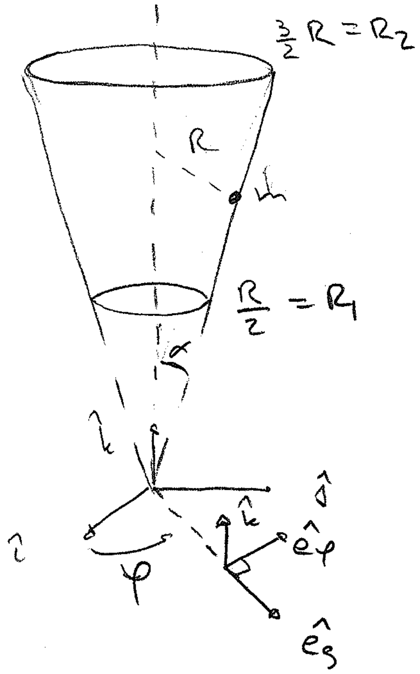
1



$\tan(\alpha) = \frac{1}{2}$

$\vec{v}(t=0) = v_0\,\hat{e}_\varphi$

$\vec{r}(t=0) = R\,\hat{e}_\rho + z(0)\,\hat{k}$

$\tan(\alpha) = \frac{R}{z(0)} \Rightarrow z(0) = \frac{R}{tg(\alpha)} = 2R$

$\vec{r}(t=0) = R\,\hat{e}_\rho + 2R\,\hat{k}$

de la misma forma $z = 2\rho$

$\vec{r}(t) = \rho\,\hat{e}_\rho + 2\rho\,\hat{k}$

$\vec{v}(t) = \dot{\rho}\,\hat{e}_\rho + \rho\dot{\varphi}\,\hat{e}_\varphi + 2\dot{\rho}\,\hat{k}$

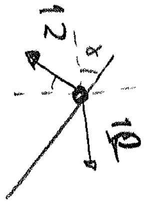
$\vec{v}(0) = v_0\,\hat{e}_\varphi \Rightarrow R\dot{\varphi}_0 = v_0$

a) $\vec{L}_0 = \vec{r}\wedge\vec{p} = \vec{r}\wedge m\vec{v} = (\rho\,\hat{e}_\rho + 2\rho\,\hat{k})\wedge m(\dot{\rho}\,\hat{e}_\rho + \rho\dot{\varphi}\,\hat{e}_\varphi + 2\dot{\rho}\,\hat{k})$

$\vec{L}_0 = m(\rho^2\dot{\varphi}\,\hat{k} - 2\rho\dot{\rho}\,\hat{e}_\varphi + 2\rho\dot{\rho}\,\hat{e}_\varphi + 2\rho^2\dot{\varphi}(-\hat{e}_\rho))$

$\vec{L}_0 = m(\rho^2\dot{\varphi}\,\hat{k} - 2\rho^2\dot{\varphi}\,\hat{e}_\rho) \Rightarrow \vec{L}_0\cdot\hat{k} = m\rho^2\dot{\varphi}$

si $\vec{M}_0\cdot\hat{k} = 0 \Rightarrow \vec{L}_0\cdot\hat{k}$ cte



Fuerzas sobre la partícula

$\vec{N} = -N\cos\alpha\,\hat{e}_\rho + N\sin\alpha\,\hat{k}$

$\vec{P} = -mg\,\hat{k}$

$\vec{M}_0 = (\rho\,\hat{e}_\rho + 2\rho\,\hat{k})\wedge(-N\cos\alpha\,\hat{e}_\rho + N\sin\alpha\,\hat{k} - mg\,\hat{k})$

$= \rho N\sin\alpha(-\hat{e}_\varphi) + mg\rho\,\hat{e}_\varphi - 2\rho N\cos\alpha\,\hat{e}_\varphi$

$\Rightarrow \vec{M}_0\cdot\hat{k} = 0 \Rightarrow \vec{L}_0\cdot\hat{k} = m\rho^2\dot{\varphi} = l_0 = $ cte.

b) Como el sistema es liso y $\vec{N}\cdot\vec{v} = 0$ $\}$ $E = T + U$ se conserva

$T = \frac{1}{2}m\vec{v}^2 = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + 4\dot{\rho}^2) = \frac{1}{2}m(5\dot{\rho}^2 + \rho^2\dot{\varphi}^2)$

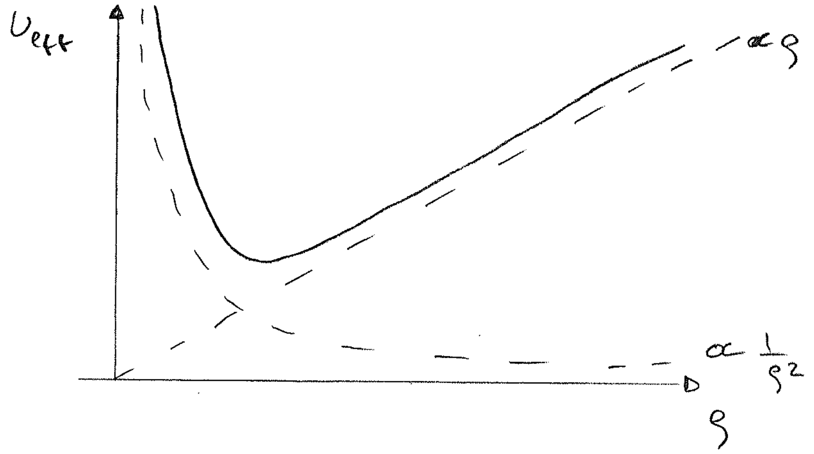
$l_0 = mR^2\dot{\varphi}_0 \Rightarrow l_0 = mR^2\frac{v_0}{R}$ $\boxed{l_0 = mRv_0}$

$\Rightarrow \boxed{\dot{\varphi} = \frac{Rv_0}{\rho^2}}$

$U = -mgz = +2mg\rho$

$E = \frac{1}{2} m \left(5\dot{\rho}^2 + \frac{R^2 v_0^2}{\rho^2} \right) + 2mg\rho$ y $\frac{dE}{dt} = 0$; $E_0 = \frac{1}{2} m v_0^2 + 2mgR$

c) $U_{eff}(\rho) = \frac{1}{2} m R^2 v_0^2 \frac{1}{\rho^2} + 2mg\rho$ $\dot{\rho}^2 = \frac{2}{5} \frac{(E_0 - U_{eff}(\rho))}{m} = f(\rho)$



como $U_{eff}(\rho) \to \infty$ para $\rho \to 0$ y $\rho \to \infty$

Las orbitas son siempre acotadas.

d) $E_0 - U_{eff}(\rho) = 0$ da los puntos de retroceso.

En la c.I. $(t=0)$ $E_0 = \frac{1}{2} m v_0^2 + 2mgR = U_{eff}(R) \Rightarrow R$ es pto de retroceso.

Para mantenerme en el cono $R_1 < \rho < R_2$ dado que R es pto de retroceso tengo 2 opciones: $R_1 < \rho \leq R$ (I)

$R \leq \rho < R_2$ (II)

Para I $E_0 - U_{eff}(R_1) = 0$

$R_1 = \frac{R}{2}$

$\frac{1}{2} m v_0^2 + 2mgR - \frac{1}{2} m \frac{R^2 v_0^2}{(R_1)^2} - 2mgR_1 = 0$

$v_0^2 + 4g(R - R_1) - v_0^2 \left(\frac{R}{R_1} \right)^2 = 0$

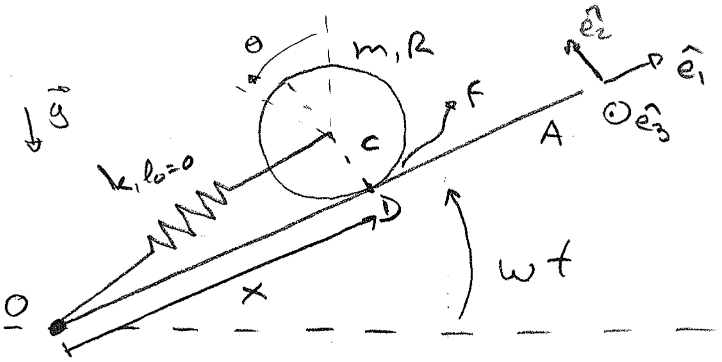
$v_0^2 (1 - 4) = -4gR\frac{1}{2}$ $v_0^{(I)} = \sqrt{\frac{2}{3} gR}$ si $v_0 > v_0^{(I)} \Rightarrow R_1 < \rho < R$

Idem (II) $E_0 - U_{eff}(R_2) = 0$

$v_0^2 \left(1 - \left(\frac{R}{R_2} \right)^2 \right) + 4g(R - R_2) = 0 \Rightarrow v_0^{(II)} = \sqrt{\frac{18}{5} gR}$ si $v_0 < v_0^{(II)}$ $R < \rho < R_2$

$\Rightarrow \boxed{\sqrt{\frac{2}{3} gR} < v_0 < \sqrt{\frac{18}{5} gR}}$

$\boxed{12}$



$k = m\omega^2$

$x(0) = 0$

$\dot{x}(0) = \frac{2}{3}\frac{g}{\omega}$

a) $\vec{r}_c = x\hat{e}_1 + R\hat{e}_2$

$\vec{v}_c = (\dot{x} - R\omega)\hat{e}_1 + x\omega\hat{e}_2$

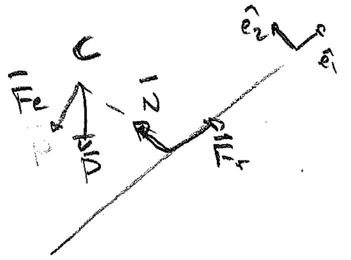
$\bar{\omega} = \omega\hat{e}_3$

$\dot{\hat{e}}_1 = \bar{\omega} \wedge \hat{e}_1 = \omega\hat{e}_2$

$\dot{\hat{e}}_2 = \bar{\omega} \wedge \hat{e}_2 = -\omega\hat{e}_1$

$\vec{a}_c = (\ddot{x} - x\omega^2)\hat{e}_1 + (2\dot{x}\omega - R\omega^2)\hat{e}_2$

$\hat{k} = \cos(\omega t)\hat{e}_2 + \sin(\omega t)\hat{e}_1$



$\vec{P} = -mg\hat{k} = -mg(\sin(\omega t)\hat{e}_1 + \cos(\omega t)\hat{e}_2)$

$\vec{N} = N\hat{e}_2$

$\vec{F}_r = F_r\hat{e}_1$

$\vec{F}_e = -k(\bar{r}_c - \bar{r}_0) = -k(x\hat{e}_1 + R\hat{e}_2)$

1ra cardinal:

$F_r - kx - mg\sin(\omega t) = m(\ddot{x} - x\omega^2)$

$N - kR - mg\cos(\omega t) = m(2\dot{x}\omega - R\omega^2)$

$k = m\omega^2$ $\Rightarrow$ $F_r - mg\sin(\omega t) = m\ddot{x}$

$N - mg\cos(\omega t) = m2\dot{x}\omega$

$\vec{\omega}_D = \dot{\theta}\hat{e}_3$ velocidad angular del disco

$\bar{v}_D = \bar{v}_c + \vec{\omega}_D \wedge (\bar{r}_D - \bar{r}_c) = (\dot{x} - R\omega)\hat{e}_1 + x\omega\hat{e}_2 + \dot{\theta}\hat{e}_3 \wedge (-R\hat{e}_2)$

$= (\dot{x} - R\omega)\hat{e}_1 + x\omega\hat{e}_2 + R\dot{\theta}\hat{e}_1 = x\omega\hat{e}_2 \Rightarrow \dot{x} + R\dot{\theta} = R\omega$

RSD

$\dot{\theta} = \frac{R\omega}{R} - \frac{\dot{x}}{R}$

$\boxed{\ddot{\theta} = -\frac{\ddot{x}}{R}}$

2da Cardinal en $\hat{e}_3$: $\frac{mR^2}{2}\ddot{\theta} = RF_r$

$F_r = -\frac{mR}{2}\frac{\ddot{x}}{R} \Rightarrow F_r = -\frac{m\ddot{x}}{2}$

$\Rightarrow -m\frac{\ddot{x}}{2} - mg\sin(\omega t) = m\ddot{x} \Rightarrow \boxed{\ddot{x} = -\frac{2}{3}g\sin(\omega t)}$

b) $\dot{x} = \frac{2}{3}\frac{g}{\omega}\cos(\omega t) + C$

$\Rightarrow N = mg\cos(\omega t) + 2m\frac{2}{3}\frac{g}{\omega}\omega\cos(\omega t$

$\dot{x}(0) = \frac{2}{3}\frac{g}{\omega} \Rightarrow C = 0$

$\boxed{N = mg\cos(\omega t)\frac{7}{3}}$

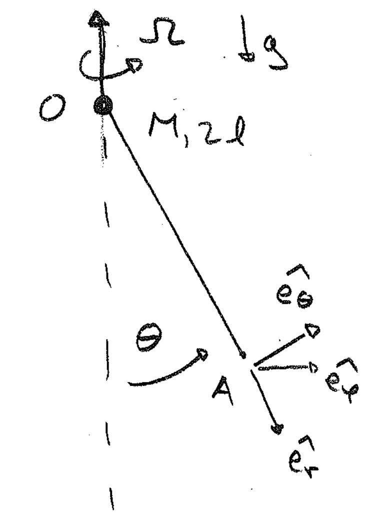
$\boxed{F_r = \frac{mg}{3}\sin(\omega t)}$

$|\vec{F}_r| \leq f|\vec{N}| \Rightarrow \frac{mg}{3}\sin(\omega t) \leq f\frac{7}{3}mg\cos(\omega t)$

$tg(\omega t) \leq 7f$

en $\frac{\pi}{4}$ $tg(\pi/4) = 1 \Rightarrow f_{mín} = \frac{1}{7}$.

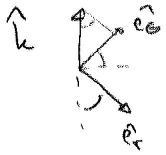
3



$0 < \theta < \frac{\pi}{2}$

$\vec{F} = F\,\hat{e}_\varphi$

$$\mathbb{I}_O^{\{\hat{e}_r, \hat{e}_\varphi, \hat{e}_\theta\}} = \frac{4}{3} m l^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$\hat{k} = \sin\theta\,\hat{e}_\theta - \cos\theta\,\hat{e}_r$

a)

$$\vec{\omega} = \Omega\hat{k} - \dot{\theta}\,\hat{e}_\varphi = \Omega\sin\theta\,\hat{e}_\theta - \Omega\cos\theta\,\hat{e}_r - \dot{\theta}\,\hat{e}_\varphi$$

$$\vec{L}_O = \mathbb{I}_O\vec{\omega} = \frac{4}{3} m l^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\Omega\cos\theta \\ -\dot{\theta} \\ \Omega\sin\theta \end{pmatrix} = \frac{4}{3} m l^2 \left(\Omega\sin\theta\,\hat{e}_\theta - \dot{\theta}\,\hat{e}_\varphi\right)$$

$$\frac{d\vec{L}_O}{dt} = \frac{4}{3} m l^2 \left(\Omega\dot{\theta}\cos\theta\,\hat{e}_\theta + \Omega\sin\theta\,\dot{\hat{e}}_\theta - \ddot{\theta}\,\hat{e}_\varphi - \dot{\theta}\,\dot{\hat{e}}_\varphi\right)$$

$$\dot{\hat{e}}_\theta = \vec{\omega}\wedge\hat{e}_\theta = (\Omega\sin\theta\,\hat{e}_\theta - \Omega\cos\theta\,\hat{e}_r - \dot{\theta}\,\hat{e}_\varphi)\wedge\hat{e}_\theta$$

$$\dot{\hat{e}}_\theta = \Omega\cos\theta\,\hat{e}_\varphi - \dot{\theta}\,\hat{e}_r$$

$$\dot{\hat{e}}_\varphi = -\Omega\sin\theta\,\hat{e}_r - \Omega\cos\theta\,\hat{e}_\theta$$

$$\frac{d\vec{L}_O}{dt} = \frac{4}{3} m l^2 \left(2\Omega\dot{\theta}\cos\theta\,\hat{e}_\theta - (\ddot{\theta} - \Omega^2\sin\theta\cos\theta)\,\hat{e}_\varphi\right)$$

$$\vec{M}_O^{ext} = +mgl\sin\theta\,\hat{e}_\varphi + 2l\hat{e}_r\wedge F\hat{e}_\varphi$$

$$= mgl\sin\theta\,\hat{e}_\varphi + 2lF\,\hat{e}_\theta$$

$$\boxed{\frac{4}{3} m l^2 (\ddot{\theta} - \Omega^2\sin\theta\cos\theta) = -mgl\sin\theta}$$ ec. de mov.

b) $F = \frac{2ml}{3}(2\Omega\dot{\theta}\cos\theta)$ hay que obtener $\dot{\theta} = f(\theta)$

(No se pide en el examen)

$$\ddot{\theta} = \Omega^2 \sin\theta \cos\theta - \frac{3}{4}\frac{g}{l}\sin\theta$$

$$\rightarrow \frac{1}{2}\dot{\theta}^2 = \frac{1}{2}\dot{\theta}_0^2 + \frac{3}{4}\frac{g}{l}(\cos\theta - \cos\theta_0) + \frac{1}{2}\Omega^2(\sin^2\theta - \sin^2\theta_0)$$

$$\dot{\theta}^2 = \dot{\theta}_0^2 + \frac{3}{2}\frac{g}{l}(\cos\theta - \cos\theta_0) + \Omega^2(\sin^2\theta - \sin^2\theta_0)$$

$$\boxed{|\vec{F}| = \frac{2}{3} m l \, (2\Omega\dot{\theta}\cos\theta)}$$

c) θ_{eq}: $\left(\frac{3}{4}\frac{g}{l} - \Omega^2\cos\theta\right)\sin\theta = 0$

$\hookrightarrow \sin\theta = 0 \rightarrow \left.\begin{matrix}\theta = 0\\ \theta = \pi\end{matrix}\right\}$ No pertenecen al intervalo considerado.

$$\cos\theta_{eq} = \frac{3}{4}\frac{g}{l\Omega^2} \quad \exists \text{ sii } \frac{3}{4}\frac{g}{l\Omega^2} < 1$$

$$\left.\frac{dF}{d\theta}\right|_{\theta_{eq}} = \cos\theta\left(\frac{3}{4}\frac{g}{l} - \Omega^2\cos\theta\right) + \Omega^2\sin^2\theta\Bigg|_{\theta_{eq}} = \Omega^2\sin^2\theta > 0$$

θ_{eq} es estable