Mathematical Characterization of the Compressive Stress-Strain Relationships of Spongy Baked Goods

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ABSTRACT

The compressive stress-strain relationships of selected breads, English muffins and sponge cake were determined with a Universal Testing Machine and described mathematically in terms of a three-parameter empirical model. These parameters gave account of the overall mechanical resistance of the sponge, the existence of a shoulder in its stress-strain curve and the strain level at which densification of the sponge becomes the dominant deformation mechanism. They were also sensitive indicators of the changes that were observed in the stress-strain relationship as a result of successive compression and could be used to distinguish between the deformation patterns of the various products.

INTRODUCTION

THE TEXTURAL PROPERTIES of breads and other baked goods have frequently been evaluated through compression tests performed with a Universal testing machine (e.g. Baker et al., 1986, 1987; Baker and Ponte 1987; Walker et al., 1987). The resulting force-deformation curves had a characteristic shape that could provide, on top of the overall force level, other parameters such as local slopes. All of these were used to characterize the product and the changes it underwent during storage. Since the force-deformation curve is affected by the test conditions, particularly the geometric features, their effects on the results were also studied (Baker et al., 1986; Walker et al., 1987).

Since bread and a few kinds of cakes have a spongy structure it is of interest to study their compressive behavior in light of mechanical theories specifically developed for solid foams and cellular solids. A comprehensive review of the theoretical relationships between the structure of cellular solids and their mechanical properties was published by Ashby (1983). It deals mainly with the effects of the cell wall length, thickness and strength on the deformability and fracture of the foam, and presents theoretical as well as experimental relationships between various mechanical parameters and the foam's relative density. (Relative density in this context is the ratio between the foam density and the absolute density of its solid matrix).

The theoretical derivations in Ashby's paper (1983) are based on elastic cell walls and uniform cell size. Although in breads and other baked products neither exist, the analysis provides at least a qualitative indication as to the role of the cellular structure in determining the mechanics of solid foams. This is particularly the case with respect to the overall shape of the compressive stress-strain relationship. Its typical shape is shown schematically in Fig. 1a. The curve has three clearly identified regions, namely, elastic or quasi-elastic deformation as a result of cell-wall bending, collapse as the cell wall buckles and yield or fracture and densification as a result of cell walls crushing together. In food testing, as in the case of engineering materials, it is often expedient to subject the specimen to repeated compression cycles. Subjecting spongy bakery products to such tests frequently results in the disappearance of the typical "shoulder" from the stress-strain curves of the second and/or subsequent compression as shown schematically in Fig. 1b.

Since a considerable amount of the foam's original resistance to deformation is contributed by air cells, the rupture of their walls during the collapse stage of the first compression weakens or eliminates this source of resistance in subsequent bites. Thus, a low density sponge with all or practically all of its cells wide open shows relatively low mechanical resistance until a considerable part of its collapsed cell wall material becomes engaged. This is manifested by a very low stress level over considerable strain and the absence of a shoulder in the stress-strain curves. Since fracture of the cell walls is an irreversible process, once the shoulder in the stress-strain curve is eliminated it will not appear again in the curves of subsequent compressions.

Although the above gives a clear description of the general deformation mechanism of foams, it provides no guidelines as to how the behavior of different foams can be compared in quantitative terms. The objective of this communication is to propose a three-parameter empirical model that can characterize the stress-strain relationships of foams in their first and any subsequent compression and to demonstrate its usefulness in the mechanical characterization of spongy bakery products.

MATERIALS & METHODS

COMMERCIAL SLICED BREAD and sponge cakes were purchased at a local supermarket. Flat, cylindrical specimens were prepared from these items using a cork borer, avoiding the crust. The specimens had a diameter of 35–40 mm and thickness (height) on the order of 20 mm. To achieve this thickness in the breads, two slices were used. (The exact dimensions were determined for each specimen with a caliper). The specimens were compressed between parallel lubricated plates at a speed of 20 $mm.min^{-1}$ using an Instron Universal Testing Machine model 1000. The Instron was interfaced with a Macintosh II computer (4MB RAM and 80MB disk) through a Strawberry Tree ACM2-12 interface board. Computer software developed by M. D. Normand was used for control of the machine during the data acquisition process and for subsequent data processing. The latter included conversion of the machine's voltage vs real time output to stress-strain relationships with any desired stress and strain definitions, and nonlinear regression fitting of the data to a large variety of selected mathematical models. The nonlinear regression package was adapted from Numerical Recipies (Press et al., 1986) by O. H. Campanella. Since nonlinear regression need not yield unique solutions, the program's fitted paramters were confirmed by comparison with those obtained by the SYSTAT package and SPSS which yielded the same results.

The proposed model

The shape of the curves shown in Fig. 1a can be described by the model

$$\sigma = \frac{C_1\epsilon}{(1 + C_2\epsilon)(C_3 - \epsilon)} \qquad (1)$$

where σ is the engineering stress and ϵ the engineering strain, i.e. the absolute deformation divided by the specimen's initial height. Since the cross-sectional area of foams usually do not expand significantly in compression, the stress can be calculated by dividing the force by the original area.

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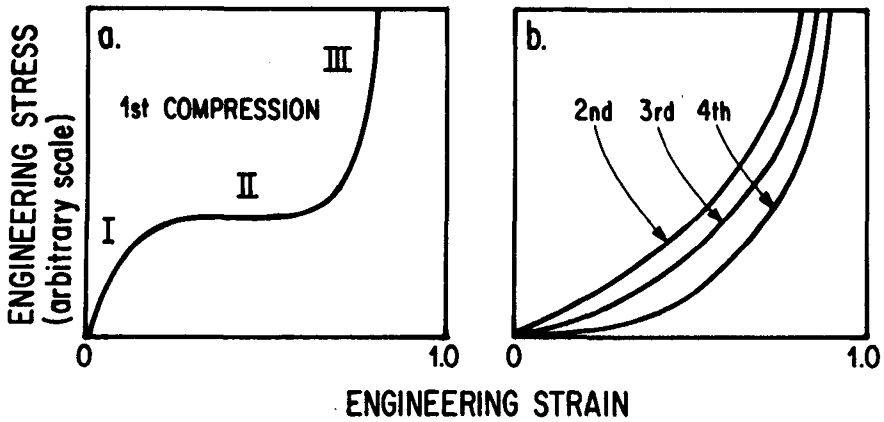

Fig. 1—Schematic view of the typical shape of compressive stress-strain curves of cellular/spongy solids. (a) The first compression curve. Region I represents deformation of the original matrix, II collapse of cell walls and III densification. (b) The typical shape of the curves in successive compressions. (Note the disappearance of shoulder).

Table 1—The shape characteristics of the compressive stress-strain relationships of various baked goods[a]

Product	Specimen height (mm)	Compression	C_1 KPa	C_2 (–)	C_3 (–)
White Bread	26	1st	0.207	44	0.79
		2nd	0.004	0	0.74
		3rd	0.003	0	0.73
Dense white bread	18	1st	1.61	39	0.79
		2nd	0.023	0	0.73
		3rd	0.018	0	0.72
Oat bread	22	1st	0.65	30	0.73
		2nd	0.014	0	0.69
		3rd	0.009	0	0.69
Pumpernickel	20	1st	0.95	27	0.75
		2nd	0.018	0	0.72
		3rd	0.014	0	0.70
English muffins	15	1st	0.30	0	1.25
		2nd	0.027	0	0.53
		3rd	0.021	0	0.52
Sponge cake	20	1st	0.13	6.5	0.75
		2nd	0.012	0	0.72
		3rd	0.009	0	0.71

[a] The constants C_1, C_2, and C_3 are the coefficients of Eq. 1. They represent, respectively, the overall stress level, the prominence of a shoulder and the strain level at which the cell wall material is being compressed.

The three constants C_1, C_2, and C_3 have the following relation to the curve's shape: The constant C_1 is basically a scale factor that determines the absolute magnitude of the stress and its units. As shown in Fig. 2, the constant C_2 is a sort of shape index whose magnitude relative to unity is related to the prominence of the shoulder. (The exact shape of the shoulder is determined by both C_2 and C_3). When $C_2 = 0$, Eq. 1 degenerates to

$$\sigma = \frac{C_1\epsilon}{C_3 - \epsilon} \tag{2}$$

a monotonically ascending relationship, with no shoulder, of the kind shown in Fig. 1b. When $C_2 >> 1$, Eq. 1 degenerates to

$$\sigma = \frac{C_1}{C_2(C_3 - \epsilon)} \tag{3}$$

If, in addition, $C_3 >> 1 > \epsilon$ the stress has a considerable plateau at a level of $C_1/(C_2C_3)$. The magnitude of C_3 determines the relationship's asymptote, that is when $\epsilon \to C_3$ $\sigma \to \infty$, as shown in Fig. 2. In the context of sponge compression this parameter represents the strain level at which densification of the cell wall material becomes the dominant deformation mechanism. Thus, for a material made of small cells with thick walls the magnitude of C_3 is expected to be relatively small. A relatively high value (i.e. close to unity) is expected for a material with an open structure and thin walls. From a purely mathematical viewpoint, the magnitude of C_3 can exceed the level of unity as shown in Fig. 2. In such cases, however, C_3 becomes merely an artificial shape characteristic with little or no physical significance.

RESULTS & DISCUSSION

THE FIT of the model (Eq. 1) to experimental engineering stress-strain relationships of various breads, English muffins and sponge cake is demonstrated in Fig. 3. The model parameters, derived by nonlinear regression, are summarized in Table 1. The figure and table show that all the breads had an initial stress-strain relationship with a prominent shoulder which was expressed by a high value of C_2, i.e. of a magnitude of 20–40. The initial curve of the sponge cake had a much less noticeable shoulder, $C_2 = 6.5$, and English muffins none at all, i.e. $C_2 = 0$. In all the breads as well as the sponge cake, the shoulder disappeared after the first compression. This feature was evident in both the curve's shape and in the zero values of C_2.

Interpretation of this observation is that the bread and sponge

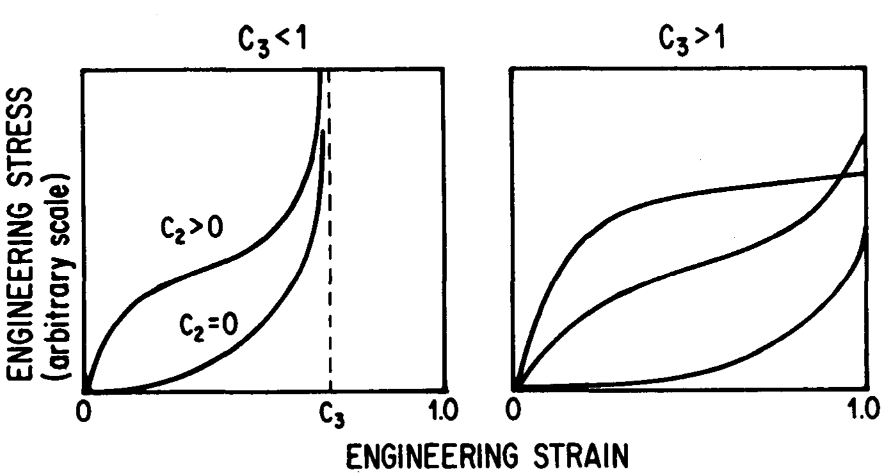

Fig. 2—Schematic presentation of the effects of the magnitude of C_1, C_2, and C_3 on the stress-strain curves produced by the model described in Eq. 1.

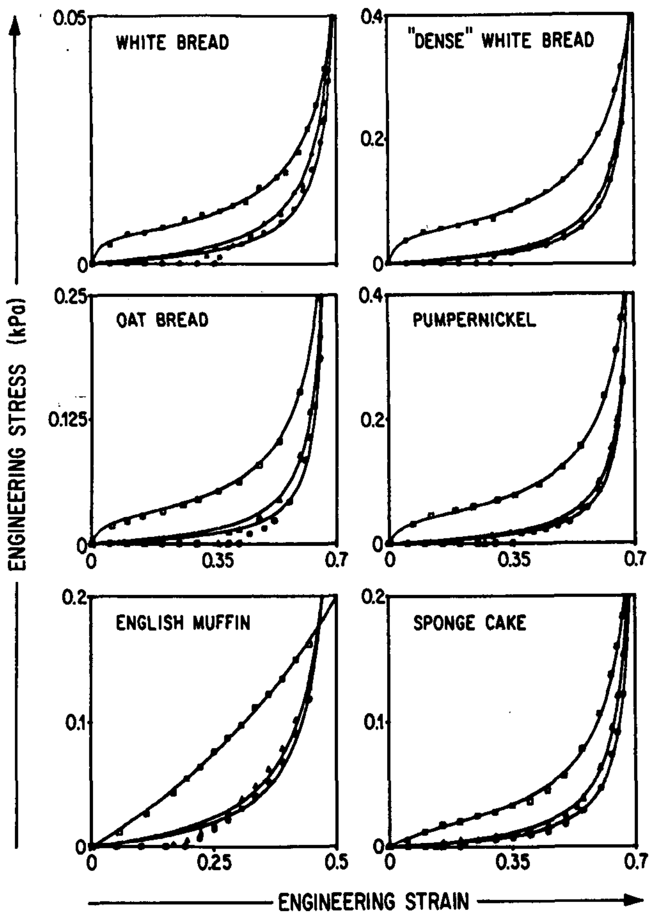

Fig. 3—Compressive stress-strain relationships of a variety of spongy baked goods. Top curve—1st compression, middle curve—2nd compression, and bottom curve—3rd compression. Dots are experimental values and solid line the fit of the model expressed by Eq. 1.

cake had closed pores that ruptured in the first compression and cell walls that collapsed or fractured, thus weakening the structure. This explanation is supported by the fact that considerable deformation after the first compression was irrecoverable and that in the subsequent compressions the stress levels were generally lower than those recorded in the first compression. The same was observed in the English muffin behavior despite the fact that the curve of the first compression had no shoulder. This also indicates that deformation beyond certain levels causes an irreversible damage to the cellular structure.

All the breads and the sponge cake appear to have a similar densification pattern. This is manifested by values of C_3 on the order of 0.73–0.79 in the first compression with a very slight but consistent decline in the second and third compressions. The English muffins had a distinctly different densification pattern. For the first compresssion, C_3 had a value greater than one indicating that the acceleration of the stress ascent started at strains higher than those reached during the test. After the presumed collapse of the initial structure, however, densification occurred at a lower strain level than that of the other products, as indicated by the lower level of C_3 ($C \approx 0.5$ as compared to 0.7).

As could be expected, the overall differences in the density and mechanical resistance of the different materials or the structures of the same material after repeated compressions were also evident from the magnitude of C_1. For the first compression it ranged from 1.6 for a dense bread to 0.13 for a sponge cake, and it was usually lower by a factor of several decades in the second and third compressions. The values of C_1 for the curves of the 2nd and 3rd compression ought to be treated cautiously since their magnitude, calculated by the described procedure, is influenced by the amount of irrecoverable deformation after the previous compression. If a better fit and a more accurate account of the role of irrecoverable deformation are desired, the initial length of the specimen ought to be determined separately and entered before each compression. Since the differences between the products were noticeable anyhow this correction was not done.

CONCLUSIONS

THE DESCRIBED three parameter empirical model gives a satisfactory account of the compression characteristics of various spongy baked goods. Consequently, stress-strain curves with very different shape characteristics can be described in terms of a single mathematical model. This enabled the expression and quantification of changes or differences in the stress-strain relationships in terms of this model's constants. Despite the fact that the described model is empirical, that is, it was not derived from any fundamental mechanical principles, its constants could still be related to the structural characteristics of spongy materials. In principle, therefore, the model has the potential of being a useful tool to characterize the deformability patterns of spongy foods, and probably other cellular materials, and to follow changes in their texture, as a result of storage, for example. Before the model is applied, however, it will be prudent to test its sensitivity to experimental conditions (e.g. deformation rate, specimen dimensions) and to evaluate the relationship between its constants and physical properties such as density, pore size distribution and sensory textural attributes.

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