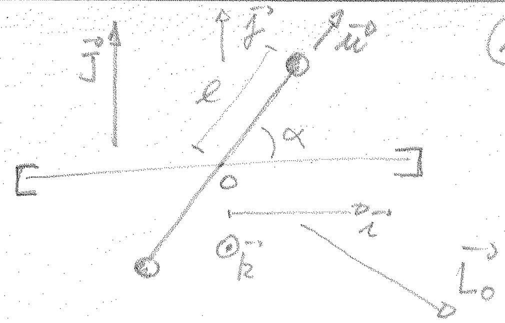
(i)



$\left.\begin{matrix}\vec{i}, \vec{j}, \vec{k} \\ \vec{u}\end{matrix}\right\}$ = solidario al rígido

$\vec{J}, \vec{K}$ } fijo

(a)

$$\mathbb{I}_o = \begin{bmatrix} (m_1+m_2)(\ell^2 \operatorname{sen}^2\alpha) & \times & \times \\ -(m_1+m_2)\ell^2 \operatorname{sen}\alpha\cos\alpha & \times & \times \\ 0 & \times & \times \end{bmatrix}$$

$$\vec{L}_o = \mathbb{I}_o \vec{\omega} \quad ; \quad \vec{\omega} = \omega \vec{i}$$

$$\vec{L}_o = (m_1+m_2)\ell^2 \operatorname{sen}^2\alpha\, \omega\, \vec{i} - (m_1+m_2)\ell^2 \operatorname{sen}\alpha\cos\alpha\, \omega\, \vec{j} =$$

$$= (m_1+m_2)\ell^2 \omega \operatorname{sen}\alpha \left(\operatorname{sen}\alpha\, \vec{i} - \cos\alpha\, \vec{j} \right)$$

(b)

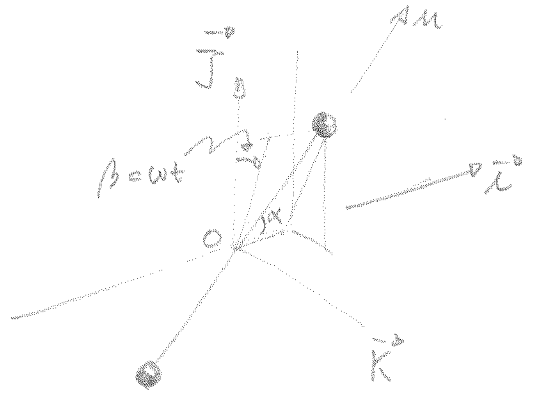
$$\dot{\vec{L}}_o = \vec{M}_o^{ext}$$

$$\dot{\vec{L}}_o = -(m_1+m_2)\ell^2\omega^2 \operatorname{sen}\alpha\cos\alpha\, \vec{k} = \vec{M}_o^{ext}$$

$$\vec{r}_G = \frac{m_1 \ell \vec{u} - m_2 \ell \vec{u}}{m_1+m_2} = \frac{m_1-m_2}{m_1+m_2}\ell \vec{u} \Rightarrow \ddot{\vec{r}}_G = -\frac{m_1-m_2}{m_1+m_2}\ell \operatorname{sen}\alpha\, \omega^2 \vec{j}$$

$$(m_1+m_2)\ddot{\vec{r}}_G = \vec{R}_o^{ext}$$

$$\vec{R}_o^{ext} = -(m_1-m_2)\ell\omega^2 \operatorname{sen}\alpha\, \vec{j}$$



$$\vec{M}_o^{ext, act} = \ell\vec{u} \times (-m_1 g)\vec{J} - \ell\vec{u} \times (-m_2 g)\vec{J} =$$

$$= \ell\vec{u} \times (m_2 - m_1) g \vec{J} =$$

$$= \left(\ell\cos\alpha\, \vec{i} + \ell \operatorname{sen}\alpha\cos\beta\, \vec{J} + \ell\operatorname{sen}\alpha\operatorname{sen}\beta\, \vec{K}\right) \times (m_2-m_1) g \vec{J} =$$

$$= (m_2-m_1) g \ell \left(\cos\alpha\, \vec{k} - \operatorname{sen}\alpha\operatorname{sen}\beta\, \vec{i} \right)$$

(b)

$$\vec{M}_O^{ext} = \vec{M}_O^{ext,act} + \vec{M}_O^{ext,react.}$$

$$\boxed{\begin{aligned}\vec{M}_O^{ext,react.} = & -(m_1+m_2)\,l^2\omega^2 \operatorname{sen}\alpha \cos\alpha\,\vec{k} \\ & -(m_2-m_1)\,lg\,\cos\alpha\,\vec{K} \\ & +(m_2-m_1)\,lg\,\operatorname{sen}\alpha\,\operatorname{sen}(\omega t)\,\vec{\imath}\end{aligned}}$$

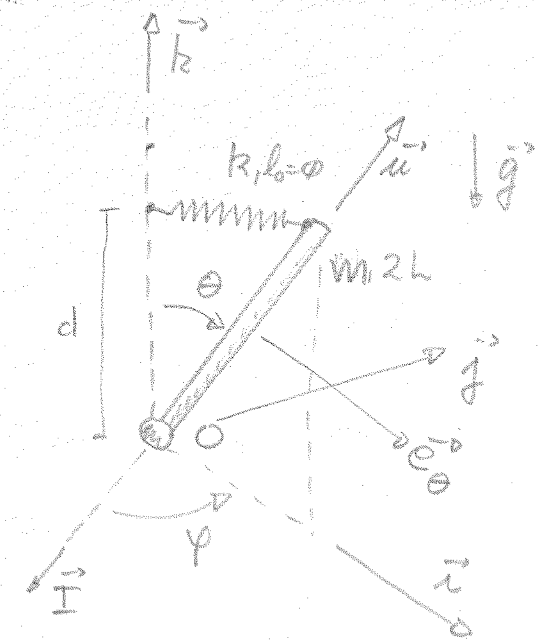
$$\vec{R}^{ext} = \vec{R}^{ext,act.} + \vec{R}^{ext,react.}$$

$$\vec{R}^{ext,act} = -(m_1+m_2)\,g\,\vec{J}$$

$$\boxed{\vec{R}^{ext,react} = -(m_1-m_2)\,l\omega^2 \operatorname{sen}\alpha\,\vec{f} + (m_1+m_2)\,g\,\vec{J}^0}$$

(iii)

ⓐ $\begin{cases} \vec{L}_o \cdot \vec{k} = c^{te} \\ E = c^{te} \end{cases}$



$$U_g = mgL\cos\theta$$

$$U_e = \frac{k}{2}\ell^2 = \frac{k}{2}\left(d^2 + 4L^2 - 4Ld\cos\theta\right)$$

$$U_e' = -2kLd\cos\theta$$

$$T = \frac{1}{2}\vec{\omega}\,\mathbb{I}_o\,\vec{\omega} \quad ; \quad \vec{\omega} = \dot{\varphi}\vec{k} + \dot{\theta}\vec{j} = \dot{\varphi}\cos\theta\,\vec{u} - \dot{\varphi}\,\text{sen}\,\theta\,\vec{e}_\theta + \dot{\theta}\,\vec{j}$$

$$\underset{(\vec{j},\vec{u},\vec{e}_\theta)}{\mathbb{I}_o} = \frac{4mL^2}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \frac{2mL^2}{3}\left(\dot{\theta}^2 + \dot{\varphi}^2\,\text{sen}^2\theta\right)$$

$$E = T + U = \frac{2mL^2}{3}\left(\dot{\theta}^2 + \dot{\varphi}^2\,\text{sen}^2\theta\right) + mgL\cos\theta - 2kLd\cos\theta$$

$$\vec{L}_o = \mathbb{I}_o\vec{\omega} = \frac{4mL^2}{3}\left(\dot{\theta}\,\vec{j} - \dot{\varphi}\,\text{sen}\,\theta\,\vec{e}_\theta\right)$$

$$\vec{L}_o \cdot \vec{k} = \frac{4mL^2}{3}\dot{\varphi}\,\text{sen}^2\theta$$

$$\frac{2}{3}\left(\dot{\theta}^2 + \dot{\varphi}^2 \operatorname{sen}^2\theta\right) + \left(g/L - \frac{2k}{m}\, d/L\right)\cos\theta = c^{te}$$

$$\dot{\varphi}\operatorname{sen}^2\theta = c^{te\,'}$$

Las ecuaciones del movimiento de un sistema corresponden formalmente a las derivadas de estas ecuaciones.

(b) $\vec{\omega}(t=0) = \omega_0 \vec{k}$; $\theta(t=0) = \pi/2$ $\Big) \Rightarrow \begin{array}{l} \dot{\theta}(0) = 0 \\ \dot{\varphi}(0) = \omega_0 \end{array}$

$$\frac{2}{3}\left(\dot{\theta}^2 + \dot{\varphi}^2 \operatorname{sen}^2\theta\right) + \left(g/L - \frac{2k}{m}\, d/L\right)\cos\theta = \frac{2\omega_0^2}{3}$$

$$\dot{\varphi}\operatorname{sen}^2\theta = \omega_0$$

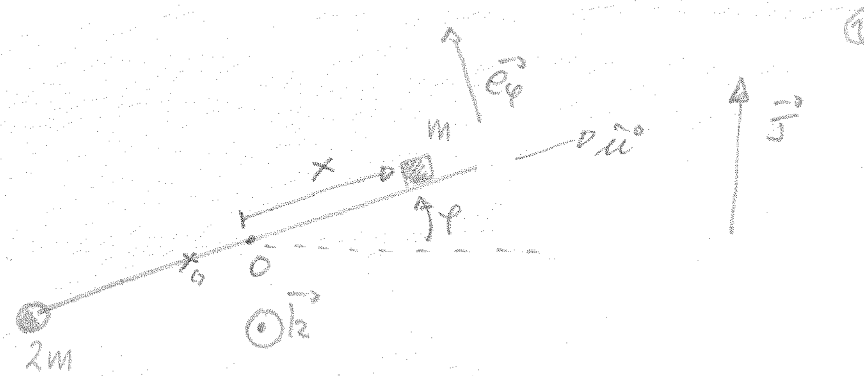
$$\frac{2}{3}\left(\dot{\theta}^2 + \frac{\omega_0^2}{\operatorname{sen}^2\theta}\right) + \left(-\right)\cos\theta = \frac{2\omega_0^2}{3}$$

Estiramiento mínimo resorte $\Rightarrow$ extremo en mov. $\theta \Rightarrow \dot{\theta} = 0$.

$$\frac{\omega_0^2}{\operatorname{sen}^2\theta} + \frac{3}{2}\left(g/L - \frac{2k}{m}\cdot d/L\right)\cos\theta = \omega_0^2$$

$$\boxed{\frac{1}{\operatorname{sen}^2\theta} + \frac{3}{2\omega_0^2}\left(g/L - \frac{2k}{m}\, d/L\right)\cos\theta = 1}$$

(a)



$$I_O = 2ml^2 + ml^2 = 3ml^2$$

$$\vec r_G = \frac{(-2ml + ml)\vec u}{3m} = -\frac{l}{3}\vec u$$

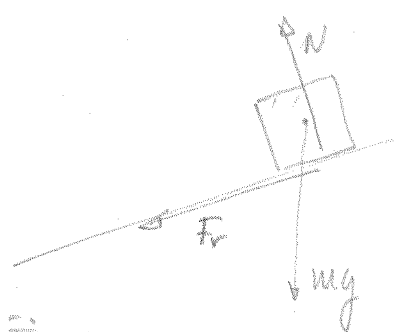
$$I_O \vec\omega = 3ml^2 \dot\varphi \vec k = \vec L_O$$

$$\dot{\vec L}_O = 3ml^2 \ddot\varphi \vec k = M^{ext}_O = 3mg\left(\frac{l}{3}\right)\cos\varphi \, \vec k$$

$$l\ddot\varphi = \frac{g}{3}\cos\varphi$$

$$\boxed{\ddot\varphi - \frac{g}{3l}\cos\varphi = 0}$$

(b)



$$m\vec a = -mg\vec J + N\vec e_\varphi - F_r \vec u$$

$$m(l\ddot\varphi \, \vec e_\varphi - l\dot\varphi^2 \vec u) = -mg\vec J + N\vec e_\varphi - F_r\vec u$$

$$\begin{cases} ml\ddot\varphi = N - mg\cos\varphi \\ -ml\dot\varphi^2 = -F_r - mg\,\mathrm{sen}\,\varphi \end{cases}$$

$$N = mg\cos\varphi + ml\ddot\varphi$$

$$F_r = ml\dot\varphi^2 - mg\,\mathrm{sen}\,\varphi$$

No deslizamiento $\Leftrightarrow$ $|F_r| \le \mu_e N$

$$\ddot\varphi = \frac{g}{3l}\cos\varphi$$

$$\dot\varphi\ddot\varphi = \frac{g}{3l}\cos\varphi\,\dot\varphi$$

$$\frac{\dot\varphi^2}{2} - \frac{\dot\varphi_0^2}{2} = \frac{g}{3l}(\mathrm{sen}\,\varphi - \mathrm{sen}\,\varphi_0)$$

$$\dot\varphi^2 = \frac{2g}{3l}\mathrm{sen}\,\varphi$$

vi

(b)

$$N = mg\cos\varphi + \frac{mg}{3}\cos\varphi = \frac{4}{3}mg\cos\varphi$$

$$F_r = \frac{2}{3}mg\,\text{sen}\varphi - mg\,\text{sen}\varphi = -\frac{mg}{3}\text{sen}\varphi$$

¿ $|F_r| = \mu_e N$?

$$\frac{mg}{3}\text{sen}\varphi_D = \frac{1}{4}\cdot\frac{4}{3}mg\cos\varphi_D$$

$$tg\,\varphi_D = 1$$

(obs.: $N > \phi$)

$$\boxed{\varphi_D = 45°}$$

(c)

$$\vec{r}_m = x\,\vec{u}$$
$$\dot{\vec{r}}_m = \dot{x}\,\vec{u} + x\dot{\varphi}\,\vec{e}_\varphi$$
$$\ddot{\vec{r}}_m = \ddot{x}\,\vec{u} + 2\dot{x}\dot{\varphi}\,\vec{e}_\varphi + x\ddot{\varphi}\,\vec{e}_\varphi - x\dot{\varphi}^2\,\vec{u}$$

$$m(x\ddot{\varphi} + 2\dot{x}\dot{\varphi}) = N - mg\cos\varphi$$

$$\dot{\vec{L}}_o' = 2m\ell^2\ddot{\varphi}\,\vec{k} = 2mg\,\ell\cos\varphi\,\vec{k} - Nx\,\vec{k}$$

$$2m\ell^2\ddot{\varphi} = 2mg\ell\cos\varphi - m(x\ddot{\varphi} + 2\dot{x}\dot{\varphi})x - mg\ell\cos\varphi$$

$\varphi = \varphi_D = 0$
$x = \ell$
$\dot{x} = 0$
$\dot{\varphi} = \sqrt{\frac{2}{3}\,g/\ell\,\text{sen}\varphi_D}$

$$2\ell\ddot{\varphi}_D = 2g\cos\varphi_D - \ell\ddot{\varphi}_D - g\cos\varphi_D$$

$$3\ell\ddot{\varphi}_D = g\cos\varphi_D$$

$$\boxed{\ddot{\varphi}_D = \frac{g}{3\ell}\cdot\frac{\sqrt{2}}{2}}$$