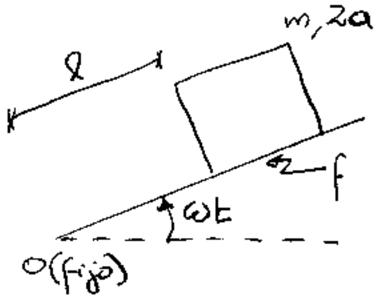
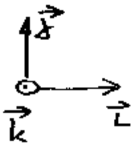
Ejercicio N°1 $\quad$ 2° Parcial Mecánica Newtoniana (5/7/2018) (1122) $\quad$ 1/6

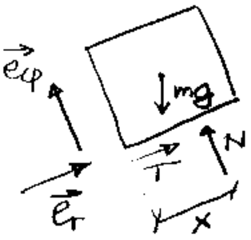


$\downarrow g$



$\dot{\ell}(0) = 0$

parte a:



1ª cardinal: $M\vec{a}_G = \vec{R}^{(ext)}$

$\vec{r}_G = (\ell+a)\vec{e}_r + a\vec{e}_\varphi$

$\vec{v}_G = (\ell+a)\omega\vec{e}_\varphi - a\omega\vec{e}_r$

$\vec{a}_G = -(\ell+a)\omega^2\vec{e}_r - a\omega^2\vec{e}_\varphi$

$\vec{R}^{(ext)} = T\vec{e}_r + N\vec{e}_\varphi - mg\vec{j}$

$\Rightarrow -m(\ell+a)\omega^2 = T - mg\underbrace{\vec{j}\cdot\vec{e}_r}_{\operatorname{sen}\omega t} \Rightarrow T = mg\operatorname{sen}\omega t - m(\ell+a)\omega^2$

en $t=0 \Rightarrow T = -m(\ell+a)\omega^2$

$-ma\omega^2 = N - mg\underbrace{\vec{j}\cdot\vec{e}_\varphi}_{\cos\omega t} \Rightarrow N = mg\cos\omega t - ma\omega^2$

en $t=0 \Rightarrow N = mg - ma\omega^2$

2ª cardinal en G: $\dot{\vec{L}}_G = \vec{M}_G^{(ext)}$

$\vec{L}_G = \mathbb{I}_G\vec{\omega} = I_{G,k}\,\omega\vec{k} \Rightarrow \dot{\vec{L}}_G = 0 \Rightarrow \vec{M}_G^{(ext)} = 0$

$(x-a)N + Ta = 0$

$x = \dfrac{N-T}{N}a$

En $t=0$: $x = \dfrac{\cancel{m}g - \cancel{m}a\omega^2 + \cancel{m}(\ell+\cancel{a})\omega^2}{\cancel{m}(g-a\omega^2)}\cdot a \Rightarrow x = \dfrac{g+\ell\omega^2}{g-a\omega^2}\cdot a$

Condición de no desprendimiento: $N \geq 0 \Rightarrow g \geq a\omega^2 \sim \boxed{\omega \leq \sqrt{\dfrac{g}{a}}}$

" $\quad$ " $\quad$ " deslizamiento: $|T| \leq f|N| = fN \Rightarrow f \geq \dfrac{\cancel{m}(\ell+a)\omega^2}{\cancel{m}(g-a\omega^2)}$

$\Downarrow$

$\boxed{f \geq \dfrac{(\ell+a)\omega^2}{g-a\omega^2}}$

" $\quad$ " $\quad$ " vuelco: $0 \leq x \leq 2a$

Se verifica siempre que $N \geq 0$

$\Rightarrow x \leq 2a \Rightarrow \dfrac{g+\ell\omega^2}{g-a\omega^2}\cancel{a} \leq 2\cancel{a} \Rightarrow \cancel{g} + \ell\omega^2 \leq \cancel{2}g - 2a\omega^2$

$\boxed{\ell \leq \dfrac{g}{\omega^2} - 2a}$

parte b: Ahora la placa desliza $\Rightarrow \ell = \ell(t)$ (con $\dot{\ell}(0) = 0$)

$\vec{v}_G = \dot{\ell}\vec{e}_r + (\ell+a)\omega\vec{e}_\varphi - a\omega\vec{e}_r$

$\vec{a}_G = \ddot{\ell}\vec{e}_r + 2\dot{\ell}\omega\vec{e}_\varphi - (\ell+a)\omega^2\vec{e}_r - a\omega^2\vec{e}_\varphi$

$$\Rightarrow m\ddot{\ell} - m(\ell+a)\omega^2 = T - mg\,\mathrm{sen}\,\omega t$$

$$2m\dot{\ell}\omega - ma\omega^2 = N - mg\cos\omega t \Rightarrow N = mg\cos\omega t + 2m\dot{\ell}\omega - ma\omega^2$$

En $t=0$: $N = m(g - a\omega^2) \geq 0 \Rightarrow g \geq a\omega^2 \sim \boxed{\omega \leq \sqrt{\frac{g}{a}}}$

Condición de no desprendimiento — Misma condición de antes

Hay deslizamiento: $|T| = f|N| = fm(g - a\omega^2) \geq 0$

La fuerza de rozamiento se opone al movimiento; y como en la parte anterior $T<0$, suponemos $T<0$ y $\ddot{\ell}>0$

$$\cancel{m}\ddot{\ell} - \cancel{m}(\ell+a)\omega^2 = -f\cancel{m}(g - a\omega^2) \quad (\text{en } t=0)$$

$$\ddot{\ell} = (\ell+a)\omega^2 - f(g - a\omega^2)$$

$$\ddot{\ell} > 0 \Rightarrow (\ell+a)\omega^2 > f(g - a\omega^2) \Rightarrow \boxed{f < \frac{(\ell+a)\omega^2}{g - a\omega^2}}$$ Condición de deslizamiento

NOTA: Observar que si se supone $T>0$ (que debería corresponder a $\ddot{\ell}<0$) da: $\ddot{\ell} = (\ell+a)\omega^2 + f(g - a\omega^2) > 0$ (si no hay desprendimiento). Lo que es una inconsistencia. El deslizamiento partiendo del reposo solo es con ℓ creciente.

O sea, no se cumple la condición de no deslizamiento de la parte anterior, lo que se quiere así sea para que deslice

2^a cardinal: $\dot{\vec{L}}_G = \vec{M}_G^{(ext)} = [(x-a)N + Ta]\vec{k}$

$$\overset{\shortparallel}{0} \quad (\vec{L}_G = \mathbb{I}_G\vec{\omega} = I_{G,\vec{k}}\,\omega\vec{k} \text{ como antes})$$

$$\Rightarrow x = \frac{N-T}{N}a = \frac{N+fN}{N}a = (f+1)a$$

Condición de no vuelco: $\underbrace{0 \leq}_{\text{se cumple siempre}} x \leq 2a \Rightarrow \cancel{a}(f+1) \leq 2\cancel{a}$

$$\Downarrow$$

$$\boxed{f \leq 1}$$

Resumen: parte a

$$\omega \leq \sqrt{\frac{g}{a}}$$

$$f \geq \frac{(\ell+a)\omega^2}{g - a\omega^2}$$

$$\ell \leq \frac{g}{\omega^2} - 2a$$

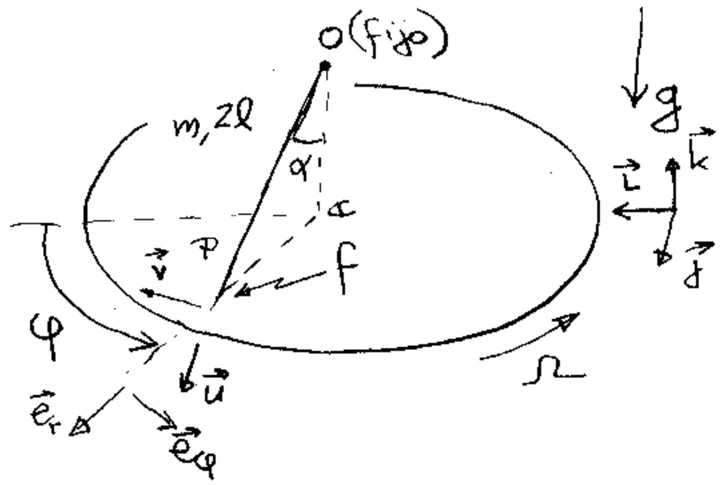
parte b:

$$\omega \leq \sqrt{\frac{g}{a}}$$

$$f < \frac{(\ell+a)\omega^2}{g - a\omega^2}$$

$$f \leq 1$$

Ejercicio Nº 2



$\dot\varphi(0)=0$

parte a: O fijo $\Rightarrow \dot{\vec L}_O = \vec M_O^{(ext)}$

$\vec L_O = \mathbb{I}_O\vec\omega$ con $\vec\omega = \dot\varphi\vec k$

Ejes principales de la barra: $\vec v, \vec u, \vec e_\varphi$ ← base directa

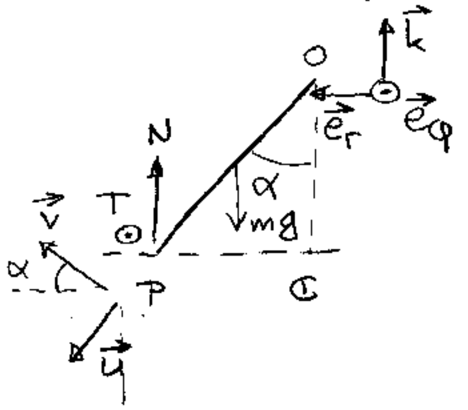
$\vec k = \operatorname{sen}\alpha\,\vec v - \cos\alpha\,\vec u$

$\vec L_O = \dot\varphi\operatorname{sen}\alpha\, I_{O,\vec v}\vec v - \dot\varphi\cos\alpha\, \underset{0}{\underset{\shortparallel}{I_{O,\vec u}}}\vec u$

$I_{O,\vec v} = \underset{\frac{m(2l)^2}{12}=\frac{ml^2}{3}}{\underset{\shortparallel}{I_{G,\vec v}}} + ml^2 = \frac{4ml^2}{3}$

$\Rightarrow \vec L_O = \frac{4ml^2}{3}\operatorname{sen}\alpha\,\dot\varphi\,\vec v$

Plano OPC (de la barra):



$$\dot{\vec L}_O = \frac{4ml^2}{3}\operatorname{sen}\alpha\,\ddot\varphi\,\vec v + \frac{4ml^2}{3}\operatorname{sen}\alpha\,\dot\varphi\,\dot{\vec v}$$

$$\vec v = \cos\alpha\,\vec e_r + \operatorname{sen}\alpha\,\vec k \Rightarrow \dot{\vec v} = \cos\alpha\,\dot{\vec e}_r = \cos\alpha\,\dot\varphi\,\vec e_\varphi$$

$$\Rightarrow \dot{\vec L}_O = \frac{4ml^2}{3}\operatorname{sen}\alpha\,\ddot\varphi\,\vec v + \frac{4ml^2}{3}\operatorname{sen}\alpha\cos\alpha\,\dot\varphi^2\,\vec e_\varphi$$

$$\vec M_O^{(ext)} = \underbrace{(\vec r_P - \vec r_O)}_{2l\vec u}\wedge T\vec e_\varphi + \underbrace{(\vec r_P - \vec r_O)}_{2l\vec u}\wedge N\vec k + \underbrace{(\vec r_G - \vec r_O)}_{l\vec u}\wedge(-mg\vec k)$$

La velocidad relativa de P respecto al plano es:

$$\vec v_R = \vec v_A - \vec v_T = 2l\operatorname{sen}\alpha\,\dot\varphi\,\vec e_\varphi - 2l\operatorname{sen}\alpha\,\Omega\,\vec e_\varphi = 2l\operatorname{sen}\alpha(\dot\varphi - \Omega)\vec e_\varphi$$

Inicialmente $\dot\varphi < \Omega \Rightarrow$ en un entorno del instante inicial

$\vec v_R\cdot\vec e_\varphi < 0 \Rightarrow$ la fuerza tangencial que se opone al movimiento es $T\vec e_\varphi$ con $T \geq 0$.

$$\vec u = \operatorname{sen}\alpha\,\vec e_r - \cos\alpha\,\vec k \Rightarrow \vec u\wedge\vec e_\varphi = \vec v$$

$$\vec u\wedge\vec k = \operatorname{sen}\alpha\,\vec e_r\wedge\vec k = -\operatorname{sen}\alpha\,\vec e_\varphi$$

$$\Rightarrow \vec M_O^{(ext)} = 2lT\vec v - 2l\operatorname{sen}\alpha\,N\vec e_\varphi + l\,mg\operatorname{sen}\alpha\,\vec e_\varphi$$

$$\Rightarrow \frac{4ml^{\cancel{2}}}{3}\operatorname{sen}\alpha\,\ddot{\varphi} = 2\cancel{l}T \Rightarrow \frac{2ml\operatorname{sen}\alpha\,\ddot{\varphi}}{3} = T$$

$$\frac{4ml^{\cancel{2}}}{3}\cancel{\operatorname{sen}}\alpha\cos\alpha\,\dot{\varphi}^2 = -2l\cancel{\operatorname{sen}\alpha}N + lmg\cancel{\operatorname{sen}\alpha}$$

$$\Rightarrow N = \frac{mg}{2} - \frac{2ml\cos\alpha\,\dot{\varphi}^2}{3}$$

En un entorno del instante inicial $(\dot{\varphi}(0)=0) \Rightarrow N > 0$

$$\Rightarrow T = fN \geq 0$$

$$\Rightarrow \frac{2\cancel{m}l\operatorname{sen}\alpha\,\ddot{\varphi}}{3} = f\frac{\cancel{m}g}{2} - 2f\frac{\cancel{m}l}{3}\cos\alpha\,\dot{\varphi}^2$$

$$\boxed{\ddot{\varphi} = \frac{3fg}{4l\operatorname{sen}\alpha} - \frac{f\dot{\varphi}^2}{\operatorname{tg}\alpha}}$$

parte b: $\dot{\varphi}^2 = u(\varphi) \Rightarrow 2\cancel{\dot{\varphi}}\ddot{\varphi} = u'\cancel{\dot{\varphi}} \Rightarrow \ddot{\varphi} = \frac{u'}{2} \Rightarrow \frac{u'}{2} = \frac{3fg}{4l\operatorname{sen}\alpha} - \frac{fu}{\operatorname{tg}\alpha}$

$\Rightarrow u' + \frac{2f}{\operatorname{tg}\alpha}u = \frac{3fg}{2l\operatorname{sen}\alpha}$ ← ec. lineal a coef. constantes no homogénea de primer orden

$$\left.\begin{array}{l} \Rightarrow u = u_h + u_p \ / \ u'_h + \frac{2f}{\operatorname{tg}\alpha}u_h = 0 \\ u_h = Ae^{-\lambda\varphi} \Rightarrow u'_h = -\lambda A e^{-\lambda\varphi} \end{array}\right\} \Rightarrow -\lambda + \frac{2f}{\operatorname{tg}\alpha} = 0 \Rightarrow \lambda = \frac{2f}{\operatorname{tg}\alpha} > 0$$

$(0 < \alpha < \frac{\pi}{2})$

$$u_p = C \text{ cte} \Rightarrow u'_p = 0 \Rightarrow \frac{2fC}{\operatorname{tg}\alpha} = \frac{3fg}{2l\operatorname{sen}\alpha} \Rightarrow C = \frac{3g}{4l\cos\alpha}$$

$$\left.\begin{array}{l} \Rightarrow \dot{\varphi}^2 = u(\varphi) = Ae^{-\lambda\varphi} + C \\ \dot{\varphi}^2(0) = 0 = u(\varphi(0)) = A + C \Rightarrow A = -C \end{array}\right\} \Rightarrow \boxed{\dot{\varphi}^2 = \frac{3g}{4l\cos\alpha}\left(1 - e^{-\lambda\varphi}\right) \ / \ \lambda = \frac{2f}{\operatorname{tg}\alpha}}$$

↳ elijo $\varphi(0) = 0$

Observar: $\dot{\varphi}^2 \leq \frac{3g}{4l\cos\alpha} = \dot{\varphi}^2_{max} \Rightarrow N \geq \frac{mg}{2} - \frac{2m\cancel{l}\cancel{\cos\alpha}}{\cancel{3}}\frac{\cancel{3}g}{4\cancel{l}\cancel{\cos\alpha}} = 0$

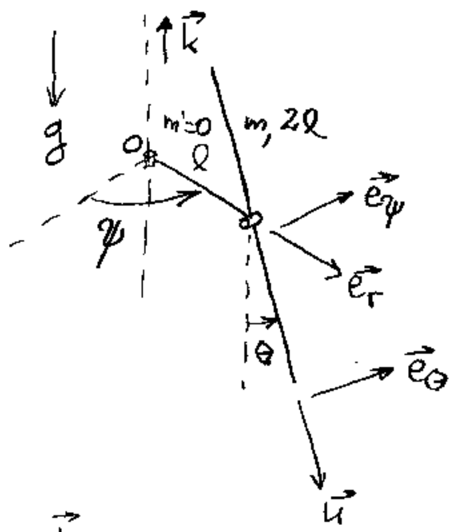
$N \geq 0 \ \forall t \Rightarrow$ la barra nunca se desprende.

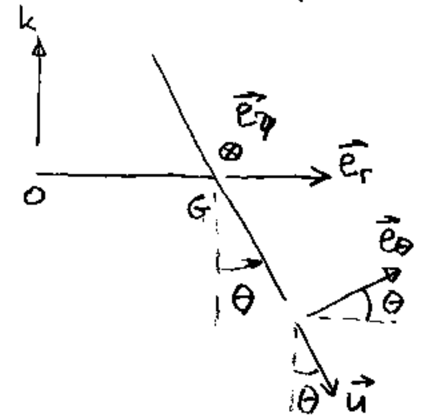
Para que siempre deslice: $\dot{\varphi} < \Omega \ \forall t$

Peor caso $\dot{\varphi}_{max} < \Omega \Rightarrow \boxed{\Omega > \sqrt{\frac{3g}{4l\cos\alpha}}}$

$= \Omega_{min}$.

Ejercicio Nº 3





parte a:

$$\vec L_O = \vec L_G + \vec p \wedge (\vec r_O - \vec r_G)$$

$$\vec L_G = \mathbb{I}_G \vec\omega$$

$$\vec\omega = \dot\psi \vec k - \dot\theta \vec e_\psi$$

$$\vec L_G = \dot\psi\, \mathbb{I}_G \vec k - \dot\theta\, \mathbb{I}_G \vec e_\psi$$

$$\vec k = -\cos\theta\, \vec u + \text{sen}\,\theta\, \vec e_\theta$$

$$\vec L_G = -\dot\psi \cos\theta \underbrace{I_{G,\vec u}}_{0} \vec u + \dot\psi\, \text{sen}\,\theta \underbrace{I_{G,\vec e_\theta}}_{\frac{m\ell^2}{3}} \vec e_\theta - \dot\theta \underbrace{I_{G,\vec e_\psi}}_{\frac{m\ell^2}{3}} \vec e_\psi$$

$$\vec L_G = \frac{m\ell^2}{3}\text{sen}\,\theta\, \dot\psi\, \vec e_\theta - \frac{m\ell^2}{3}\dot\theta\, \vec e_\psi$$

$$\left.\begin{aligned} \vec p &= m\vec v_G = m\ell\dot\psi\, \vec e_\psi \\ \vec r_O - \vec r_G &= -\ell\, \vec e_r \end{aligned}\right\} \vec p \wedge (\vec r_O - \vec r_G) = +m\ell^2\dot\psi\, \vec k$$

$$\Rightarrow \boxed{\vec L_O = \frac{m\ell^2}{3}\text{sen}\,\theta\,\dot\psi\, \vec e_\theta - \frac{m\ell^2}{3}\dot\theta\, \vec e_\psi + m\ell^2\dot\psi\, \vec k}$$

parte b: $\dot{\vec L}_O = \underbrace{\vec p \wedge \dot{O}}_{0} + \vec M_O^{(ext)} = \vec M_O^{(reac)} + (\vec r_G - \vec r_O)\wedge(-mg\vec k)$

$$\Rightarrow \dot{\vec L}_O \cdot \vec k = 0 \Rightarrow \vec L_O \cdot \vec k = cte$$

$$\vec L_O \cdot \vec k = \frac{m\ell^2}{3}\text{sen}\,\theta\,\dot\psi \underbrace{\vec e_\theta \cdot \vec k}_{\text{sen}\,\theta} + m\ell^2\dot\psi = cte = L$$

$$\frac{m\ell^2}{3}\left(3 + \text{sen}^2\theta\right)\dot\psi = L \Rightarrow \boxed{\left(3 + \text{sen}^2\theta\right)\dot\psi = \frac{3L}{m\ell^2} = L'}$$

Cantidad conservada

Ec. de mov: $\boxed{\left(3 + \text{sen}^2\theta\right)\ddot\psi + 2\,\text{sen}\,\theta\cos\theta\,\dot\theta\,\dot\psi = 0}$

EL sistema es conservativo porque las únicas fuerzas externas son el peso (conservativo) y el momento de reactivas en O (de potencia nula) $\Rightarrow T + U = E$

$$T = \underbrace{\frac{1}{2}m\vec v_G^{\,2}}_{\frac{m\ell^2\dot\psi^2}{2}} + \underbrace{\frac{1}{2}\vec\omega \cdot \mathbb{I}_G\vec\omega}_{\frac{1}{2}\left(\dot\psi\vec k - \dot\theta\,\vec e_\psi\right)\cdot\left(\frac{m\ell^2}{3}\text{sen}\,\theta\,\dot\psi\,\vec e_\theta - \frac{m\ell^2}{3}\dot\theta\,\vec e_\psi\right)}$$

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$$\Rightarrow T = \frac{m\ell^2\dot{\psi}^2}{2} + \frac{m\ell^2 \operatorname{sen}^2\theta\,\dot{\psi}^2}{6} + \frac{m\ell^2}{6}\dot{\theta}^2$$

$$U = \text{cte} \Rightarrow \frac{m\ell^2}{6}\left(3+\operatorname{sen}^2\theta\right)\dot{\psi}^2 + \frac{m\ell^2}{6}\dot{\theta}^2 = E$$

$$\boxed{\left(3+\operatorname{sen}^2\theta\right)\dot{\psi}^2 + \dot{\theta}^2 = \frac{6E}{m\ell^2} = e}$$ Cantidad conservada

$$\Rightarrow \underbrace{\cancel{2}\dot{\psi}\ddot{\psi}\left(3+\operatorname{sen}^2\theta\right)}_{-2\operatorname{sen}\theta\cos\theta\,\dot{\theta}\dot{\psi}} + \cancel{2}\operatorname{sen}\theta\cos\theta\,\dot{\theta}\,\dot{\psi} + \cancel{2}\dot{\theta}\ddot{\theta} = 0$$

$$\Rightarrow \boxed{\ddot{\theta} = \operatorname{sen}\theta\cos\theta\,\dot{\psi}^2}$$ Ec. de mov