C_{40}^{50} A) C_{40}^{50} B) C_{49}^{52} ...

$5 \times C_3^5 - 2C_2^5 = 5 \times \frac{5 \times 4}{2!} - \cancel{2} \times \frac{5 \times \cancel{4}}{\cancel{2}!} \ldots$

$C_3^5 = \frac{5!}{3!\,2!}$

M01

6 preguntas MO → 5 posibles respuestas, contando en blanco.

Formas posibles de entregar MO: $5^6 = 15625$ ← nidos

estudiantes: 15625+1 ← palomas

resp: 15626 ← D

MO2

a_m: cant de palabras de largo m formadas por $\{A, B, C\}$ sin 3 A seguidas. Lo que nos importa es a_5
Hallar una recurrencia para a_m.

			B,C	A	A
				B,C	A
1	2	$m-3$	$m-2$	$m-1$	m

$$a_m = 2(a_{m-1} + a_{m-2} + a_{m-3})$$

Caso 1

el m es B o C
en este caso hay

$$\underbrace{2}_{\text{B o C}}\underbrace{a_{m-1}}_{\text{las } m-1 \text{ anteriores}}$$

Caso 2

el m es A
el $m-1$ es B o C

$$\underbrace{2}_{\text{B o C}}\underbrace{a_{m-2}}_{\text{las } m-2 \text{ anteriores}}$$

Caso 3

el m es A
el $m-1$ es A

$$\underbrace{2}_{\text{B o C } m-2}\underbrace{a_{m-3}}_{m-3 \text{ anteriores}}$$

$$a_n = 2(a_{n-1} + a_{n-2} + a_{n-3})$$

$a_1 = 3$ A, B, C $\underset{3}{\_}$

$a_2 = 9 = 3^2$ AA, AB, ... $\underset{3}{\_}\ \underset{3}{\_}$

$a_3 = 3^3 - 1$
$= 26$ ~~AAA~~ AAB AAC ... $\underset{3}{\_}\ \underset{3}{\_}\ \underset{3}{\_}$ − (AAA)

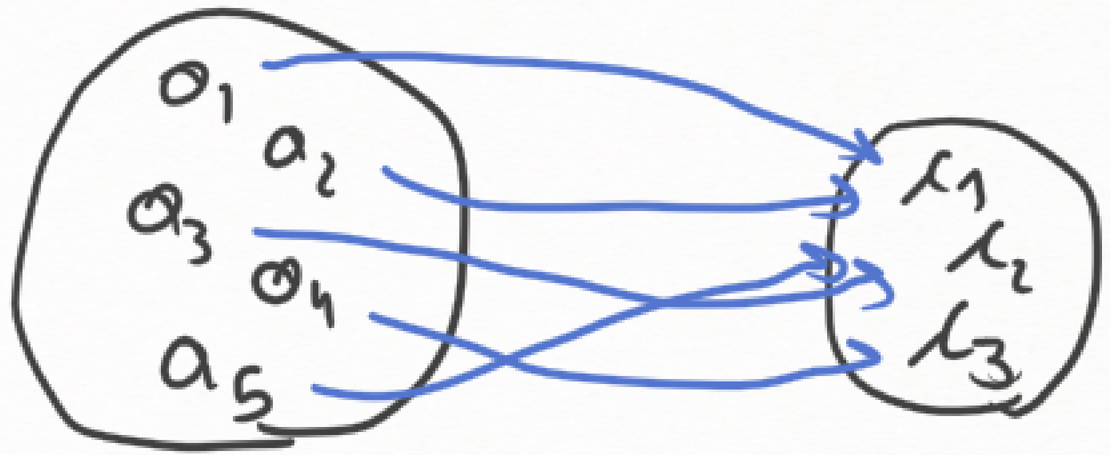
$n = 4$

$$\boxed{a_4 = 2(a_3 + a_2 + a_1)} = 2(26 + 9 + 3) = 2 \times 38 = 76$$

$$\boxed{a_5 = 2(a_4 + a_3 + a_2)} = 2(\underbrace{\underbrace{76 + 26}_{102} + 9}_{111}) = 2(111) = 222$$

$n = 5$

5 amigos   3 carpas = ninguna vacía



si las 3 carpas fueran ≠

cada distribución es una función sobreyectiva

$$S(5,3) = \frac{1}{3!} Sob(5,3)$$

las 3 carpas son =

resp $\frac{m}{6}$

$$Sob(5,3) = \sum_{i=0}^{2} (-1)^i C_i^3 (3-i)^5$$

C_1: nadie va a la carpa 1, C_2: nadie... 2

$$Sob(5,3) = (-1)^0 C_0^3 \times 3^5 - C_1^3 2^5 + C_2^3 1^5$$

$$= 3^5 - 3 \times 2^5 + 3 = m$$

M04

1 C
1 B
1 O
2 A
2 L

Caso 1: se eligen 3 letras $\neq$

$$C^5_3 \times 3!$$

elegir 3 letras de C B O A L

ordenarlas

Caso 2: se eligen 2 letras $=$

Caso 2.1 se elige 2 veces la A $\quad 4 \times 3$

otra letra

$\boxed{AAx \quad AxA \quad xAA}$

Caso 2.2 se elige 2 veces la L : 4×3

idem

total : $C^5_3 3! + 2 \cdot 4 \cdot 3 = \dfrac{5!}{2!\cancel{3!}}\cancel{3!} + 24$

$= 5 \cdot 4 \cdot 3 + 24 = 60 + 24 = 84$

MOS

perms de 123456 que no dejan en su lugar 2, 4, 6

C_1: 2 está en su lugar C_2: 4 está en su lugar C_3: 6 está en su lugar

$$\text{resultado} = \text{total} - C_1 \cup C_2 \cup C_3$$

$$= \underbrace{\text{total} - \left(|C_1| + |C_2| + |C_3|\right) + \left(\overset{|C_1 \cap C_2|}{|C_1 C_2|} + |C_1 C_3| + |C_2 C_3|\right) - \overset{|C_1 \cap C_2 \cap C_3|}{|C_1 C_2 C_3|}}$$

$$\text{total} = 6!$$

→ $|C_1| = 5!$ (2 está fijo y el resto permutan) $|C_2| = |C_3| = 5!$

$|C_1 C_2| = 4!$ (2 y 4 están fijos) $|C_1 C_3| = |C_2 C_3| = 4!$

$|C_1 C_2 C_3| = 3!$ (2, 4 y 6 fijos) $3! = 6$ $4! = 24$ $5!$

$$\text{res} = 6! - 3 \cdot 5! + 3 \cdot 4! - 3! = \ldots$$

M06

$\frac{1}{3}\ 0\ 0\ -\frac{2}{3}\ 0\ 0\ \ \frac{4}{3}\ 0\ 0\ -\frac{8}{3}\ 0\ 0\ -$

$1+y+y^2 \cdots$

$y \qquad (-2x^3)^2 \qquad (-2x^3)^3$

$y=-2x^3$

$$\frac{1}{3} - \frac{2x^3}{3} + \frac{4x^6}{3} - \frac{8x^9}{3} + \cdots = \frac{1}{3}\left(1 - 2x^3 + 4x^6 - 8x^9 + \cdots\right)$$

$$= \frac{1}{3}\left(1+y+y^2+y^3+\cdots\right) = \frac{1}{3} \times \frac{1}{1-y} = \frac{1}{3}\,\frac{1}{1+2x^3} = \frac{1}{3+6x^3}$$

$$a_n = a_{n-1} + n^2 \qquad a_0 = 0$$

sol. homogenea, particular, etc

$$a_n = a_{n-1} \Rightarrow a_n^h = c \cdot 1^n = c$$ ← sol. gral homogenea

$$a_n - a_{n-1} = 0$$

pol car. $\lambda - 1 = 0 \Rightarrow \lambda = 1 \Rightarrow a_n^h = c \cdot 1^n$

Particular

ord 2

Notas rel recurrencia h7

$$a_n - a_{n-1} = n^2 \cdot 1^n \qquad a_n^p = n^1 (\alpha n^2 + \beta n + \gamma)$$

1 raiz del pol car.

$$\boxed{a_n - a_{n-1} = n^2}$$

$$\alpha n^3 + \beta n^2 + \gamma n - \alpha (n-1)^3 - \beta (n-1)^2 - \gamma (n-1) = n^2$$

$$\boxed{a_n^p = \alpha n^3 + \beta n^2 + \gamma n}$$

Hallar α, β, γ

$$\alpha n^3 + \beta n^2 + \gamma n - \left(\alpha (n-1)^3 + \beta (n-1)^2 + \gamma (n-1)\right) = n^2$$

$$\cancel{\alpha n^3} + \cancel{\beta n^2} + \cancel{\gamma n} - \left(\cancel{\alpha n^3} - 3\alpha n^2 + 3\alpha n - \alpha + \cancel{\beta n^2} - 2\beta n + \beta + \cancel{\gamma n} - \gamma\right) = n^2$$

$$3\alpha n^2 - 3\alpha n + \alpha + 2\beta n - \beta + \gamma = n^2 \qquad (n-1)^3 = n^3 - 3n^2 + 3n - 1$$

$$\underline{3\alpha n^2} + (-3\alpha + 2\beta) n + \alpha - \beta + \gamma = 1 n^2 + 0 n + 0$$

$$\begin{cases} 3\alpha = 1 \\ -3\alpha + 2\beta = 0 \\ \alpha - \beta + \gamma = 0 \end{cases} \Rightarrow \alpha = \frac{1}{3} \Rightarrow -1 + 2\beta = 0 \Rightarrow \beta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{2} + \gamma = 0 \Rightarrow \gamma = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$a_n^p = \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n = n\left(\frac{n^2}{3} + \frac{n}{2} + \frac{1}{6}\right) = n\left(\frac{2n^2 + 3n + 1}{6}\right)$$

$$= \frac{n(2n+1)(n+1)}{6} \qquad a_n^p = \frac{n(2n+1)(n+1)}{6}$$

$$a_n = c + \frac{n(2n+1)(n+1)}{6}$$

Hallamos la c con $a_0 = 0$

$$a_0 = c + \frac{0 \times (2 \times 0+1)(0+1)}{6} = c \Rightarrow c = 0$$

$$\boxed{a_n = \frac{n(2n+1)(n+1)}{6}}$$ (a)

$$a_n = n^2 + a_{n-1} = n^2 + (n-1)^2 + a_{n-2} = n^2 + (n-1)^2 + (n-2)^2 + a_{n-3} = \cdots$$

$$= n^2 + (n-1)^2 + \cdots + 2^2 + 1 + \underbrace{a_0}_{=0}$$

$$\frac{n(2n+1)(n+1)}{6} = a_n = \sum_{i=0}^{n} i^2$$

$$a_n = a_{n-1} + n^2$$

$$a_0 = 0$$

$$a_1 = a_0 + 1^2 = 1^2$$

(b)

$$a_2 = a_1 + 2^2 = 1^2 + 2^2$$

$$a_3 = a_2 + 3^2 = 1^2 + 2^2 + 3^2$$

$$a_n = \sum_{i=0}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

(c) $\sum_{i=0}^{n} i^2 = \dfrac{n(2n+1)(n+1)}{6}$ por IC

$P(n): \sum_{i=0}^{n} i^2 = \dfrac{n(2n+1)(n+1)}{6}$ PB, PI

PB $P(0)$:

$$\sum_{i=0}^{0} i^2 = 0$$

$$\frac{n(2n+1)(n+1)}{6} = \frac{0(2\cdot 0+1)(0+1)}{6} = 0$$

PI H $\sum_{i=0}^{n} i^2 = \dfrac{n(2n+1)(n+1)}{6}$ T $\sum_{i=0}^{n+1} i^2 = \dfrac{(n+1)(2(n+1)+1)(n+2)}{6}$

$$\sum_{i=0}^{n+1} i^2 = \sum_{i=0}^{n} i^2 + (n+1)^2 \overset{H}{=} \frac{n(2n+1)(n+1)}{6} + (n+1)^2$$

$$= (n+1)\left(\frac{n(2n+1)}{6} + n+1\right) = (n+1)\left(\frac{n(2n+1)+6(n+1)}{6}\right)$$

$$= \frac{(n+1)}{6}(2n^2+7n+6) \overset{?}{=} \frac{(n+1)}{6}(2(n+1)+1)(n+2)$$

$$\Longleftrightarrow 2n^2+7n+6 = (2(n+1)+1)(n+2) = (2n+3)(n+2) = 2n^2+7n+6 \checkmark$$