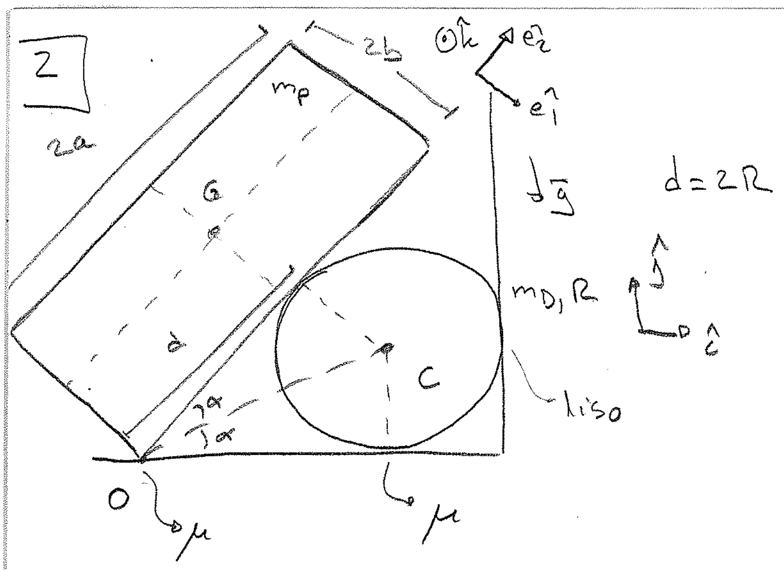
$$\ddot{\theta}(0) = \frac{3}{11}\frac{g}{a} \rightarrow ma\frac{3}{11}\frac{g}{a} = -T \rightarrow \boxed{T = -\frac{3}{11}mg}$$

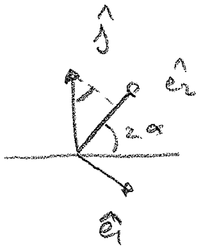
$$N = mg\left(1 - \frac{3}{11}\right) = \frac{8}{11}mg \quad N > 0 \checkmark$$

$$|\vec{T}| \leq \mu|\vec{N}| \rightarrow \frac{3}{11}mg \leq \mu\frac{8}{11}mg \rightarrow \boxed{\mu \geq \frac{3}{8}}$$

$$\boxed{\mu_{min} = \frac{3}{8}}$$

2

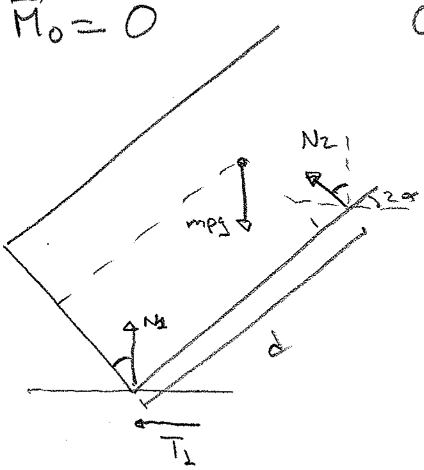




$$\hat{j} = -\cos(2\alpha)\,\hat{e}_1 + \sin(2\alpha)\,\hat{e}_2$$

a) Cardinales para la placa:

$\vec{M}_O = 0$



$$0 = \vec{M}_O^{(ext)} = (a\hat{e}_2 - b\hat{e}_1) \wedge (-m_p g\,\hat{j}) + N_2 d\,\hat{k}$$

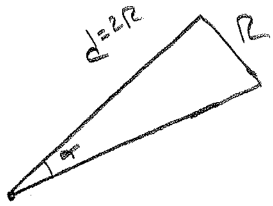
$$0 = -m_p g\left(+a\cos(2\alpha)\hat{k} - b\sin(2\alpha)\hat{k}\right) + N_2 d\,\hat{k}$$

$$N_2 d = m_p g\left(a\cos(2\alpha) - b\sin(2\alpha)\right)$$

$$N_2 = \frac{m_p g}{d}\left(a\cos(2\alpha) - b\sin(2\alpha)\right)$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$



$$\sin\alpha = \frac{R}{\sqrt{5}R} = \frac{1}{\sqrt{5}} \qquad \sin(2\alpha) = \frac{4}{5}$$

$$\Rightarrow$$

$$\cos\alpha = \frac{2R}{\sqrt{5}R} = \frac{2}{\sqrt{5}} \qquad \cos(2\alpha) = \frac{3}{5}$$

$$N_2 = \frac{m_p g}{2R}\left(\frac{3}{5}a - \frac{4}{5}b\right) \rightarrow N_2 = \frac{m_p g}{10R}\left(3a - 4b\right) > 0$$

$$\hookrightarrow \boxed{3a > 4b}$$

1ra cardinal:

$\hat{i}$) $-T_1 - N_2\sin(2\alpha) = 0 \rightarrow T_1 = -\frac{4}{5}N_2$

$\hat{j}$) $N_1 + N_2\cos(2\alpha) - m_p g = 0 \qquad N_1 = m_p g - \frac{3}{5}N_2$

$$T_1 = -\frac{4}{5}\frac{m_p g}{10R}(3a - 4b)$$

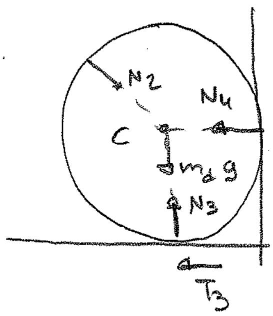
$$N_1 = m_p g\left(1 - \frac{3}{5}\frac{1}{10R}(3a - 4b)\right)$$

$N_1 > 0 \rightarrow \frac{3}{50R}(3a-4b) < 1$

$|\vec{T}_1| \leq \mu |\vec{N}_1| \rightarrow \frac{4}{5}\frac{m_P g}{10R}(3a-4b) \leq m_P g \mu \left(1 - \frac{3}{5}\frac{1}{10R}(3a-4b)\right)$

$\frac{m_P g}{50R}\left(4(3a-4b) + 3(3a-4b)\mu\right) \leq m_P g \mu$

$(3a-4b) \leq \frac{50R\mu}{4+3\mu}$



b) 2ᵈᵃ en C

$\vec{M}_c^{ext} = RT_3(-\hat{k}) = 0 \rightarrow \boxed{T_3 = 0}$

1ʳᵃ cardinal. $N_3 - m_d g - N_2\cos(2\alpha) = 0$

$N_2\sin(2\alpha) - N_4 = 0$

$N_4 = \frac{4}{5}N_2 = \frac{4}{5}\frac{m_P g}{10R}(3a-4b)$ $\quad N_4 > 0$ pues $N_2 > 0$

es la misma condición

$N_3 = N_2\frac{3}{5} + m_d g$ $\quad N_3 = \frac{3}{5}\frac{m_P g}{10R}(3a-4b) + m_d g > 0$

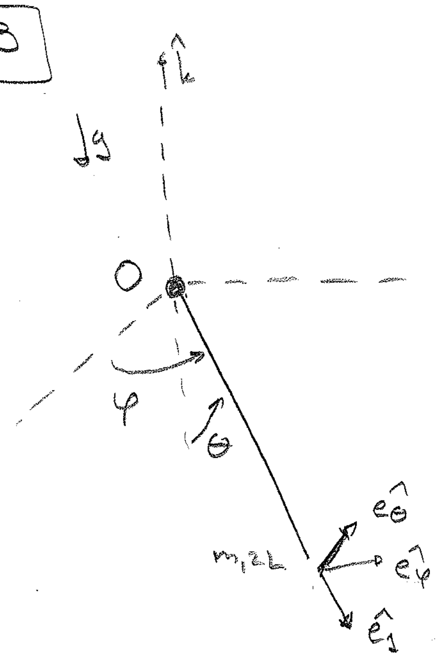
pues $3a > 4b$.

Resumen: $\boxed{3a > 4b}$

$\boxed{(3a-4b) < \frac{50R}{3}}$

y $(3a-4b) \leq \frac{50R\mu}{4+3\mu}$

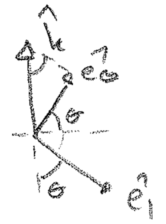
3



C.I. $\vec{v}_G(0) = v_1 \hat{e}_\varphi + v_2 \hat{e}_\theta \quad v_1, v_2 > 0$

$\varphi(0) = 0 \quad \theta(0) = \frac{\pi}{4}$

En O tenemos una articulación esférica lisa.



$\hat{k} = \sin\theta\, \hat{e}_\theta - \cos\theta\, \hat{e}_1$

a) Sabemos que $\vec{M}_O^{ext} = 0$ la única fuerza que hace momento desde O es el peso $\Rightarrow \vec{M}_O^{ext} = L\hat{e}_1 \wedge (-mg\hat{k})$

$\vec{M}_O^{ext} = +mgL\sin\theta\, \hat{e}_\varphi$

$\Rightarrow \vec{M}_O^{(ext)} \cdot \hat{k} = 0$

2da Cardinal

$$\frac{d\vec{L}_O}{dt} = \vec{M}_O^{ext} \quad \Rightarrow \frac{d(\vec{L}_O\cdot\hat{k})}{dt} = \underbrace{\frac{d\vec{L}_O}{dt}}_{\vec{M}_O^{ext}}\cdot\hat{k} + \vec{L}_O\cdot\overset{0}{\overset{\shortparallel}{\frac{d\hat{k}}{dt}}} = \vec{M}_O^{ext}\cdot\hat{k} = 0$$

$\Rightarrow \vec{L}_O \cdot \hat{k}$ es constante en el tiempo (cantidad conservada)

Potencia de Fuerza Neta sin incluir el peso que es conservativo.

$$P = \underbrace{\vec{F}\cdot\vec{v}_O}_{\substack{\shortparallel \\ 0 \text{ pues } \vec{v}_O = 0}} + \vec{M}_O\cdot\vec{\omega}$$

$\vec{M}_O^{ext}\cdot\vec{\omega} = 0$ pues el peso es el único que hace momentos desde O

$\Rightarrow P = 0 \quad \Rightarrow$ solo tengo fuerzas conservativas o de potencia nula $\Rightarrow$ se conserva la energía E

$E = T + U$ cte en el tiempo.

$\vec{L}_0 = \mathbb{I}_0 \vec{\omega}$ $\quad \vec{\omega} = \dot{\varphi}\hat{k} - \dot{\theta}\hat{e_\varphi}$

$\vec{\omega} = -\dot{\varphi}\cos\theta\,\hat{e_1} + \dot{\varphi}\sin\theta\,\hat{e_\theta} - \dot{\theta}\,\hat{e_\varphi}$

$$\mathbb{I}_0^{\{\hat{e_1}, \hat{e_\varphi}, \hat{e_\theta}\}} = \frac{4ML^2}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vec{L}_0 = \frac{4}{3}ML^2\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\dot{\varphi}\cos\theta \\ -\dot{\theta} \\ \dot{\varphi}\sin\theta \end{pmatrix} \Rightarrow \vec{L}_0 = \frac{4}{3}mL^2\left(-\dot{\theta}\,\hat{e_\varphi} + \dot{\varphi}\sin\theta\,\hat{e_\theta}\right)$$

$$\vec{L}_0 \cdot \hat{k} = \frac{4}{3}mL^2\dot{\varphi}\sin^2\theta = L_z \rightarrow \frac{dL_z}{dt} = 0 = \frac{4}{3}mL^2\left(\ddot{\varphi}\sin^2\theta + 2\dot{\varphi}\dot{\theta}\sin\theta\cos\theta\right)$$

$$\rightarrow \dot{\varphi} = \frac{3}{4}\frac{L_z}{mL^2}\frac{1}{\sin^2\theta}$$

Energía: $E = T + U \quad T = \frac{1}{2}\vec{\omega}\cdot\mathbb{I}_0\vec{\omega} \quad U = -mgL\cos\theta$

$$T = \left(-\dot{\varphi}\cos\theta\,\hat{e_1} + \dot{\varphi}\sin\theta\,\hat{e_\theta} - \dot{\theta}\,\hat{e_\varphi}\right)\cdot\left(\frac{4}{3}mL^2\right)\left(\dot{\varphi}\sin\theta\,\hat{e_\theta} - \dot{\theta}\,\hat{e_\varphi}\right)\frac{1}{2}$$

$$= \frac{2}{3}mL^2\left(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2\right)$$

$$E = \frac{2}{3}mL^2\left(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2\right) - mgL\cos\theta = E_0$$

condición inicial.

$\vec{r}_G = L\hat{e_1}$

$$\vec{v}_G = L\dot{\hat{e}}_1 = L\dot{\varphi}\sin\theta\,\hat{e_\varphi} + L\dot{\theta}\,\hat{e_\theta} \rightarrow L\dot{\varphi}_0\sin(\theta(0)) = v_1$$

$$L\dot{\theta}_0 = v_2$$

$\dot{\varphi}_0 \frac{L}{\sqrt{2}} = v_1 \qquad v_2 = L\dot{\theta}_0$

$$L_z = \frac{4}{3} m L^2 \dot{\varphi}_0 \sin^2(\theta(0)) = \frac{4}{3} m L^2 \sqrt{2}\frac{v_1}{L}\frac{1}{2}$$

$$\boxed{L_z = \frac{2\sqrt{2}}{3} v_1 L m}$$

$$\rightarrow \dot{\varphi} = \frac{3}{4}\frac{L_z}{mL^2}\frac{1}{\sin^2\theta} \rightarrow \dot{\varphi} = \frac{\sqrt{2}}{2}\frac{v_1}{L}\frac{1}{\sin^2\theta}$$

$$E_0 = \frac{2}{3} m L^2 \left(\dot{\varphi}_0^2 \frac{1}{2} + \dot{\theta}_0^2 \right) - \frac{mgL}{\sqrt{2}}$$

$$= \frac{2}{3} m L^2 \left(\frac{v_1^2}{L^2} + \frac{v_2^2}{L^2} \right) - \frac{mgL}{\sqrt{2}}$$

$$\boxed{E_0 = \frac{2}{3} m \left(v_1^2 + v_2^2 \right) - \frac{mgL}{\sqrt{2}}}$$

$$E = \frac{2}{3} m L^2 \left(\dot{\varphi}^2 \sin^2\theta + \dot{\theta}^2 \right) - mgL\cos\theta = E_0$$

$$E_0 = \frac{2}{3} m L^2 \left(\left(\frac{3}{4}\frac{L_z}{mL^2} \right)^2 \frac{1}{\sin^2\theta} + \dot{\theta}^2 \right) - mgL\cos\theta$$

$$\rightarrow \boxed{\dot{\theta}^2 = \frac{3}{2}\frac{E_0}{mL^2} + \frac{3}{2}\frac{g}{L}\cos\theta - \frac{9}{16}\frac{L_z^2}{m^2L^4}\frac{1}{\sin^2\theta}}$$

resta sustituir E_0 y L_z.