RESEARCH NOTE

THE COMPRESSIVE BEHAVIOR OF SOLID FOOD SPECIMENS WITH SMALL HEIGHT TO DIAMETER RATIOS[1]

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ABSTRACT

Cylindrical specimens of height to diameter ratios between 0.12 to 1.0 of potato flesh, bologna sausage and process American cheese were uniaxially compressed to failure. The flatter the specimen the stiffer it appeared. This was also true with respect to strength. The magnitude of strain at failure also increased with the specimen's flatness. Application of two correction procedures for the calculation of a dimensionally independent modulus did not always yield consistent results, demonstrating that the stiffness-strength-shape relationships can depend not only on the material but also on the particular character of the end constraints.

INTRODUCTION

It has long been recognized that the effective stiffness and strength of constrained specimens in uniaxial compression is considerably higher than those of specimens compressed between lubricated plates. This phenomenon has several technological implications, notably in the cushioning of heavy machines and buildings with rubber (Lindley 1978), and in metal processing (Dieter 1976). The theoretical aspects of the mechanical behavior of constrained specimens have been investigated by different methods, and there are various expressions that describe the relationship between the ap-

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parent modulus and the geometry of the constrained specimen (Lindley 1979; Gent and Lindley 1959). Since the effects of end constraints can be significant, they must be taken into account in the interpretation of compressive test results as demonstrated by Hamerle and McClure (1971), Culioli and Sherman (1976), Atkin and Sherman (1984) and Bagley *et al.* (1985a,b).

Ideally, a specimen is considered constrained when it is bonded (e.g. glued) to the plates between which it is compressed. In practice, a specimen can also be constrained or partially constrained if there is considerable friction between its ends and the supporting plates. In both cases, the area in contact with the plates cannot expand freely thus causing bulging of the specimen's middle section. This phenomenon, known as "barrelling", provides a visual indication of the existence of end constraints. It also shows that the middle section of the specimen develops tensile stresses that can cause failure in tension rather than in shear, as is the case in certain types of cheese (see also Culioli and Sherman 1976).

Estimation of the magnitude of end effects in compressive testing of foods by the application of theoretical considerations is difficult, since most solid foods have rheological characteristics for which theoretical solutions do not yet exist. Notable exceptions are rubbery food materials (e.g. gels). For these an empirical correction formula originally devised for bonded rubber blocks yielded reasonable results (Peleg *et al.* 1981). A second difficulty stems from the fact that in most routine testing procedures the specimen is not bonded to the supporting plates and the end effects originate from friction. The latter can change within large bounds depending on the roughness of the plate's surface and the type of food, particularly with respect to its lubrication capacity as a result of released moisture or fat.

Irrespective of the kind and origin of the end constraints, their effect becomes increasingly significant as the specimen's shape is made flatter. Flatness (or its reverse) is usually expressed in terms of height (or length) to diameter ratio which is a particularly convenient measure for cylindrical specimens loaded axially. It can also be quantified by a more general shape factor (Lindley 1978) defined as:

$$\text{Shape factor} = \frac{\text{Loaded area}}{\text{Force free area}} \tag{1}$$

For an axially loaded cylinder the shape factor (S) is given by:

$$S = \frac{D}{4H} \tag{2}$$

where D and H are the specimen's original diameter and length, respectively.

Under common testing conditions of food materials, especially when the compressed specimens are very flat (e.g. as in the form of a slice), it is not clear if the observed mechanical behavior can be expressed solely as a function of the shape factor (or the height to diameter ratio) or whether height and diameter can each play an independent role. This communication reports empirical relationships between the apparent deformability modulus and failure parameters, and the physical dimensions of flat specimens of selected food materials, and an attempt to develop a correction procedure in order to calculate a dimension free "independent" deformability modulus, and to express the dimensions and friction effects in terms of a constant of an empirical formula.

MATERIALS AND METHODS

Potato, bologna sausage and American process cheese were purchased at a local store. They were first sliced to different thicknesses by an adjustable laboratory meat slicer, and then bored using cork borers of different diameters to obtain flat, cylindrical specimens with various diameters and heights. The normal dimensions of the specimens and their corresponding shape factor and height to diameter ratio are listed in Table 1. The actual dimensions of each specimen were determined by caliper measurements. The results presented in the figures are based on shape factor calculation with the actual dimensions also given at the side of each figure. All the specimens were equilibrated at room temperature in a desiccator with water to minimize dehydration.

The specimens were compressed by an Instron Universal Testing Machine, model TM, at a constant deformation rate of 0.5 $cm.min.^{-1}$. The apparent deformability modulus was determined as the engineering stress divided by engineering strain at 20% deformation. The specimens were tested in an alternating order to minimize the risk of systematic error stemming from differences in the time between the sample preparation and testing. Each reported data point is the mean value of at least four deformation tests performed on specimens with the same dimensions. The coefficient of variance between these measurements was generally in the range of 10-20%. The higher figure generally represents the "flatter" specimens. Each compression test was performed between flat, smooth metal plates and between surfaces coated with emery cloth (No. 120) to increase friction. The latter

Table 1. Nominal[1] Shape factor and height to Diameter Ratio (in parentheses) of the Specimens used in the investigation[2]

Height (cm)	Diameter (cm)				
	1.0	1.3	1.5	1.9	2.1
1.0	0.25 (1.0)	0.33 (0.77)	0.38 (0.67)	0.48 (0.53)	0.53 (0.48)
0.8	0.31 (0.8)	0.41 (0.62)	0.47 (0.53)	0.59 (0.42)	0.66 (0.38)
0.5	0.50 (0.5)	0.65 (0.38)	0.75 (0.33)	0.95 (0.26)	1.05 (0.24)
0.4	0.63 (0.4)	0.81 (0.31)	0.94 (0.27)	1.2 (0.21)	1.3 (0.19)
0.25	1.0 (0.25)	1.3 (0.19)	1.5 (0.17)	1.9 (0.13)	2.1 (0.12)

1) The term nominal is used since the actual dimensions of the specimen, particularly their height, had variations in the order of up to 0.3 mm.

2) Note that the overlap was intentional.

was replaced frequently to avoid coating by the specimen material. All the tests were repeated with fresh material.

RESULTS AND DISCUSSION

Examples of the relationship between the deformability modulus of potato flesh, bologna sausage and American cheese and the shape factor of the specimen are shown in Fig. 1-3. These reaffirm that, in general, the flatter the specimen, as measured by its shape factor, the stiffer it appears. This is particularly the case when there is considerable friction between the specimen ends and the machine plates. The lack of any clear trend and the considerable scatter in Fig. 2 (left) appears to be a result of self-lubrication, an explanation confirmed in other tests, and by a similar behavior observed in process American cheese, another fat rich material. In all cases where friction was noticeable, the dimensional effect was not only significant as could be expected but also of considerable magnitude, i.e. in the range of 80% to 8 fold depending on the material.

Failure Conditions

Examples of failure conditions of specimens with different dimensions are presented in Fig. 4-5. Since failure conditions in process

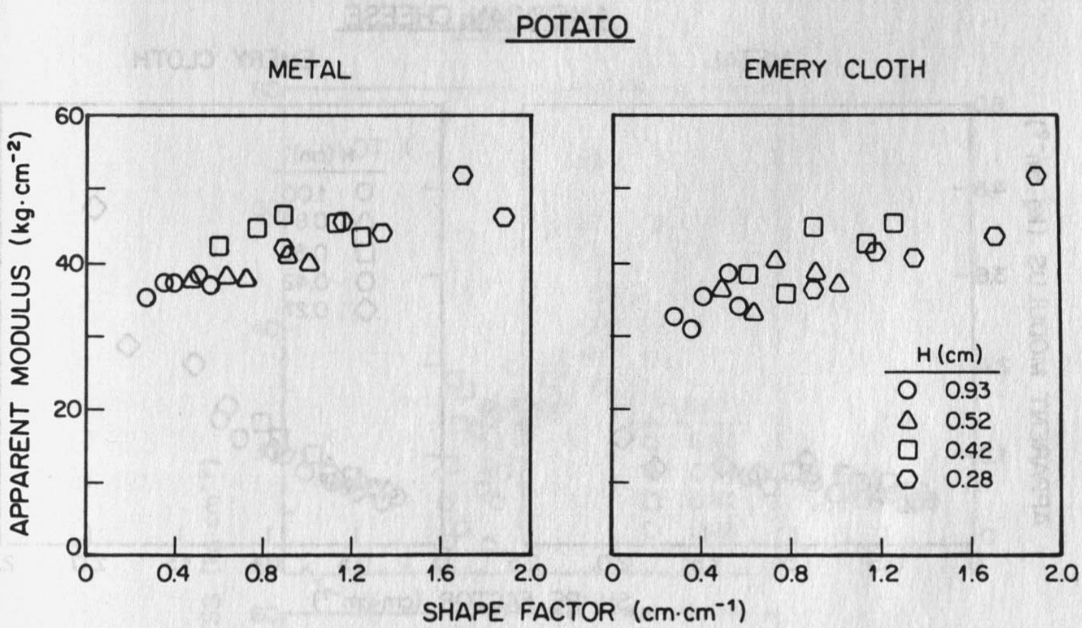

FIG. 1. THE RELATIONSHIP BETWEEN APPARENT COMPRESSIVE DEFORMABILITY MODULUS OF POTATO FLESH
Left — between smooth metal plates, right — between plates coated with emery cloth.

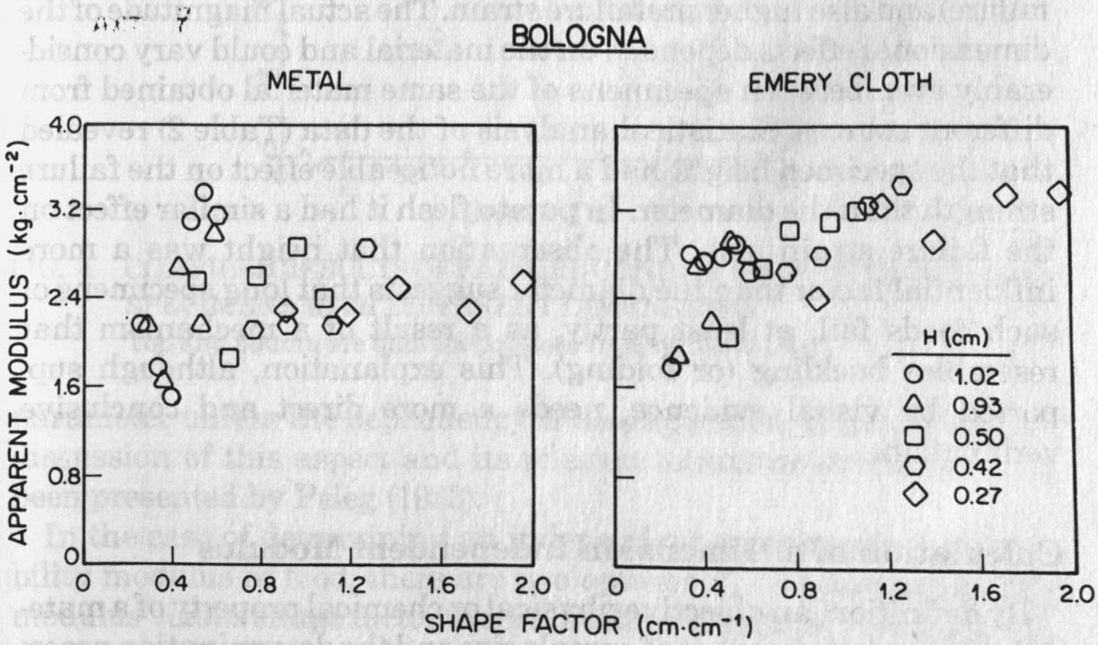

FIG. 2. THE RELATIONSHIP BETWEEN APPARENT COMPRESSIVE DEFORMABILITY MODULUS OF BOLOGNA SAUSAGE
Left — between smooth metal plates, right — between plates coated with emery cloth.

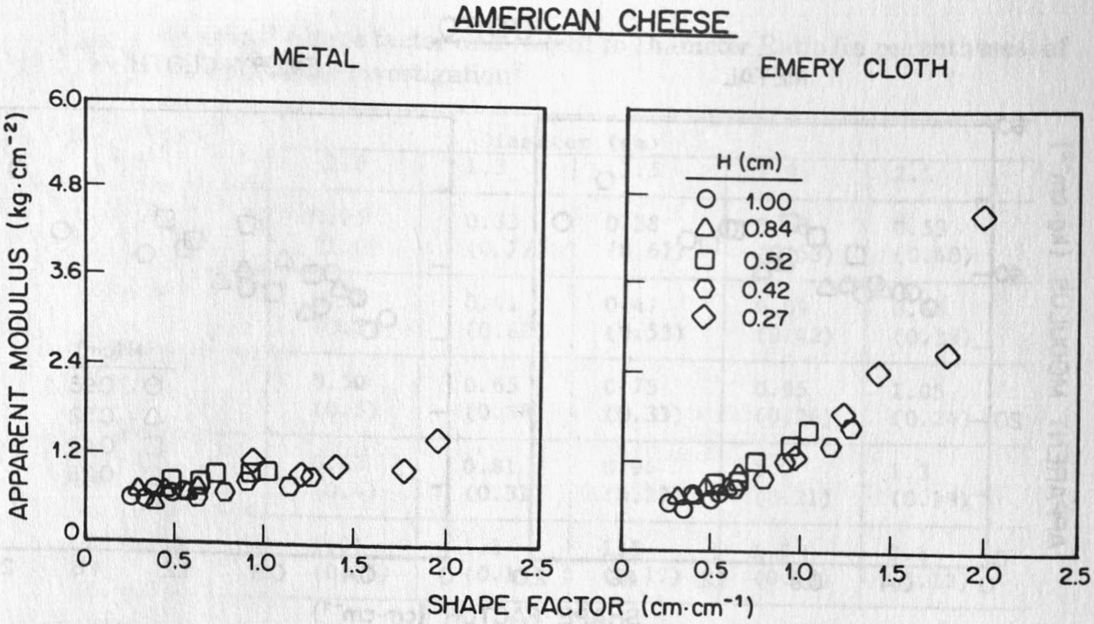

FIG. 3. THE RELATIONSHIP BETWEEN APPARENT COMPRESSIVE DEFORMABILITY MODULUS OF PROCESS AMERICAN CHEESE
Left — between smooth metal plates, right — between plates coated with emery cloth.

American cheese cannot be clearly determined, only the potato and bologna sausage data are reported. The figures clearly demonstrate that flat specimens exhibit higher ultimate strength (i.e. stress at failure) and also higher prefailure strain. The actual magnitude of the dimensional effects depended on the material and could vary considerably even between specimens of the same material obtained from different sources. Statistical analysis of the data (Table 2) revealed that the specimen height had a more noticeable effect on the failure strength than the diameter. In potato flesh it had a similar effect on the failure strain also. The observation that height was a more influential factor than the diameter suggests that long specimens of such foods fail, at least partly, as a result of a mechanism that resembles buckling (or folding). This explanation, although supported by visual evidence, needs a more direct and conclusive verification.

Calculation of a Dimension Independent Modulus

By definition, an objective physical or chemical property of a material must be independent of sample size and the determination procedure. Where an inherent dependency does exist, as in the case of response properties, there is little meaning to the magnitude of a

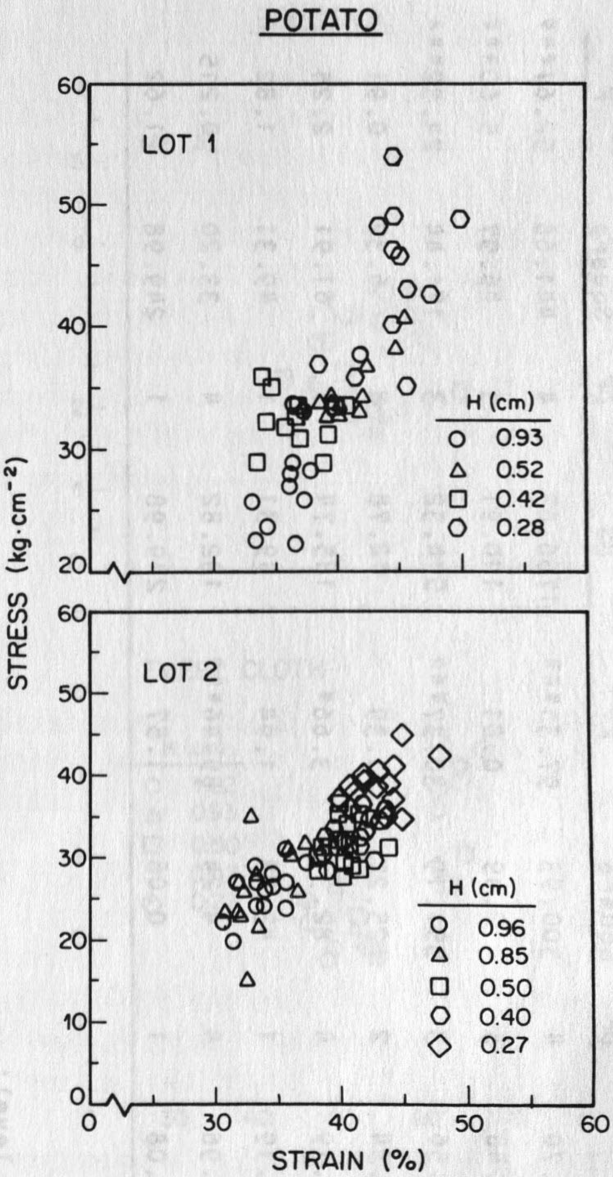

FIG. 4. COMPILED RESULTS OF FAILURE CONDITIONS OF POTATO FLESH SPECIMENS WITH DIFFERENT DIMENSIONS
Top and bottom are data for potatoes from different lots.

parameter unless the dependency is also specified. A more detailed discussion of this aspect and its relation to texture evaluation has been presented by Peleg (1983).

In the case of determining an independent compressive deformability modulus of food, there are two options: (1) To extrapolate the modulus versus shape factor to zero (Dieter 1976) thus "shedding off" dimensional effects. (2) To apply a correction formula so that the transformed relationship will become a straight line parallel to the shape factor axis (see below). For the first option, one needs to know

Table 2. Statistical analysis of the effects of diameter and height on the failure conditions of flat potato flesh and bologna sausage specimens.

		Failure Stress (strength)				Failure Strain			
Material	Source of Variation	SS	DF	Mean Square	F	SS	DF	Mean Square	F
Potato	Height	1203.70	4	300.93	97.31***	1766.63	4	441.66	52.67***
	Diameter	7.48	3	2.49	0.81	140.91	3	46.97	5.60***
Potato	Height	1653.56	3	551.19	30.37***	514.32	3	171.44	24.32***
	Diameter	50.54	2	25.27	1.39	12.78	2	6.39	0.91
Bologna	Height	329.19	4	82.3	3.66*	185.74	3	61.91	2.28
	Diameter	329.19	1	43.68	1.94	49.31	1	49.31	1.82
Bologna	Height	16.96	4	4.24	85.46***	132.82	4	33.20	0.215
	Diameter	0.08	1	0.08	1.57	249.98	1	249.98	1.62

***, significant at 0.1 % level;
**, significant at 1% level;
*, significant at 5% level.

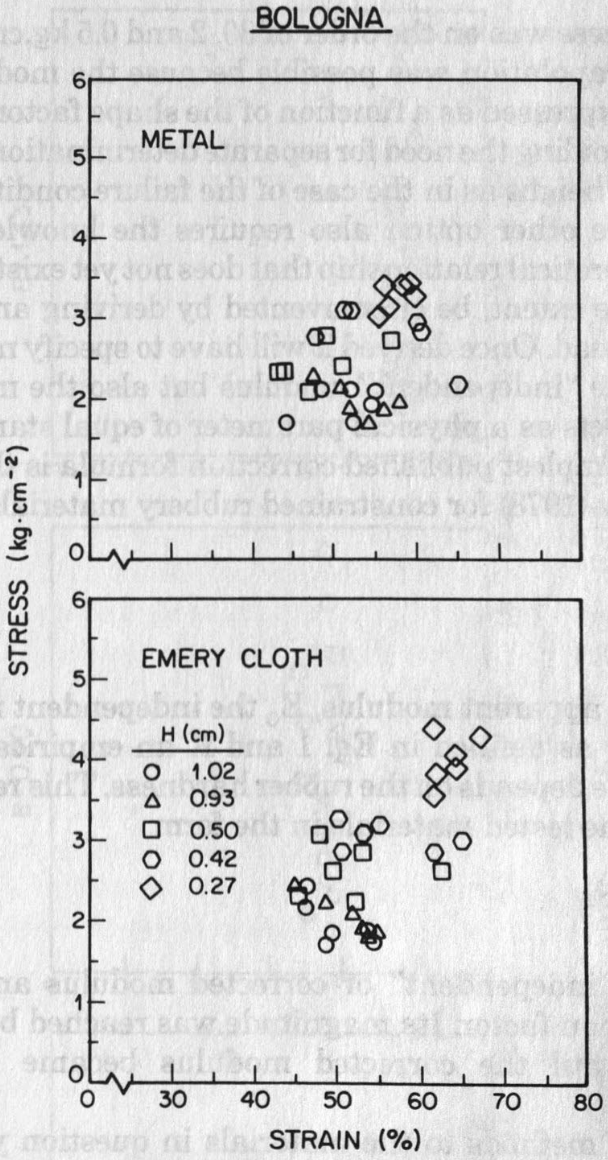

FIG. 5. COMPILED RESULTS OF FAILURE CONDITIONS OF BOLOGNA SAUSAGE

Top — between smooth metal plates, bottom — between plates coated with emery cloth.

from theoretical considerations the kind of relationship that exists between the shape factor and the modulus. Unfortunately, the nature of such relationships is still unknown in foods, particularly when friction intensity cannot be qualified but can be a factor too. The simplest form of extrapolation is through linear regression which, although on shaky theoretical grounds, can lead to the establishment of an "extrapolated modulus" as a rough approximation. For the materials reported in this communication, its value for potatoes, bologna and cheese was on the order of 30, 2 and 0.5 kg.cm^{-2}, respectively. This extrapolation was possible because the modulus could reasonably be expressed as a function of the shape factor alone (see Fig. 1-3), thus avoiding the need for separate determination of the role of diameter and height as in the case of the failure conditions.

Exercising the other option also requires the knowledge of an appropriate theoretical relationship that does not yet exist. The problem can, to some extent, be circumvented by deriving an empirical relationship instead. Once derived it will have to specify not only the magnitude of the "independent" modulus but also the mode of the dimensional effects as a physical parameter of equal standing.

Perhaps the simplest published correction formula is the one proposed by Lindley (1978) for constrained rubbery materials, i.e.

$$E_{AP} = \frac{E_O}{1+2KS^2} \tag{3}$$

where E_{AP} is the apparent modulus, E_O the independent modulus, S the shape factor as defined in Eq. 1 and K an empirical constant whose magnitude depends on the rubber hardness. This relationship was applied to the tested materials in the form

$$E = E_{AP}(1 + BS^2) \tag{4}$$

where E is the "independent" or corrected modulus and B is an empirical correction factor. Its magnitude was reached by trial and error (Fig. 6) until the corrected modulus became dimension independent.

Application of methods to the materials in question yielded the following:

Material	Independent Modulus (kg.cm^{-2})	Correction Factor (B)
Potato flesh	34-47	0-0.13
Bologna sausage	1.9-2.6	0.12-0.40
American cheese	0.72-2.6	0.25-1.45

The higher B values represented the conditions where friction was a dominant factor.

Comparison of the "independent" moduli calculated in the two ways revealed that although they were related on a large scale (Fig. 7) there could be a considerable difference between the values calculated

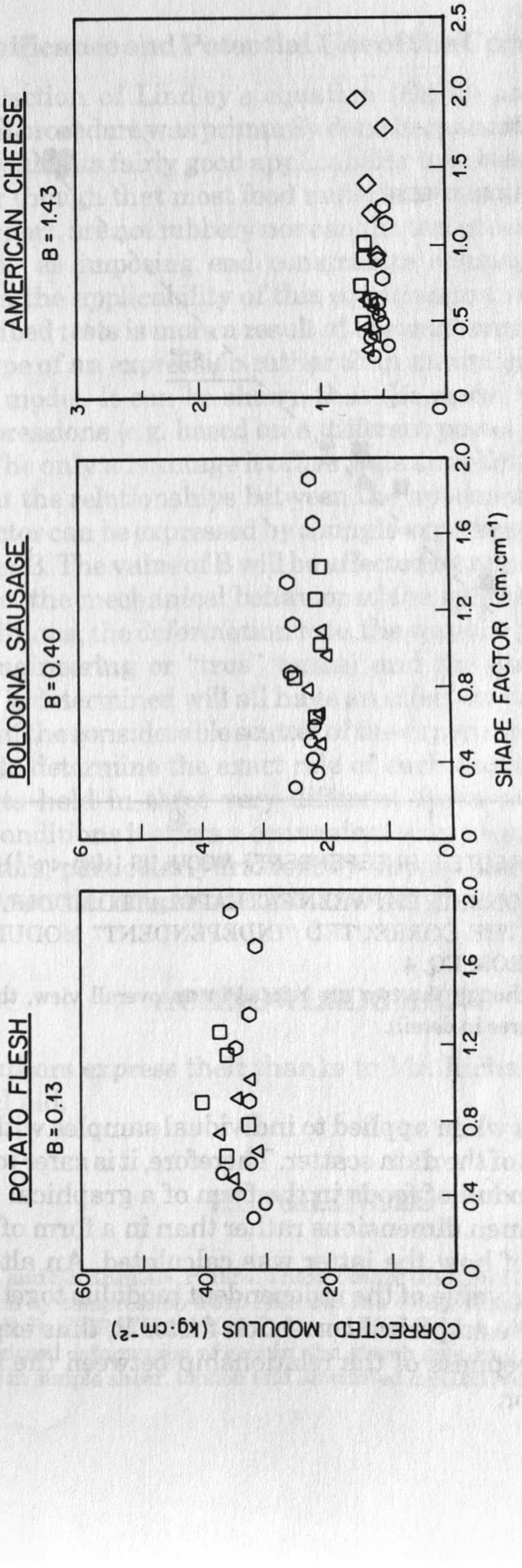

FIG. 6. EXAMPLES OF CORRECTED MODULUS VS SHAPE FACTOR RELATIONSHIPS EMPLOYING A MODIFIED VERSION OF A FORMULA (EQ. 4) ORIGINALLY PROPOSED FOR BONDED RUBBER SPECIMENS (LINDLEY 1978).

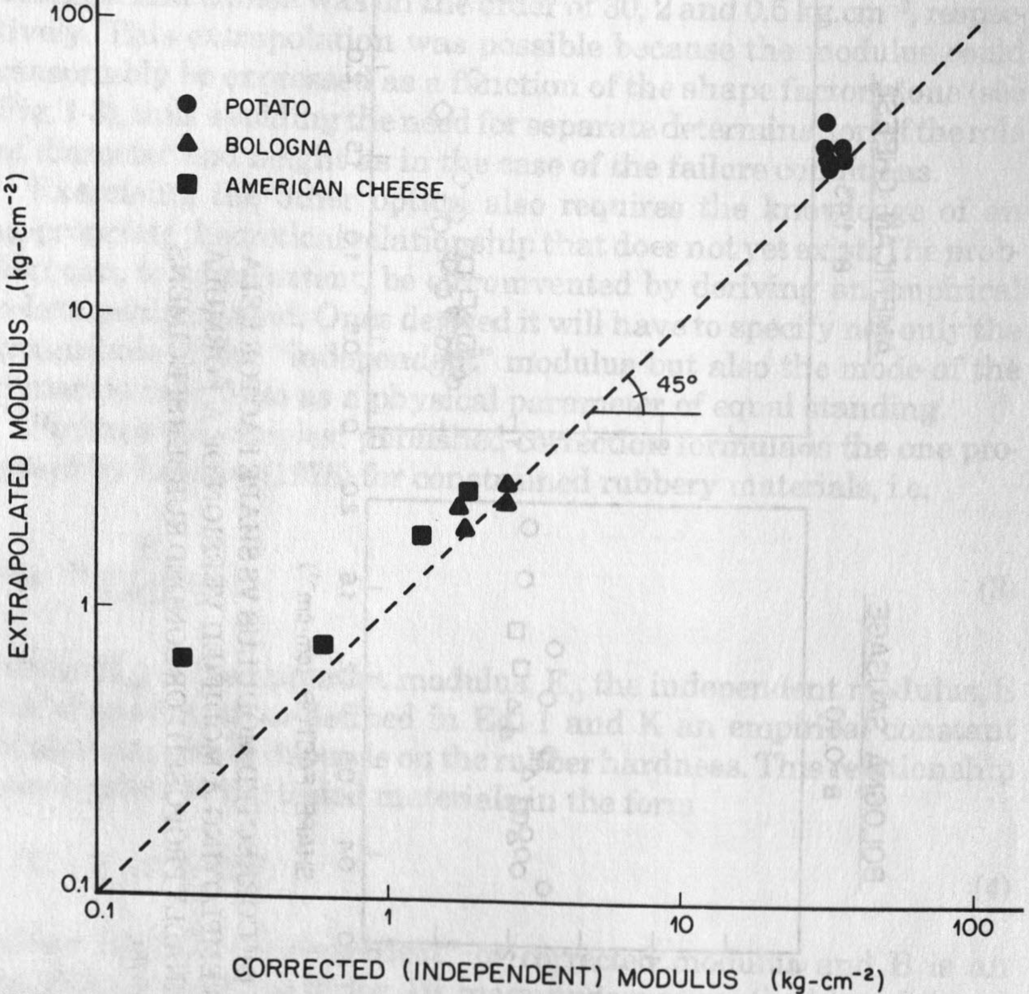

FIG. 7. THE RELATIONSHIP BETWEEN EXTRAPOLATED MODULUS (i.e. AT S = O) AND THE CORRECTED "INDEPENDENT" MODULUS CALCULATED FROM EQ. 4
(Note that although the two are related in an overall view, they do not necessarily agree in detail).

by the two methods when applied to individual samples within each material as a result of the data scatter. Therefore, it is safer to present the compressive moduli of foods in the form of a graphical relationship with the specimen dimensions rather than in a form of a single value irrespective of how the latter was calculated. An alternative way is to specify the value of the independent modulus together with the magnitude of the empirical correction factor B, thus expressing numerically the steepness of the relationship between the modulus and the shape factor.

The Significance and Potential Use of the Correction Factor B

The selection of Lindley's equation (Eq. 3) as the basis of the correction procedure was primarily done because of its mathematical simplicity and its fairly good applicability to rubbery food materials. It is clear though that most food materials, including those selected for this report, are not rubbery nor can the test procedure employed be considered as imposing end constraints comparable to bonding. Therefore, the applicability of this equation to the data obtained in the described tests is more a result of the mathematical properties of such a type of an expression rather than an indication of its fit as a physical model. It can be shown that the model is not unique and other expressions (e.g. based on a different power) will yield similar results. The only advantage it offers is its simplicity and the convenience that the relationships between the apparent modulus and the shape factor can be expressed by a single experimentally determined parameter B. The value of B will be affected by any and all the factors that affect the mechanical behavior of the specimen. Different friction conditions, the deformation rate, the way the modulus is defined (i.e. in engineering or "true" terms) and the strain at which the modulus is determined will all have an effect on the magnitude of B. Because of the considerable scatter of the experimental data it may be difficult to determine the exact role of each factor. Since the model appears to hold in three very different foods and under different friction conditions it offers a convenient way to quantify the effect of such factors, particularly friction, by simply assessing the relationship between the magnitude of B and the given parameter.

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ANNOUNCEMENT

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Participation

Applications for attendance are invited. Those wishing to receive the preliminary program of the Conference and further information are kindly requested to write to the address below.

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