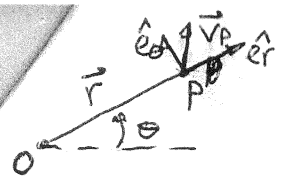


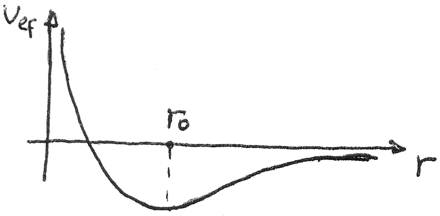
a) $\vec{F} = F(r).\hat{e}_r \Rightarrow \vec{L}_O = (P-O)\wedge\vec{P} \Rightarrow \dot{\vec{L}}_O = (\dot{P}-\underbrace{\dot{O}}_{=0})\wedge\vec{P} + (P-O)\wedge\vec{F}$

$\Rightarrow \dot{\vec{L}}_O = \underbrace{\vec{V}_P\wedge\vec{P}}_{=0} + \underbrace{r.\hat{e}_r \wedge F(r).\hat{e}_r}_{=0} \Rightarrow \dot{\vec{L}}_O = 0 \Rightarrow \vec{L}_O = cte = \ell.\hat{K}$

La conservación de $\vec{L}_O$ implica que el plano definido por $\vec{R}$ y $\vec{V}_O$ es el plano donde se mueve la partícula.

b) $\ell = m R.V_0 \Rightarrow V_{ef} = \dfrac{\ell^2}{2mr^2} + V(r) = \dfrac{mR^2V_0^2 - K_2}{2r^2} - \dfrac{K_1}{r}$

si $\quad V_0^2 > \dfrac{K_2}{mR^2} \quad \Rightarrow$



c) $u'' + u = -\dfrac{m}{\ell^2}\cdot\dfrac{F(u)}{u^2} = \dfrac{-m}{m^2R^2V_0^2}\left[\dfrac{-K_1u^2 - K_2u^3}{u^2}\right] \Rightarrow u''+u = \dfrac{K_1 + K_2 u}{mR^2V_0^2}$

$\Rightarrow u'' + u\underbrace{\left[1 - \dfrac{K_2}{mR^2V_0^2}\right]}_{=w^2} = \dfrac{K_1}{mR^2V_0^2} = 1/p \qquad u = \dfrac{1}{r}$

$u_H = A.\cos(w.\theta) + B.\operatorname{sen}(w\theta)$
$u_P = \dfrac{1}{pw^2}$

$\Rightarrow u = \dfrac{1}{pw^2} + A.\cos(w\theta) + B.\operatorname{sen}(w\theta)$

$u(0) = \dfrac{1}{R} = \dfrac{1}{pw^2} + A \Rightarrow A = \dfrac{1}{R} - \dfrac{1}{pw^2} \quad \Big\| \quad u'(0) = -\dfrac{m}{\ell}.\underbrace{\dot{r}(0)}_{=0} \Rightarrow B.w = 0 \Rightarrow B = 0$

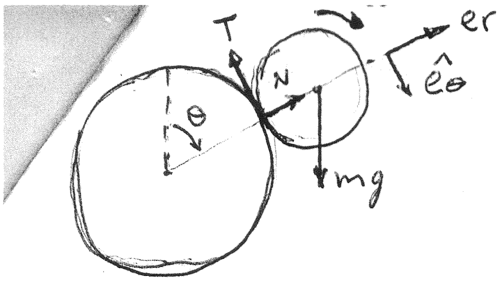
$$\Rightarrow \boxed{r(\theta) = \dfrac{1}{\dfrac{1}{pw^2} + \left(\dfrac{1}{R} - \dfrac{1}{pw^2}\right)\cos(w\theta)}}$$

d) órbita circular para $\dfrac{1}{R} = \dfrac{1}{pw^2} \Rightarrow pw^2 = R \Rightarrow \dfrac{mR^2V_0^2}{K_1}.\left(1 - \dfrac{K_2}{mR^2V_0^2}\right) = R$

$\Rightarrow mR^2V_0^2 - K_2 = RK_1 \Rightarrow R^2 - \dfrac{K_1}{mV_0^2}R - \dfrac{K_2}{mV_0^2} = 0 \Rightarrow R = \dfrac{1}{2}\left[\dfrac{K_1}{mV_0^2} \pm \sqrt{\dfrac{K_1^2}{m^2V_0^4} + \dfrac{4K_2}{mV_0^2}}\right]$

$R = \dfrac{K_1}{2mV_0^2}\left[1 \pm \sqrt{1 + \dfrac{4K_2mV_0^2}{K_1^2}}\right]$, el signo $\ominus$ no es físicamente posible $\Rightarrow$

$$\Rightarrow \boxed{R = \dfrac{K_1}{2mV_0^2}\left[1 + \sqrt{1 + \dfrac{4K_2mV_0^2}{K_1^2}}\right]}$$



a) R.S.D $\longrightarrow (R+r)\dot{\theta} - r.w = 0 \longrightarrow w = 4\dot{\theta}$

$m(R+r)\ddot{\theta} = -T + mg.\text{sen}\theta \Longrightarrow 4mr\ddot{\theta} = -T + mg.\text{sen}\theta$

$-m(R+r)\dot{\theta}^2 = N - mg.\cos\theta \Longrightarrow -4mr.\dot{\theta}^2 = N - mg.\cos\theta$

$\frac{mr^2}{2}\dot{w} = r.T \longrightarrow \frac{mr}{2}\dot{w} = T$

$\frac{mr}{2}.4\ddot{\theta} = T \longrightarrow 4mr\ddot{\theta} = -2mr\ddot{\theta} + mg.\text{sen}\theta \Longrightarrow \boxed{\ddot{\theta} = \frac{1}{6}(g/r)\text{sen}\theta}$

b) Preintegro ec de mov $\longrightarrow \ddot{\theta}.\dot{\theta} = \frac{1}{6}(g/r)\,\text{sen}\theta.\dot{\theta} \longrightarrow \frac{\dot{\theta}^2}{2} = \frac{1}{6}(g/r)[1-\cos\theta]$

$N = mg.\cos\theta - 4mr.\dot{\theta}^2 = mg.\cos\theta - 4mr.\frac{1}{3}(g/r)[1-\cos\theta] = \frac{mg}{3}[7\cos\theta - 4]$

$T = 2mr\ddot{\theta} = 2mr.\frac{1}{6}(g/r)\text{sen}\theta = \frac{mg}{3}.\text{sen}\theta$

empieza a deslizar cuando $T = f_s.N \Longrightarrow \cancel{\frac{mg}{3}}.\text{sen}\theta_d = f_s\cancel{\frac{mg}{3}}[7\cos\theta_d - 4]$

$\Longrightarrow \boxed{\text{sen}\,\theta_d = f_s[7\cos\theta_d - 4]}$

c) Despues que comienza a deslizar

$$\left.\begin{array}{l} 4mr\ddot{\theta} = -T + mg.\text{sen}\theta \\ -4mr\dot{\theta}^2 = N - mg.\cos\theta \\ \frac{mr}{2}\dot{w} = T \\ T = \underbrace{f_d}_{\approx 0}.N \approx 0 \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} 4mr\ddot{\theta} = mg.\text{sen}\theta \\ -4mr\dot{\theta}^2 = N - mg.\cos\theta \\ \dot{w} = 0 \end{array}\right\} \Longrightarrow \boxed{\begin{array}{l} \ddot{\theta} = \frac{1}{4}(g/r)\,\text{sen}\theta \\ \dot{w} = 0 \end{array}}$$

ec mov

$\dot{\theta}^2 = \dot{\theta}^2(\theta_d) - \frac{1}{2}(g/r)[\cos\theta - \cos\theta_d]$ $\quad\|\quad$ $\dot{\theta}^2(\theta_d) = \frac{1}{3}(g/r)[1 - \cos\theta_d]$

$\dot{\theta}^2 = (g/3r)[1 - \cos\theta_d] - (g/2r)[\cos\theta - \cos\theta_d] = (g/3r)[1 + \frac{1}{2}\cos\theta_d] - (g/2r)\cos\theta$

$N = mg.\cos\theta - 4mr.[(g/3r)[1 + \frac{1}{2}\cos\theta_d] - (g/2r)\cos\theta] = mg[3\cos\theta - \frac{4}{3}(1 + \frac{1}{2}\cos\theta_d)]$

Pierde contacto $N = 0 \Longrightarrow 3\cos\theta_D = \frac{4}{3}(1 + \frac{1}{2}\cos\theta_d) \Longrightarrow \boxed{\cos\theta_D = \frac{4}{9}(1 + \frac{1}{2}\cos\theta_d)}$

$$\vec{w} = \Omega.\hat{e_r} + \dot{\varphi}\,\hat{j} + w_T.\hat{k}$$

$$\hat{k} = \cos\varphi.\hat{e_r} + \operatorname{sen}\varphi.\hat{e_\varphi}$$

$$\boxed{\vec{w} = (\Omega + w_T.\cos\varphi)\,\hat{e_r} + w_T.\operatorname{sen}\varphi.\hat{e_\varphi} + \dot{\varphi}\,\hat{j}}$$

$$\boxed{\vec{L_G} = \frac{ma^2}{2}(\Omega + w_T.\cos\varphi)\,\hat{e_r} + \frac{ma^2}{4}.w_T.\operatorname{sen}\varphi.\hat{e_\varphi} + \frac{ma^2}{4}.\dot{\varphi}\,\hat{j}}$$

b) $\dot{\vec{L_G}}.\hat{j} = 0$

$$\dot{\vec{L_G}} = \frac{ma^2}{2}(\Omega - w_T.\operatorname{sen}\varphi.\dot{\varphi})\,\hat{e_r} + \frac{ma^2}{2}(\Omega + w_T.\cos\varphi)\,\dot{\hat{e_r}} + \frac{ma^2}{4}.w_T.\cos\varphi.\dot{\varphi}.\hat{e_\varphi}$$

$$+ \frac{ma^2}{4}.w_T.\operatorname{sen}\varphi.\dot{\hat{e_\varphi}} + \frac{ma^2}{4}\ddot{\varphi}\,\hat{j} + \frac{ma^2}{4}\dot{\varphi}\,\dot{\hat{j}}$$

$$\dot{\hat{e_r}} = (\dot{\varphi}\,\hat{j} + w_T.\hat{k}) \wedge \hat{e_r} = -\dot{\varphi}\,\hat{e_\varphi} + w_T.\operatorname{sen}\varphi.\hat{j}$$

$$\dot{\hat{e_\varphi}} = (\dot{\varphi}\,\hat{j} + w_T.\hat{k}) \wedge \hat{e_\varphi} = \dot{\varphi}\,\hat{e_r} - w_T.\cos\varphi\,\hat{j}$$

$$\dot{\hat{j}} = (\dot{\varphi}\,\hat{j} + w_T.\hat{k}) \wedge \hat{j}$$

$$\dot{\vec{L_G}}.\hat{j} = \frac{ma^2}{2}(\Omega + w_T.\cos\varphi)\,w_T.\operatorname{sen}\varphi + \frac{ma^2}{4}.w_T.\operatorname{sen}\varphi(-w_T.\cos\varphi) + \frac{ma^2}{4}\ddot{\varphi} = 0$$

$$\frac{ma^2}{4}\ddot{\varphi} + \frac{ma^2}{2}\Omega.w_T.\operatorname{sen}\varphi + \frac{ma^2}{4}.w_T^2.\operatorname{sen}\varphi.\cos\varphi = 0$$

$$\Rightarrow \boxed{\ddot{\varphi} + 2\Omega.w_T.\operatorname{sen}\varphi + w_T^2.\operatorname{sen}\varphi.\cos\varphi = 0}$$

c) $\Omega \gg w_T \rightarrow \ddot{\varphi} + 2\Omega.w_T.\operatorname{sen}\varphi = 0$

eq Relativo $\ddot{\varphi} = 0 \Rightarrow \operatorname{sen}\varphi_{eq} = 0 \rightarrow \varphi_{eq} = \begin{cases} 0 \\ \pi \end{cases}$