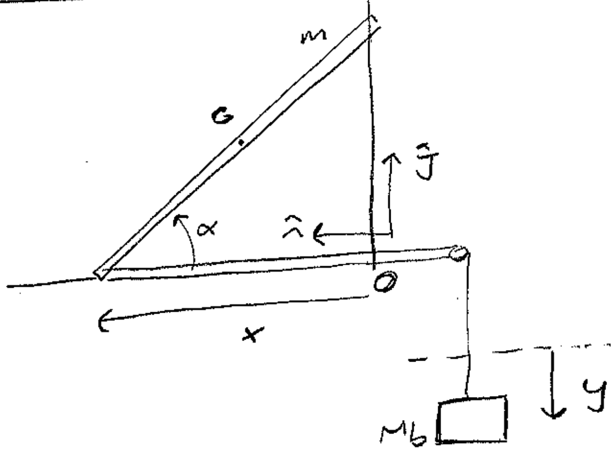
Ejercicio 1



Sea y medida desde la posición inicial de M_b; como el hilo es de largo constante:

$$x + y = x(0) + y(0) = 2\ell\cos\alpha_0$$

$$x = 2\ell\cos\alpha \;:\; y = 2\ell(\cos\alpha_0 - \cos\alpha)$$

a) $U = mg\ell\,\text{sen}\,\alpha - M_b g y = mg\ell\,\text{sen}\,\alpha - M_b g 2\ell(\cos\alpha_0 - \cos\alpha)$

El sistema permanece en reposo si la configuración es de equilibrio:

$$\left.\frac{dU}{d\alpha}\right|_{\alpha_0} = 0 : \left.\left(mg\ell\cos\alpha - M_b g 2\ell\,\text{sen}\,\alpha\right)\right|_{\alpha_0} = 0 : \; M_b = \frac{m}{2\,\text{tg}\,\alpha_0} = \boxed{\frac{m}{2}}$$

b) $M_b = m$;

La energía del sistema se conserva:

$$E = E(0) = U(0) = mg\ell\,\text{sen}\,\alpha_0 = mg\ell\frac{1}{\sqrt{2}}$$

$$E = \frac{1}{2}m\dot{y}^2 - mgy + \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\dot{\alpha}^2 + mg\ell\,\text{sen}\,\alpha \;;\; I_G = \frac{m\ell^2}{3}$$

$$\vec{v}_G : \; G - O = \ell\cos\alpha\,\hat{\imath} + \ell\,\text{sen}\,\alpha\,\hat{j} \;:\; \vec{v}_G = -\ell\,\text{sen}\,\alpha\,\dot{\alpha}\,\hat{\imath} + \ell\cos\alpha\,\dot{\alpha}\,\hat{j}$$

$$\Rightarrow v_G^2 = \ell^2\dot{\alpha}^2 ;$$

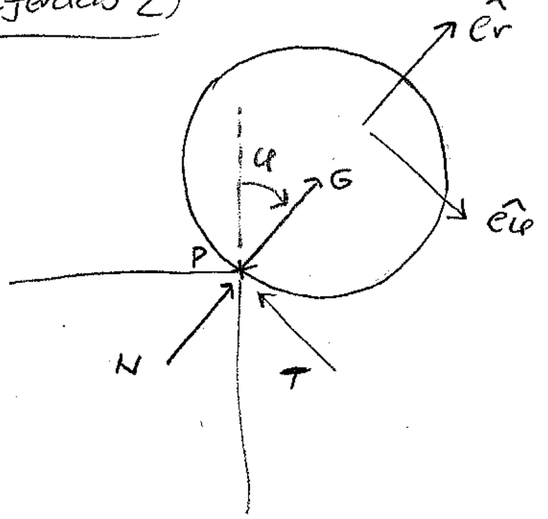
$$y = 2\ell(\cos\alpha_0 - \cos\alpha) = 2\ell\left(\frac{1}{\sqrt{2}} - \cos\alpha\right) \;:\; \dot{y} = 2\ell\,\text{sen}\,\alpha\,\dot{\alpha}$$

$$\Rightarrow E = 2m\ell^2\text{sen}^2\alpha\,\dot{\alpha}^2 - mg2\ell\left(\frac{1}{\sqrt{2}} - \cos\alpha\right) + \frac{1}{2}m\ell^2\dot{\alpha}^2 + \frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\dot{\alpha}^2 + mg\ell\,\text{sen}\,\alpha$$

$$E(\alpha = \pi/2) = m\ell^2\dot{\alpha}^2_{\frac{\pi}{2}}\left(2 + \frac{1}{2} + \frac{1}{6}\right) - mg2\ell\left(\frac{1}{\sqrt{2}}\right) + mg\ell = E(\alpha_0) = \frac{mg\ell}{\sqrt{2}}$$

$$\Rightarrow \boxed{\dot{\alpha}(\alpha = \pi/2) = \left(\frac{9 - 3\sqrt{2}}{8\sqrt{2}}\,g/\ell\right)^{1/2}}$$

Ejercicio 2)



a) A partir de la conservación de la energía:

$$E = \frac{1}{2} I_P \dot{\varphi}^2 + MgR\cos\varphi = E_0 = MgR$$

con $I_P = I_G + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$$\Rightarrow \frac{3}{4}MR^2\dot{\varphi}^2 + MgR\cos\varphi = MgR \Leftrightarrow$$

$$\Leftrightarrow \boxed{\dot{\varphi}^2 = \frac{4}{3}\, g/R\,(1-\cos\varphi)}$$

b) 1era Cardinal: $\begin{cases} N - Mg\cos\varphi = M\vec{a}_G\cdot\hat{e_r} = -MR\dot{\varphi}^2 \\ Mg\,\text{sen}\varphi - T = M\vec{a}_G\cdot\hat{e_\varphi} = MR\ddot{\varphi} \end{cases}$

$$\Rightarrow N = Mg\cos\varphi - MR\dot{\varphi}^2 = Mg\cos\varphi - \frac{4}{3}Mg(1-\cos\varphi) = \frac{Mg}{3}(7\cos\varphi - 4)$$

$T = Mg\,\text{sen}\varphi - MR\ddot{\varphi}$; derivando la ecuación de movimiento: $\ddot{\varphi} = \frac{2}{3}\, g/R\, \text{sen}\varphi$

$$\Rightarrow T = Mg\,\text{sen}\varphi - \frac{2}{3}Mg\,\text{sen}\varphi = \frac{Mg}{3}\text{sen}\varphi$$

mientras el disco no desliza: $|T| \le f_E|N|$: $f_E \ge \frac{|T|}{|N|} = \left|\frac{\text{sen}\varphi}{7\cos\varphi - 4}\right|$

si comienza a deslizar para $\varphi = \pi/6$: $f_E = \frac{\text{sen}\,\pi/6}{7\cos\pi/6 - 4} = \boxed{\frac{1}{7\sqrt{3}-8}}$

c) las cardinales al disco son: $\begin{cases} N - Mg\cos\varphi = -MR\dot{\varphi}^2 \quad (i) \\ Mg\,\text{sen}\varphi - T = MR\ddot{\varphi} \quad (ii) \\ I_G\dot{\omega} = RT \quad (iii) \end{cases}$, con $T = f_0 N$ (iv)

Eliminando T entre (ii) y (iii): $Mg\,\text{sen}\varphi - \frac{I_G}{R}\dot{\omega} = MR\ddot{\varphi}$:

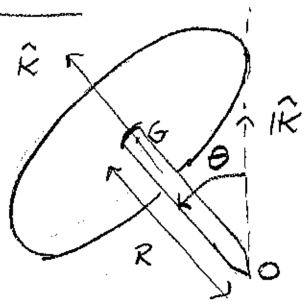
$$\boxed{\dot{\omega} = 2(g/R\,\text{sen}\varphi - \ddot{\varphi})}$$

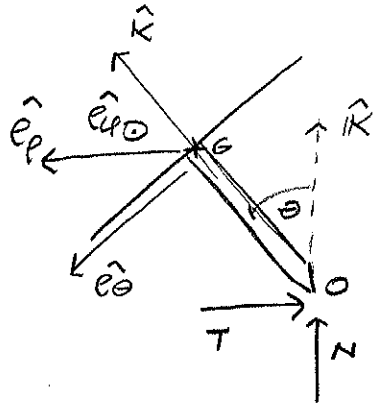
usando (iv) en (ii) y eliminando N entre (ii) y (i):

$$Mg\,\text{sen}\varphi - f_0(Mg\cos\varphi - MR\dot{\varphi}^2) = MR\ddot{\varphi}$$

$$\boxed{\ddot{\varphi} - f_0\dot{\varphi}^2 = g/R\,(\text{sen}\varphi - f_0\cos\varphi)}$$

Ejercicio 3)





a) $\vec{a}_G = -\omega^2 R\,\text{sen}\theta\,\hat{e_\rho}$: $(N - Mg)\hat{IK} - T\hat{e_\rho} = -M\omega^2 R\,\text{sen}\theta\,\hat{e_\rho}$

$$\Rightarrow \boxed{\begin{aligned} N &= Mg \\ T &= M\omega^2 R\,\text{sen}\theta \end{aligned}}$$

b) $\vec{L_O} = \mathbb{I}_O\,\vec{\omega}$; $\vec{\omega} = \Omega\hat{k} + \omega\hat{IK}$

El tensor de inercia con respecto a O es diagonal en la base $\{\hat{k}, \hat{e_\theta}, \hat{e_\varphi}\}$:

$$\mathbb{I}_O^{\{\hat{k},\hat{e_\theta},\hat{e_\varphi}\}} = \begin{pmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 + MR^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 + MR^2 \end{pmatrix} = \frac{MR^2}{4}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

En la misma base $\vec{\omega}$ queda:

$$\vec{\omega} = \Omega\hat{k} + \omega\cos\theta\,\hat{k} - \omega\,\text{sen}\theta\,\hat{e_\theta} = (\Omega + \omega\cos\theta)\hat{k} - \omega\,\text{sen}\theta\,\hat{e_\theta}$$

$$\Rightarrow \vec{L_O} = \mathbb{I}_O\vec{\omega} = \boxed{\frac{MR^2}{4}\left[2(\Omega + \omega\cos\theta)\hat{k} - 5\omega\,\text{sen}\theta\,\hat{e_\theta}\right]}$$

c) 2da Cardinal desde O:

$\vec{M_O}^{(ext)} = \dot{\vec{L_O}}$: $MgR\,\text{sen}\theta\,\hat{e_\varphi} = \dot{\vec{L_O}}$;

$$\dot{\vec{L_O}} = \frac{d'\vec{L_O}}{dt} + \vec{\omega}_{\{\hat{k},\hat{e_\theta},\hat{e_\varphi}\}} \times \vec{L_O} = \omega\hat{IK} \times \vec{L_O} =$$

$$= \frac{MR^2}{4}\omega\left[2(\Omega + \omega\cos\theta)\,\text{sen}\theta - 5\omega\,\text{sen}\theta\cos\theta\right]\hat{e_\varphi} = \frac{MR^2\omega}{4}\left[(2\Omega - 3\omega\cos\theta)\,\text{sen}\theta\right]\hat{e_\varphi}$$

$$\Rightarrow \cancel{M}gR\,\text{sen}\theta = \frac{\cancel{M}R^2}{4}\left[(2\Omega - 3\omega\cos\theta)\,\text{sen}\theta\right]\omega \quad : (\theta \neq 0)$$

$$\boxed{4g/R = (2\Omega - 3\omega\cos\theta)\omega}$$