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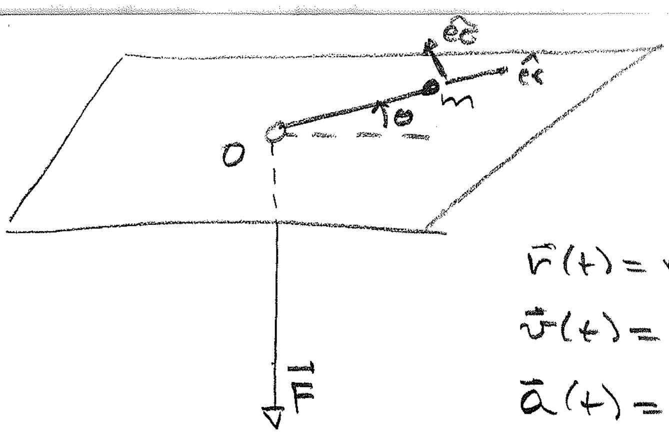

C.I.: $\vec{v}(0) = v_0 \hat{e_\theta} - v_1 \hat{e_r}$

$v_0, v_1 > 0 \qquad r(0) = R$

$\vec{r}(t) = r\hat{e_r}$

$\vec{v}(t) = r\dot{\theta}\hat{e_\theta} + \dot{r}\hat{e_r}$

$\vec{a}(t) = (\ddot{r} - r\dot{\theta}^2)\hat{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e_\theta}$

a) si $\dot{r}(t) = cte \Rightarrow \dot{r} = -v_1$ y $r(t) = R - v_1 t$

Conservación de $\vec{L}_0$: $\vec{L}_0 = \vec{r} \wedge \vec{p} = r\hat{e_r} \wedge m(r\dot{\theta}\hat{e_\theta} + \dot{r}\hat{e_r})$

$\vec{L}_0 = mr^2\dot{\theta}\hat{k}$

$l = mr^2\dot{\theta} = mR^2\dot{\theta}_0$

$\dot{\theta}_0 = \frac{v_0}{R}$

$\Big\} \; l = mRv_0 \Rightarrow \dot{\theta} = \frac{Rv_0}{r^2} \rightarrow \boxed{\dot{\theta} = \frac{Rv_0}{(R-v_1t)^2}}$

Ley horaria: $\boxed{r(t) = R - v_1 t}$

$\theta(t) = Rv_0 \int \frac{1}{(R-v_1t)^2} dt = -\frac{Rv_0}{v_1}\int \frac{1}{u^2} du = \frac{Rv_0}{v_1}\left(\frac{1}{R-v_1t} - \frac{1}{R}\right)$

c.v. $u = R - v_1 t$

$du = -v_1 dt$

$\boxed{\theta(t) = \frac{v_0 t}{(R - v_1 t)}}$

b) 2da Ley de Newton

$m(\ddot{r} - r\dot{\theta}^2) = -F$

$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$

para el movimiento requerido $\quad -mr\dot{\theta}^2 = -F$

$F = m(R - v_1 t)\frac{(Rv_0)^2}{(R-v_1t)^4}$

$F = \frac{mR^2v_0^2}{(R - v_1 t)^3}$

c) Trayectoria $r(\theta)$ $\qquad \theta(t) = \frac{v_0 t}{(R - v_1 t)} = \frac{v_0 t}{r}$

$r = R - v_1 t$

$t = \frac{R - r}{v_1}$

$\theta = \frac{v_0}{v_1}\frac{(R-r)}{r} \rightarrow r\theta + r\frac{v_0}{v_1} = \frac{Rv_0}{v_1}$

$\boxed{r(\theta) = \frac{R\, v_0/v_1}{\theta + \frac{v_0}{v_1}}}$

Por Binet:

$$a_r = -\frac{\ell^2 u^2}{m^2}\left[u + u''\right]$$

$$u = \frac{1}{r}\ ;\ a_r = -\frac{F}{m} = -\frac{mR^2v_0^2}{mr^3} = -\frac{mR^2v_0^2}{m}u^3 = -\frac{\ell^2}{m^2}u^3$$

$$-\frac{\ell^2}{m^2}u^3 = -\frac{\ell^2}{m^2}u^2\left[u+u''\right] \Rightarrow \quad u = u + u'' \rightarrow u'' = 0$$

$$u(\theta) = A\theta + B$$

$$u(0) = \frac{1}{R} = B$$

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 = v_1^2 + \frac{R^2v_0^2}{r^2} = v_1^2 + R^2v_0^2u^2$$

$$v^2 = \frac{\ell^2}{m^2}\left[u^2 + u'^2\right] \qquad v_1^2 + \cancel{R^2v_0^2u^2} = \cancel{R^2v_0^2u^2} + u'^2R^2v_0^2$$

$$u' = \frac{v_1}{Rv_0} = A$$

$$\boxed{u(\theta) = \frac{1}{R}\left(\frac{v_1}{v_0}\theta + 1\right)} \rightarrow \boxed{r(\theta) = \frac{Rv_0/v_1}{\theta + v_0/v_1}}$$

d) $F = m(Rv_0)^2\dfrac{1}{r^3}$

$$W = \int_R^{R/2}\vec{F}\cdot d\vec{r} = \int_R^{R/2} -\frac{m(Rv_0)^2}{r^3}\hat{e}_r\cdot dr\hat{e}_r = -m(Rv_0)^2\int_R^{R/2}\frac{1}{r^3}dr = \frac{m}{2}(Rv_0)^2\left(\frac{1}{(R/2)^2} - \frac{1}{R^2}\right)$$

$$W = \frac{m}{2}(Rv_0)^2\left(\frac{4}{R^2} - \frac{1}{R^2}\right) = \frac{mv_0^2}{2}3 = \frac{3}{2}mv_0^2$$

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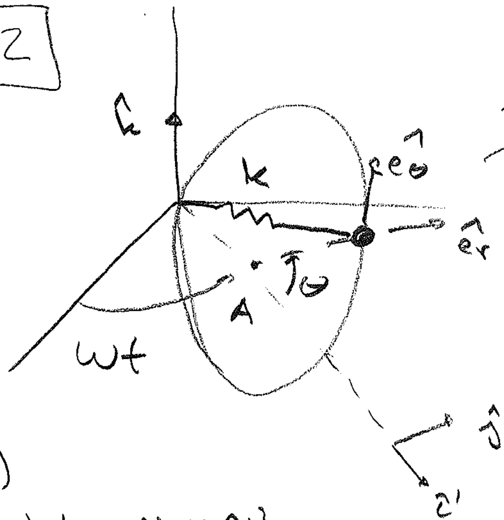
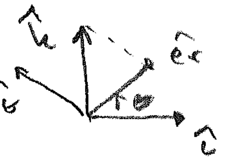


$\cancel{\downarrow \vec{g}}$



$\hat{k} = \sin\theta\, \hat{e}_r + \cos\theta\, \hat{e}_\theta$

$\hat{i}' = \cos\theta\, \hat{e}_r - \sin\theta\, \hat{e}_\theta$

a)

$S' = \{A, \hat{i}', \hat{j}', \hat{k}'\}$

$\vec{\omega} = \omega \hat{k}$

$\vec{r}' = R\hat{e}_r$

$\vec{v}' = R\dot{\theta}\, \hat{e}_\theta$

$\vec{a}' = R\ddot{\theta}\, \hat{e}_\theta - R\dot{\theta}^2 \hat{e}_r$

b)

$\vec{a} = \vec{a}' + \vec{a}_T + \vec{a}_c$

$\vec{a}_c = 2\vec{\omega} \wedge \vec{v}' = 2\omega \hat{k} \wedge R\dot{\theta}\hat{e}_\theta = -2R\omega\dot{\theta}\sin\theta\, \hat{j}'$

$\vec{a}_T = \vec{a}_A + \dot{\vec{\omega}} \wedge \vec{r}' + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}')$

$\vec{a}_A = -R\omega^2 \hat{i}'$

$\dot{\vec{\omega}} = 0$

$\vec{\omega} \wedge \vec{r}' = \omega\hat{k} \wedge R\hat{e}_r = R\omega\cos\theta\, \hat{j}'$

$\vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}') = \omega\hat{k} \wedge (R\omega\cos\theta\, \hat{j}') = -R\omega^2\cos\theta\, \hat{i}'$

$\vec{a} = R\ddot{\theta}\, \hat{e}_\theta - R\dot{\theta}^2 \hat{e}_r - R\omega^2 \hat{i}' - R\omega^2\cos\theta\, \hat{i}' - 2R\omega\dot{\theta}\sin\theta\, \hat{j}'$

c)

$\vec{F}_N = \vec{F}_e + \vec{N}$

$\vec{N} = N_1 \hat{e}_r + N_2 \hat{j}'$

$\vec{F}_e = -k(R\hat{i}' + R\hat{e}_r)$

$(\vec{F}_N = m\vec{a}) \cdot \hat{e}_\theta \Rightarrow kR\sin\theta = mR\ddot{\theta} + R\omega^2(1+\cos\theta)\sin\theta$

$mR\ddot{\theta} + Rm\left[\omega^2\cos\theta + \left(\omega^2 - \frac{k}{m}\right)\right]\sin\theta = 0$

d) $\sin\theta = 0 \quad \theta_1 = 0$ y $\theta_2 = \pi$

$\omega^2\cos\theta + \left(\omega^2 - \frac{k}{m}\right) = 0 \Rightarrow \cos\theta_3 = \dfrac{\frac{k}{m} - \omega^2}{\omega^2}$ $\quad \exists$ si $\frac{k}{m} < 2\omega^2$

$$f(\theta) = \left[\left(\omega^2 - \frac{k}{m}\right) + \omega^2\cos\theta\right]\sin\theta$$

$$\frac{df}{d\theta} = \cos\theta\left[\left(\omega^2 - \frac{k}{m}\right) + \omega^2\cos\theta\right] - \omega^2\sin^2\theta$$

$$\left.\frac{df}{d\theta}\right|_{\theta_1=0} = \left(\omega^2 - \frac{k}{m}\right) + \omega^2 = 2\omega^2 - \frac{k}{m} \Rightarrow \theta_1 = 0 \text{ es estable si } 2\omega^2 > \frac{k}{m}$$

$$\left.\frac{df}{d\theta}\right|_{\theta_2=\pi} = -\left[\left(\omega^2 - \frac{k}{m}\right) - \omega^2\right] = \frac{k}{m} > 0 \Rightarrow \theta_2 = \pi \text{ estable}$$

$$\left.\frac{df}{d\theta}\right|_{\theta_3} = \cos\theta_3 \underbrace{\left[\left(\omega^2 - \frac{k}{m}\right) + \omega^2\cos\theta_3\right]}_{=0} - \omega^2\sin^2\theta < 0 \quad \theta_3 \text{ inestable.}$$