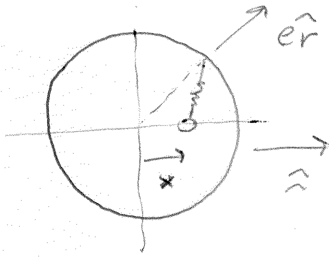
Examen Mecánica Newtoniana 30/07/10

Ej1)



1) Ecuación de movimiento:

$$-k(x\hat{x} - R\hat{e}_r)\cdot\hat{x} = m(\ddot{x} - x\omega^2)$$

$$\boxed{\ddot{x} + (\omega_0^2 - \omega^2)x = \omega_0^2 R \,\text{sen}\,\varphi_0} \qquad \omega_0^2 = k/m$$

2) a) $\ddot{x} = 0$: $(\omega_0^2 - \omega^2)x_{eq} = \omega_0^2 R\,\text{sen}\,\varphi_0$

si $\omega \neq \omega_0$: $x_{eq} = \dfrac{\omega_0^2 R\,\text{sen}\,\varphi_0}{\omega_0^2 - \omega^2}$; como la ec. de movimiento es de la forma: $\ddot{x} + f(x) = 0$, con $f'(x) = \omega_0^2 - \omega^2$, x_{eq} es estable (inestable) para $\omega_0 > \omega$ $(<)$

si $\omega = \omega_0$: no hay posición de equilibrio

b) $\omega_0 > \omega$: $\ddot{x} + (\omega_0^2 - \omega^2)x = \omega_0^2 R\,\text{sen}\,\varphi_0$

con $x(0) = 0$
$\dot{x}(0) = 0$

$$\Rightarrow x(t) = \frac{\omega_0^2 R\,\text{sen}\,\varphi_0}{(\omega_0^2 - \omega^2)}\left(1 - \cos\sqrt{\omega_0^2 - \omega^2}\,t\right) ; \quad x_{min} = 0$$

$$x_{max} = \frac{2\omega_0^2 R\,\text{sen}\,\varphi_0}{\omega_0^2 - \omega^2}$$

si $x_{max} \leq R$, la partícula no escapa del diámetro

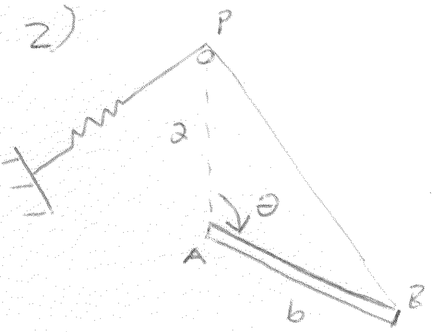
$\omega = \omega_0$: $\ddot{x} = \omega_0^2 R\,\text{sen}\,\varphi_0$ $\quad (x(0) = 0,\ \dot{x}(0) = 0)$

$\Rightarrow x(t) = \omega_0^2 R\,\text{sen}\,\varphi_0 \dfrac{t^2}{2}$: la partícula siempre escapa

$\omega > \omega_0$: $\ddot{x} - (\omega^2 - \omega_0^2)x = \omega_0^2 R\,\text{sen}\,\varphi_0$ $\quad (x(0) = 0,\ \dot{x}(0) = 0)$

$\Rightarrow x(t) = \dfrac{\omega_0^2 R\,\text{sen}\,\varphi_0}{(\omega^2 - \omega_0^2)}\left(\text{ch}\sqrt{\omega^2 - \omega_0^2}\,t - 1\right)$: como $\text{ch}\,\alpha t$ es divergente con t, la partícula escapa siempre

Ej 2)



1) A partir de la conservación de la energía tenemos:

$$\frac{1}{2}K(\overline{BP})^2 + Mg\frac{b}{2}\cos\theta + \frac{1}{2}I_A\dot{\theta}^2 = cte$$

$$(\overline{BP})^2 = (a - b\cos\theta)^2 + (b\,\text{sen}\,\theta)^2$$

$$(\overline{BP})^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\Rightarrow \frac{1}{2}K(a^2+b^2-2ab\cos\theta) + Mg\frac{b}{2}\cos\theta + \frac{1}{2}\left(\frac{Mb^2}{3}\right)\dot{\theta}^2 = cte$$

$$\boxed{\left(Mg\frac{b}{2} - Kab\right)\cos\theta + \frac{1}{2}\frac{Mb^2}{3}\dot{\theta}^2 = cte}$$

2) De acuerdo a las condiciones iniciales:

$$\left(Mg\frac{b}{2} - Kab\right)\cos\theta + \frac{1}{2}\frac{Mb^2}{3}\dot{\theta}^2 = Mg\frac{b}{2} - Kab + \frac{1}{2}\frac{Mb^2}{3}\Omega_0^2$$

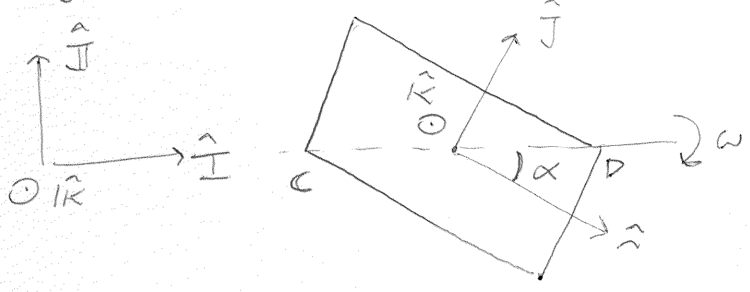
$$\frac{1}{2}\frac{Mb^2}{3}\dot{\theta}^2 = \left(Mg\frac{b}{2} - Kab\right)(1-\cos\theta) + \frac{1}{2}\frac{Mb^2}{3}\Omega_0^2 > 0 \quad \forall\theta$$ de modo que la barra no se detenga

- Si $\frac{Mg}{2} - Ka \geq 0$ alcanza con $\boxed{\Omega_0^2 > 0}$

- Si $\frac{Mg}{2} - Ka < 0 \Rightarrow \frac{1}{2}\frac{Mb}{3}\Omega_0^2 > 2\left(Ka - \frac{Mg}{2}\right)$

$$\boxed{\Omega_0^2 > \frac{6}{Mb}(2Ka - Mg)}$$

Ej 3)



1) $\mathbb{I}_o^{\{\hat{i},\hat{j},\hat{k}\}} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_1+I_2 \end{pmatrix}$

$$I_1 = \int_{-a}^{a} dx \int_{-a/2}^{a/2} dy \, \frac{M}{2a^2} y^2 = \frac{Ma^2}{12}$$

idem: $I_2 = \frac{M}{12}(2a)^2$

$$\mathbb{I}_o^{\{\hat{i},\hat{j},\hat{k}\}} = \frac{Ma^2}{12} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$\vec{L}_o = \mathbb{I}_o \vec{\omega}$; $\vec{\omega} = \omega \hat{I} = \omega [\cos\alpha \, \hat{i} + \operatorname{sen}\alpha \, \hat{j}]$, $\cos\alpha = \frac{2}{\sqrt{5}}$, $\operatorname{sen}\alpha = \frac{1}{\sqrt{5}}$

$$\boxed{\vec{L}_o = \frac{Ma^2\omega}{12} [\cos\alpha \, \hat{i} + 4\operatorname{sen}\alpha \, \hat{j}]}$$

2) $\dot{\vec{L}}_o \cdot \hat{I} = \vec{M}_o^{(ext)} \cdot \hat{I} = 0 \Rightarrow \vec{L}_o \cdot \hat{I} = cte$:

$\hat{I} \cdot \mathbb{I}_o \vec{\omega} = \hat{I} \cdot \mathbb{I}_o \omega \hat{I} = \underbrace{(\hat{I} \cdot \mathbb{I}_o \hat{I})}_{I_{o,\hat{I}} = cte.} \omega = cte \Rightarrow \omega = cte.$ ✓

3) Sean $R_{C_{2,3}}$ las componentes según $\hat{J}, \hat{K}$ de la reacción en C (idem en D)

$\vec{L}_C = \vec{L}_o + \underbrace{\vec{P}}_{=0} \times (C-O) = \vec{L}_o$

$\dot{\vec{L}}_C = \vec{M}_C^{(ext)}$

$\dot{\vec{L}}_o = \vec{\omega} \times \mathbb{I}_o \vec{\omega} = \frac{Ma^2\omega^2}{12} (4\operatorname{sen}\alpha\cos\alpha - \operatorname{sen}\alpha\cos\alpha) \hat{K} = \frac{Ma^2\omega^2}{10} \hat{K}$

$\dot{\vec{L}}_o = \frac{Ma^2\omega^2}{10} (\cos\omega t \, \hat{J} + \operatorname{sen}\omega t \, \hat{K})$

$\vec{M}_C^{(ext)} = R_{D_2} a\sqrt{5} \, \hat{K} - R_{D_3} a\sqrt{5} \, \hat{J} - mg \, a\frac{\sqrt{5}}{2} \, \hat{K}$

$\Rightarrow$ $\boxed{R_{D_3} = -\frac{Ma\omega^2}{10\sqrt{5}} \cos\omega t \qquad R_{D_2} = \frac{Mg}{2} + \frac{Ma\omega^2}{10\sqrt{5}} \operatorname{sen}\omega t}$

1era cardinal según $\hat{J}, \hat{K}$:

$R_{C_2} + R_{D_2} = Mg$

$R_{C_3} + R_{D_3} = 0$

$\Rightarrow$ $\boxed{R_{C_2} = \frac{Mg}{2} - \frac{Ma\omega^2}{10\sqrt{5}} \operatorname{sen}\omega t \qquad R_{C_3} = \frac{Ma\omega^2}{10\sqrt{5}} \cos\omega t}$