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A model for the compressibility of food–finger(s) arrays

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Synopsis

In a compressed in-series array of bodies having the same cross-sectional area, the force is the same and the deformation the sum of that of the individual components. This concept, with certain simplifying assumptions, was used to develop a simple mathematical model with which the compressive force–deformation relationships of a finger–object array can be predicted on the basis of the compressive behavior of the individual components. The models' applicability is demonstrated with experimental compression data of a human's index finger and thumb and food specimens having concave upward or downward force–deformation curves (hot dog, cheddar cheese, and cream cheese), tested alone and in an in-series array. This was done by representing the force–deformation curve of each component by a two-parameter mathematical model whose form had been specially selected so that the force–deformation relationship of the array could be calculated by an explicit algebraic expression.

INTRODUCTION

The texture of many solid foods, as well as other materials, is frequently assessed, at least initially, by squeezing them with a finger or between the fingers. This resembles, at least superficially, a compression test. The main differences between such a sensory assessment and a compression test performed by man-made machines are the test geometry and the fact that the fingers can have deformability that is comparable or even higher than that of the tested object (Fig. 1). This is in sharp contrast with the man-made testing machines, whose own deformability is negligible when operated in their designed load range. The effect of the testing machine's small deformation on the results of mechanical tests has been primarily studied in the context of eliminating

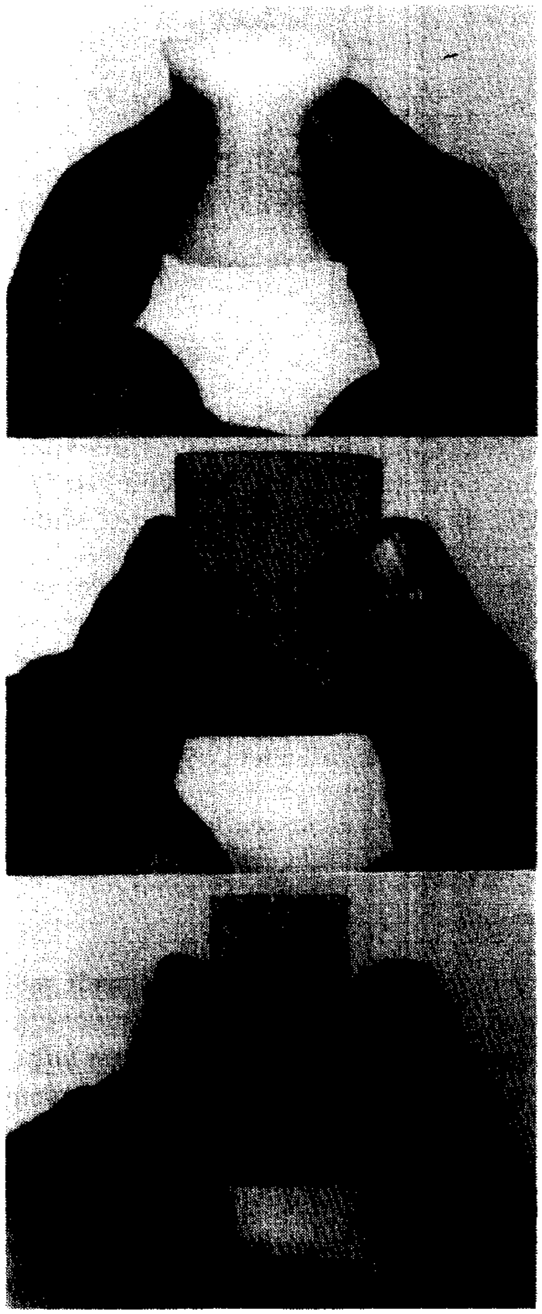

FIG. 1. Three situations in "hardness" assessment by compression when using the fingers; soft object: practically all the deformation is in the tested object (left); intermediate object: both object and finger are substantially deformed (center); and hard object: practically all the deformation is in the finger or fingers (right).

instrumental artifacts (e.g., Saraf and Porter, 1987). When the machine's own deformation is large, as in the case of the fingers, it has two main impacts. The system's mechanical sensitivity is considerably affected and becomes force dependent (Peleg and Campanella, 1989) and the resulting force–deformation relationships are considerably altered (Peleg, 1983; Campanella and Peleg, 1988).

Manners in which the finger deformability and properties can theoretically affect the outcome of the squeezing test in which a liquid or solid specimen is compressed between the fingers have been previously studied assuming an ideal geometry and oversimplified rheological models (Peleg, 1983; Campanella and Peleg, 1987, 1988). Such analyses, however, only show the kind of rheological behavior that the finger–object system can exhibit, but hardly provide quantitative information on the magnitude of the effects.

Because of the complexity of the fingers geometry and internal structure and the rheological properties of both fingers and foods, analysis of their combined deformation as a contact stress problem must be an extremely difficult task. Because the resulting deformations are large and the fingers compressible, solving such a problem even by a finite-element analysis still requires many simplifying assumptions regarding the finger geometry and structure and arbitrary selection of rheological constants. An alternative approach is to look at the food–finger(s) system as a whole and try to quantify its behavior in terms of phenomenological models. Such an approach does not address the rheological behavior in detail, but it can lead to a quantitative assessment or estimation of the differences between the force–deformation of foods as recorded by testing machines and those produced during squeezing by a finger or between the fingers. Such information can be used to explain discrepancies between sensory and instrumental evaluations of "texture" or even be used to "correct" instrumental data, so that they will give a better account of the behavior of the object during its sensory evaluations.

THE MODEL

During sensory assessment of hardness with the finger or fingers, the object and the finger(s) form an in-series array. Ideally, that is, when two bodies in-series have the same cross-sectional area and lateral stresses at the contact area can be ignored, the force–deformation of their array can be calculated by assuming that the force along the array is the same and the total deformation is additive (Swyngedau *et al.*, 1991).

On the basis of experimental evidence, it is known that the compressive force–deformation curve of the fingers is concave upward. It can be represented by a power-law model (Peleg and Campanella, 1989) or alternatively by an expression of the kind

$$F=k_1x_1/(1-b_1x_1), \tag{1}$$

where F is the force, x_1 the absolute deformation in length units ($0\leqslant x_1<1/b_1$), and k_1 and b_1 constants. [In the following discussion the subscript 1 will refer to the finger(s) and the subscript 2 to the tested object.]

Equation (1) can be rearranged and written in the form

$$x_1=F/(k_1+b_1F), \tag{2}$$

which makes it more convenient to deal with systems in which deformations are added. In order to calculate the constants k_1 and b_1, this model, in either form, can be fitted to the force deformation data either by nonlinear regression or by linear regression of the transformed relationship F/x_1 vs F (Swyngedau *et al.*, 1991).

The deformability of a tested object can also be expressed by a similar expression, i.e.,

$$F=k_2x_2/(1-b_2x_2) \quad \text{or} \quad x_2=F/(k_2+b_2F). \tag{3}$$

Most commonly the food's compressive force–deformation curve is concave upward, in which case $b_2>0$. For highly yielding materials, certain cheeses, for example, the compressive force-deformation curve is concave downward in which case $b_2<0$. If the food's force–deformation relationship is linear $b_2=0$ and Eq. (3) is reduced to

$$F=k_2x_2 \quad \text{or} \quad x_2=F/k_2. \tag{4}$$

The deformation of the finger–food array, X, at any given force is given by the sum of the component's deformation, i.e.,

$$X=x_1+x_2 \tag{5}$$

or

$$X=F/(k_1+b_1F)+F/(k_2+b_2F). \tag{6}$$

The format of Eqs. (1) and (3) enables the expression of the force-deformation relationship of the array to be written as an explicit function $[F=F(X)]$, i.e.,

$$F=\{(k_1b_2+k_2b_1)X-k_1-k_2 + \sqrt{[(k_1+k_2)-(k_1b_2+k_2b_1)X]^2+4(b_1+b_2-b_1b_2X)k_1k_2X}\} \times[2(b_1+b_2-b_1b_2X)]^{-1}. \tag{7}$$

[Although the quadratic equation from which Eq. (7) is derived has two real solutions, only the positive root satisfies the conditions that $F(0)=0$ and $F(X>0)>0$.] Equation (7), despite its somewhat cumbersome structure, is an explicit algebraic expression and can easily be used to calculate the force–deformation curve of arrays for which the k's and b's are known or can be estimated. If the force–deformation relationship of the tested object is linear ($b_2=0$), Eq. (7) is reduced to

$$F=\frac{k_2b_1X-k_1-k_2+\sqrt{(k_1+k_2-k_2b_1X)^2+4k_1k_2b_1X}}{2b_1}, \tag{8}$$

a relationship that can also be derived directly from Eq. (6).

Since Eqs. (1)–(4) are based on force–deformation relationships, their constants, namely the k's and b's, ought to correspond to objects with exactly the same dimensions. If the available data are of a specimen with different dimensions or are given in the form of stress–strain relationships, the problems can be solved by converting the stress–strain relationship to the pertinent force–deformation relationship using a desired object's specific dimensions, i.e.,

$$x_2=H_{02}\epsilon \tag{9}$$

and

$$F=A_{02}\sigma, \tag{10}$$

where H_{02} and A_{02} are the specimen height and area and ϵ and σ the engineering strain and engineering stress, respectively. A similar substitution can be derived for a true stress versus Hencky's strain relationship in case the information regarding the object's behavior is given in this form (Swyngedau *et al.*, 1991).

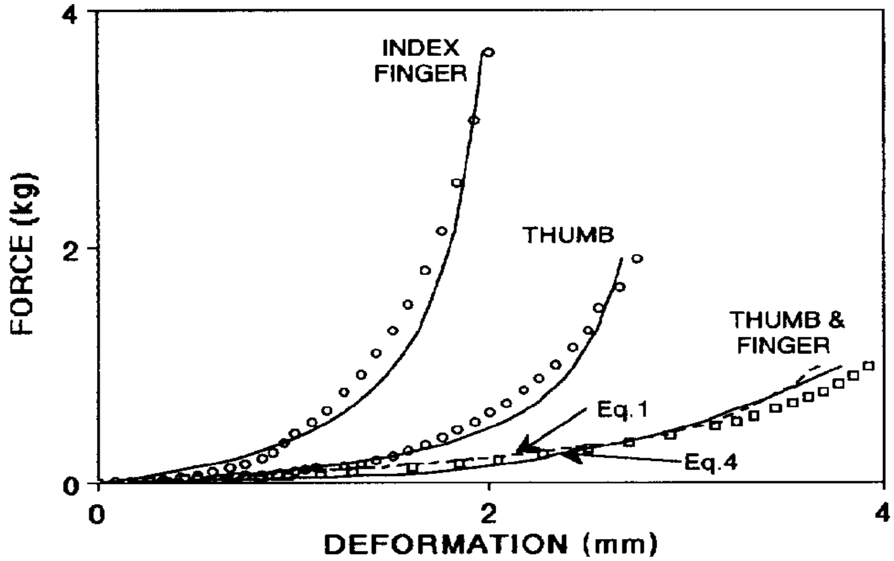

FIG. 2. Experimental compressive force–deformation relationships of a human's index finger, thumb, and their in-series combination. Note the fit of Eq. (1) to the deformation data of the fingers alone and in combination with the prediction using Eq. (4). (Open circles and squares are the experimental data.)

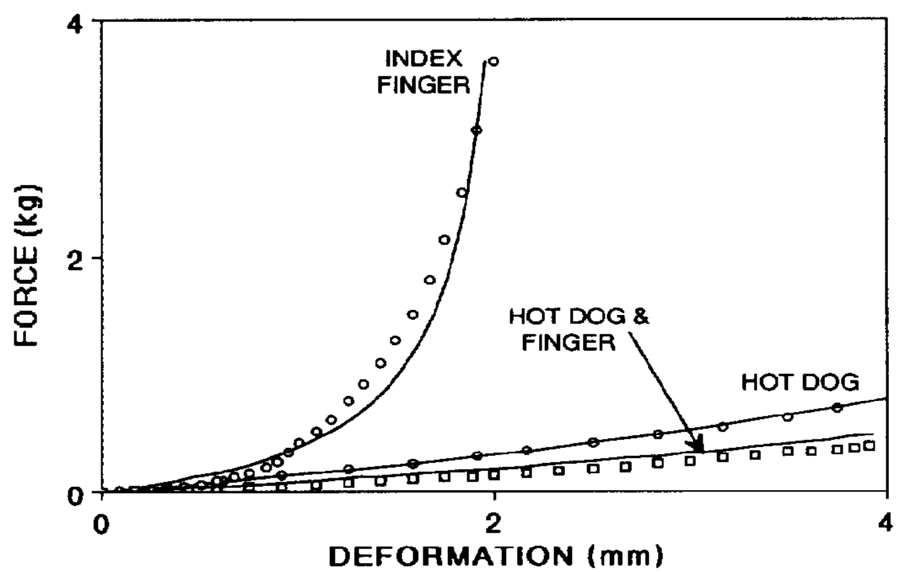

FIG. 3. Prediction of the force–deformation relationships of a finger–hot dog combination using Eq. (4). [Open squares are the experimental data and the solid line the prediction of Eq. (4).]

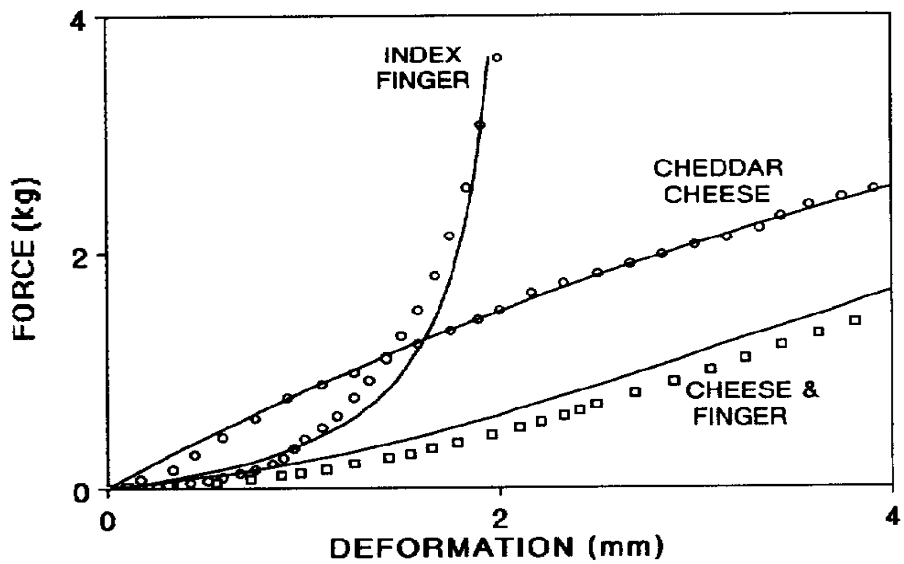

FIG. 4. Prediction of the force–deformation relationships of a finger–cheddar cheese combination using Eq. (11). [Open squares are the experimental data and the solid line the prediction of Eq. (11).]

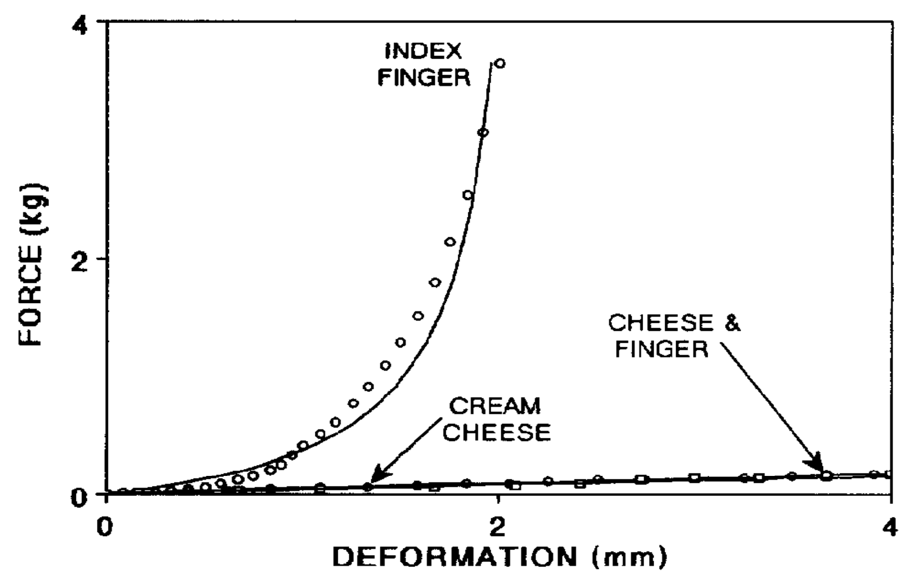

FIG. 5. Prediction of the force–deformation relationships of a finger–cream cheese combination using Eq. (11). [Open squares are the experimental data and the solid line the prediction of Eq. (11).] Note that because the cream cheese is much softer than the finger, its force–deformation relationship when pressed by a finger is not very different from that obtained by a rigid testing machine.

TABLE I. Compressive deformability constants of fingers and selected foods.[a]

Object	Dimensions[b]	k (kgf mm^{-1})	b (mm^{-1})	MSE[c]
Index finger	...	0.20	0.45	0.016
Thumb	...	0.08	0.33	0.010
Index finger and thumb	...	0.05	0.20	0.024
Hot dog	$D = 21$ mm $H = 20$ mm	0.14	0.08	0.004
Cheddar cheese	$D = 19$ mm $H = 20$ mm	0.92	-0.11	0.003
Cream cheese	$D = 20$ mm $H = 20$ mm	0.06	-0.10	0.007

[a]Determined by nonlinear regression of digitized force–deformation relationships obtained at a deformation rate of 10 mm min^{-1}.
[b]Cylindrical samples of which D and H are the diameter and height, respectively.
[c]MSE is the mean square error.

DEMONSTRATION OF THE MODEL'S CAPABILITY

Examples of compressive force–deformation relationships of the index finger, thumb, and three food materials are shown in Figs. 2–5. They were determined between two parallel metal plates at a deformation rate of 10 mm min^{-1} using an Instron Universal Testing Machine, model 1000. The results were confirmed by performing the tests in triplicate with a fresh food sample in each test. At the employed level of deformation (strain) rate the rheological behavior of the fingers and foods is not strongly rate dependent. So although the effective strain rate at which the arrays were tested is lower than that of the components when tested separately, the deformation rate was not adjusted. The Instron was interfaced with a Macintosh II microcomputer. The software included the SYSTAT package so that the model's constants, the k's and b's, could be calculated directly by nonlinear regression. The fit of the models [Eqs. (1) or (3)] is also shown in the figures and the magnitude of the constants k and b is listed in Table I. The figures and the magnitude of the mean square error (MSE), which is a measure of the goodness of fit, demonstrate that the proposed mathematical models can satisfactorily describe the various deformability patterns of the fingers and foods, irrespective of whether they have an upward or downward concavity.

Figure 2 shows not only the force-deformation relationships of an index finger and thumb but also that of their combination, that is, when they were compressed together. As can be seen from the figure and Table I, the force–deformation of the combination could also be described by Eq. (1). It could also be predicted, however, using Eq. (7), demonstrating that the model was valid for this system. The successful prediction also confirms that the effect of any lateral stresses between the fingers can be neglected in this kind of analysis. It also demonstrates that the force–deformation relationship can be satisfactorily described by more than one kind of a mathematical model.

The force–deformation relationships of the index finger combined with a specimen of hot dog, cheddar cheese, and cream cheese are shown in Figs. 3–5. Also shown in the figures are the predictions of the model expressed by Eq. (7). As in the case of the index finger-thumb array, the fit was highly satisfactory, as is also evident from the magnitude of the MSEs shown in Table I. The figures also show that in the case of the hot dog and cheese, Figs. 3 and 4, the finger deformability had a profound effect on the resulting force–deformation relationships. That is, the force-deformation of these specimens when pressed between the fingers is very different from that obtained with a rigid testing machine. Because the deformability of the cream cheese was much higher than that of the finger, Fig. 5, the latter had practically no effect on the resulting force–deformation relationship. This demonstrates that agreement between force–deformation relationships obtained sensorily and by machines exists only in the case of very soft foods. It ought to be mentioned though that in all the tested arrays the test geometry was fairly flat and resembled, at least to some extent, that of a standard compression test or, in other words, in the described tests the finger deformed but did not penetrate the food or vice versa. The models would probably not work so well if penetration does take place or when the object's geometry is grossly distorted. In such cases the assumptions on which the model is based would be violated and the forces would have to be accounted for by more complicated models than the one described. It should also be mentioned that since the deformation rate was the same in all the tests the strain rate in the array was lower than that of the food specimens tested alone. But, as could be expected, this factor had practically no effect on the results. The demonstration of the model applicability (Figs. 3–5) is based on the behavior of finger-food arrays, with force-deformation relationships that correspond to those produced during pressing an object against a hard surface. In principle, the method can also be used to predict the force–deformation or force–time relationship when an object is pressed between the index finger and thumb. In such a case the constants of the finger–thumb combination (Table I) will replace k_1 and b_1 in Eq. (7), which will otherwise be used in the same way. It is obvious that the combined deformability of the finger and thumb is much larger than that of the index finger or thumb alone and, therefore, the force–deformation curve of the index finger–food–thumb system will be much flatter than that produced with the index finger alone.

POTENTIAL APPLICATIONS

It has been demonstrated that the force–deformation relationship generated in a "soft machine" like the fingers can be very different from that obtained by a man-made "hard machine" when the tested object is relatively hard to the fingers. It was also shown that even when the test geometry was only approximately flat, the force–deformation relationship of the finger–object array could be predicted with reasonable accuracy. Although the examples given are of a finger and selected foods, the concept and probably the mathematical procedure can be extended to nonfood objects whose hardness or softness is also assessed by squeezing with a finger or between the fingers. The concept can also be useful in the interpretation of experiments in which psychophysical relationships are determined through compression with the fingers or hand, using calibrated springs or rubbery objects with known mechanical properties. A psychophysical relationship is the relationship between the stimulus intensity as measured instrumentally and its sensorily perceived intensity (Scot-Blair, 1958; Stevens, 1975). Over a considerable range of stimulus intensity, it can be described by a power-law relationship (Stevens, 1975), i.e.,

$$\Psi = K\phi^n, \tag{11}$$

where ϕ is the stimulus magnitude, ψ its perceived magnitude, and K and n constants. For mechanical stimuli, n is in the range 0.4–2 (Harper and Stevens, 1964; Stevens, 1975; Moskowitz *et al.*, 1974). This and previous work indicate, however, that the magnitude of the objective mechanical stimulus can be significantly affected by the sensory system's own deformability, a factor that ought to be taken into account in the interpretation of psychophysical relationships.

The mechanical sensitivity of soft machines and other rheological characteristics of their output have been discussed elsewhere (Peleg and Campanella, 1989). These studies as well as this work only address the mechanics of the compression process and not the perception mechanism of the information that they generate. It can be argued though that whether the latter is examined in the form of overall psychophysical relationships, or in terms of temporal events at the mechanoreceptor level, the source of information is still the deformability of the finger–object *array* and not that of the compressed object only. The former it seems, however, can be reasonably estimated or reproduced on the basis of the latter using the described mathematical procedure.

The latter can also be used to assess the role of the finger(s) hardness on the "performance" of the hand as a testing machine. In this work, however, the fingers of only one individual were employed. Therefore, it would be an interesting topic for further research to establish how the properties of fingers, in terms of k_1 and b_1 in Eq. (1), vary among humans. Once determined, though, they can be incorporated into the software of testing machines to produce, using Eq. (7) [or (8)], modified force–deformation or force–time relationships that are closer to those actually generated in sensory assessments.

ACKNOWLEDGMENTS

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