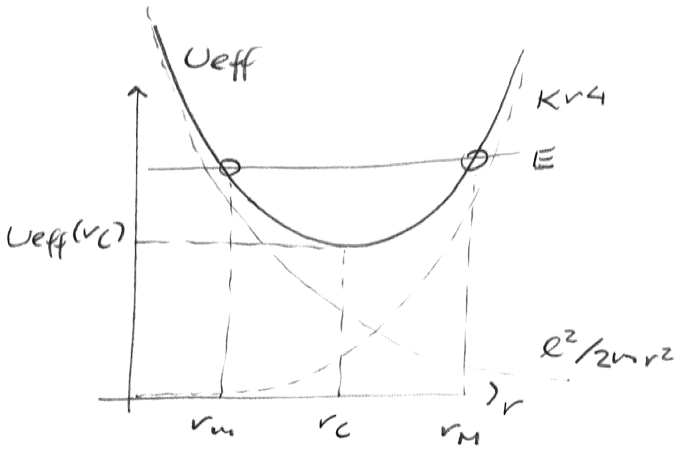
Ejercicio 1

a) $E = \frac{1}{2} m\dot{r}^2 + \underbrace{\frac{\ell^2}{2mr^2} + Kr^4}_{U_{eff}(r)}$



La trayectoria circular corresponde a $r = r_c\ \forall t$, siendo r_c donde se da el mínimo de U_{eff}, que se consigue si $E = U_{eff}(r_c)$. En este caso $r_c = a$:

$$\begin{cases} \left.\frac{dU_{eff}}{dr}\right|_{r=a} = 0 \\ E = U_{eff}(a) \end{cases}$$

$$\left.\frac{dU_{eff}}{dr}\right|_a = \left(-\frac{\ell^2}{mr^3} + 4Kr^3\right)_a = 0 : \quad \boxed{\ell^2 = 4Kma^6}$$

$$E = U_{eff}(a) = \frac{\ell^2}{2ma^2} + Ka^4 \xrightarrow[(\ell^2 = 4Kma^6)]{} \boxed{E = 3Ka^4}$$

Periodo del movimiento circular: $\ell = mr^2\dot{\theta} \underset{(r=a)}{=} ma^2\dot{\theta}$: $\dot{\theta} = \frac{\ell}{ma^2}$;

$$\dot{\theta} = 2\pi/T \leftrightarrow T = \frac{2\pi}{\dot{\theta}} = \frac{2\pi ma^2}{\ell} = \frac{2\pi ma^2}{\sqrt{4Kma^6}} \Rightarrow \boxed{T = \frac{\pi}{a}\sqrt{\frac{m}{K}}}$$

b) Los acercamientos máximo y mínimo al centro de fuerzas (r_m, r_M) corresponden a $\dot{r} = 0$:

$$E = U_{eff}(r), \text{ donde } E = 2(3Ka^4) = 6Ka^4$$

$$\Rightarrow 6Ka^4 = \frac{\ell^2}{2mr^2} + Kr^4 \underset{(\ell^2 = 4Kma^6)}{=} \frac{2Ka^6}{r^2} + Kr^4 :$$

$$\boxed{r^6 - 6a^4r^2 + 2a^6 = 0}$$ r_m, r_M son solución

Tiempo de tránsito entre r_m y r_M: $\frac{1}{2}m\dot{r}^2 = E - U_{eff}(r)$: $\dot{r} = \pm\sqrt{\frac{2}{m}(E - U_{eff})}$

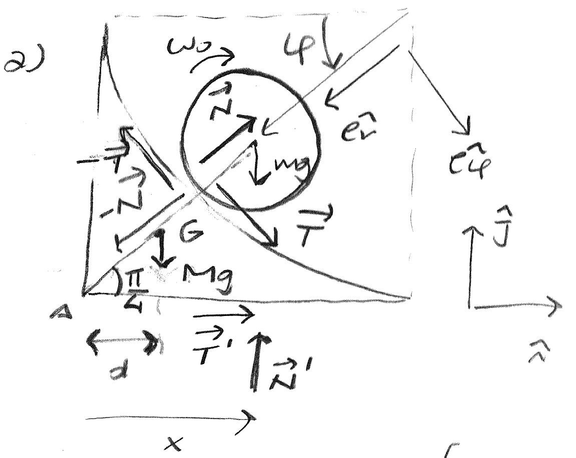
$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}(E - U_{eff}(r))} \rightarrow dt = \frac{dr}{\pm\sqrt{\frac{2}{m}(E - U_{eff}(r))}}$$

Integrando en variables separadas con un sg definido para $\dot{r}$ (≥ 0 entre r_m y r_M):

$$\boxed{T_{tránsito} = \int_{r_m}^{r_M} \frac{dr}{\sqrt{\frac{2}{m}(E - U_{eff}(r))}}}$$

Ejercicio 2

a)



Para que la placa permanezca en reposo se debe verificar:

(I) $N' \geq 0$ (no se desprende del piso)

(II) $|T'| \leq f_E |N'|$ (no desliza respecto al piso)

(III) $0 \leq x \leq R$ (no vuelca)

1era Cardinal a la placa
$$\begin{cases} \hat{\imath}) \ T' = N\cos\frac{\pi}{4} + T\,\text{sen}\,\frac{\pi}{4} = \frac{1}{\sqrt{2}}(N+T) \quad (i) \\ \hat{j}) \ N' + T\cos\frac{\pi}{4} = Mg + N\,\text{sen}\,\frac{\pi}{4} : N' = Mg + \frac{1}{\sqrt{2}}(N-T) \quad (ii) \end{cases}$$

2da Cardinal a la placa desde A : $xN' + (\sqrt{2}R - R)T = dMg$ (iii)

Fricción dinámica : $T = f_D N$ (iv)

1era Cardinal al disco en la dirección $\hat{e_r}$) $mg\,\text{sen}\,\varphi - N = -m(R-r)\dot{\varphi}^2$,

que en $t=0$ ($\varphi = \pi/4$, $\dot{\varphi} = 0$) se reduce a $N = \dfrac{mg}{\sqrt{2}} = \sqrt{2}Mg$ (v)

Sustituyendo (v) en (iv) $T = f_D\sqrt{2}Mg$ (iv'); luego (v) y (iv') en (i), (ii), (iii)

permiten despejar T', N', x
$$\begin{cases} T' = (1+f_D)Mg \\ N' = (2-f_D)Mg \\ x = \dfrac{d - f_D(\sqrt{2}-1)\sqrt{2}R}{(2-f_D)} \end{cases}$$

$$\Rightarrow \begin{cases} (I) \ (2-f_D)Mg \geq 0 : \boxed{f_D \leq 2} \\ (II) \ (1+f_D)Mg \leq f_E(2-f_D)Mg : \boxed{f_E \geq \dfrac{1+f_D}{2-f_D}} \\ (III) \ \boxed{0 \leq \dfrac{d - f_D(\sqrt{2}-1)\sqrt{2}R}{(2-f_D)} \leq R} \end{cases}$$

b) 1era Cardinal al disco según $\hat{e_r}$) nos daba: $N = mg\,\text{sen}\,\varphi + m(R-r)\dot{\varphi}^2$ (1)

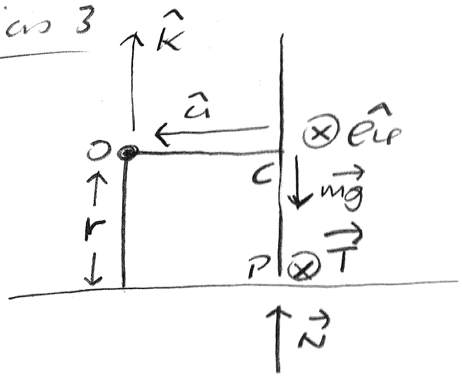
1era Cardinal al disco según $\hat{e_\varphi}$) $mg\cos\varphi + T = m(R-r)\ddot{\varphi}$ (2); usando (iv)

$mg\cos\varphi + f_D N = m(R-r)\ddot{\varphi} \xrightarrow{(1)} \boxed{mg\cos\varphi + f_D(mg\,\text{sen}\,\varphi + m(R-r)\dot{\varphi}^2) = m(R-r)\ddot{\varphi}}$

2da Cardinal al disco desde C : $\boxed{I_C\dot{\omega} = -rT = -rf_D N = -rf_D(mg\,\text{sen}\,\varphi + m(R-r)\dot{\varphi}^2)}$

$\left(I_C = \frac{1}{2}mr^2\right)$

Ejercicio 3



a) $\vec{\omega} = \dot{\varphi}\hat{k} + \dot{\psi}\hat{u}$;

$\vec{v}_P(t=0) = -r\dot{\psi}_0 \hat{e}_\varphi$: $\vec{T} = +T\hat{e}_\varphi$,

$T = fN$.

2da Cardinal al rígido desde O: $\dot{\vec{L}}_O = \vec{M}_O^{(ext)}$;

$\vec{L}_O = \mathbb{I}_O \vec{\omega}$; $I_O\{\hat{u}, \hat{e}_\varphi, \hat{k}\} = \begin{pmatrix} \frac{mr^2}{2} & 0 & 0 \\ 0 & \frac{mr^2}{4} + mr^2 & 0 \\ 0 & 0 & \frac{mr^2}{4} + mr^2 \end{pmatrix}$:

$$I_O\{\hat{u}, \hat{e}_\varphi, \hat{k}\} = \frac{mr^2}{4}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} ;$$

luego: $\vec{L}_O = \frac{mr^2}{4}(2\dot{\psi}\hat{u} + 5\dot{\varphi}\hat{k})$

$\dot{\vec{L}}_O = \frac{mr^2}{4}(2\ddot{\psi}\hat{u} + 2\dot{\psi}\dot{\hat{u}} + 5\ddot{\varphi}\hat{k})$; $\dot{\hat{u}} = \vec{\omega} \times \hat{u} = -\dot{\varphi}\hat{e}_\varphi$

$$\boxed{\dot{\vec{L}}_O = \frac{mr^2}{4}(2\ddot{\psi}\hat{u} - 2\dot{\psi}\dot{\varphi}\hat{e}_\varphi + 5\ddot{\varphi}\hat{k})} \quad (i)$$

$$\vec{M}_O^{(ext)} = \underbrace{(C-O)}_{-r\hat{u}} \times (-mg\hat{k}) + \underbrace{(P-O)}_{-r\hat{u} - r\hat{k}} \times (N\hat{k} + fN\hat{e}_\varphi)$$

$$= \boxed{r(mg-N)\hat{e}_\varphi + rfN\hat{k} - rfN\hat{u}} \quad (ii)$$

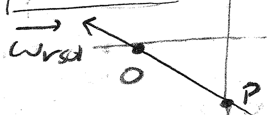
Igualando las componentes (i) y (ii):
$$\begin{cases} \frac{mr^2}{2}\ddot{\psi} = -rfN & (I) \\ -\frac{mr^2}{2}\dot{\psi}\dot{\varphi} = r(mg-N) & (II) \\ \frac{5mr^2}{4}\ddot{\varphi} = rfN & (III) \end{cases}$$

Igualando (I) y (III) $\boxed{\ddot{\psi} = -\frac{5}{2}\ddot{\varphi}}$ (A)

Eliminando N de (III) y sust. en (II): $\boxed{-\frac{mr^2}{2}\dot{\psi}\dot{\varphi} = rmg - \frac{5mr^2\ddot{\varphi}}{4f}}$ (B)

b) Cuando el rígido comienza a rodar sin deslizar, el eje instantáneo de rotación coincide con PO:

$\Leftrightarrow \boxed{\dot{\varphi}_{rsd} = \dot{\psi}_{rsd}}$; integrando (A) en el tiempo: $\dot{\psi} - \dot{\psi}_0 = -\frac{5}{2}\dot{\varphi}$



$\Rightarrow \dot{\psi}_{rsd} - \dot{\psi}_0 = -\frac{5}{2}\dot{\psi}_{rsd}$: $\dot{\psi}_{rsd} = \frac{2}{7}\dot{\psi}_0$: $\boxed{\vec{\omega}_{rsd} = \frac{2}{7}\dot{\psi}_0(\hat{u} + \hat{k})}$