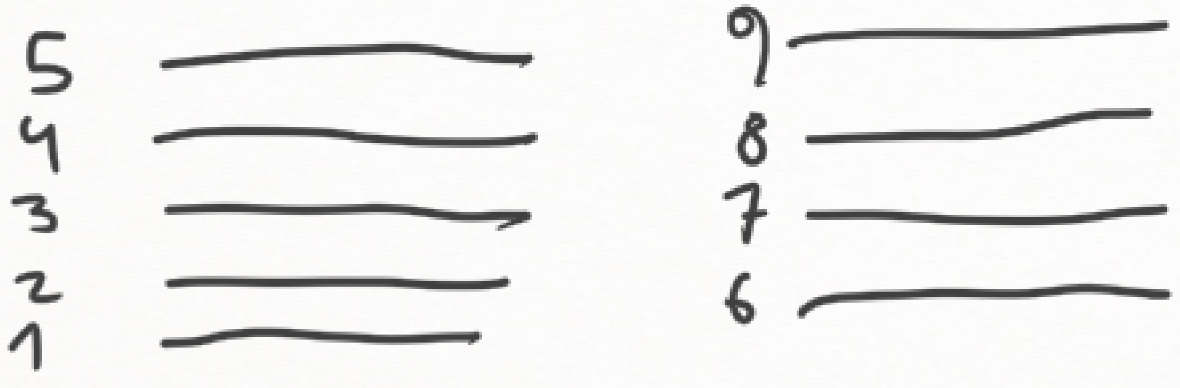
Ej 1

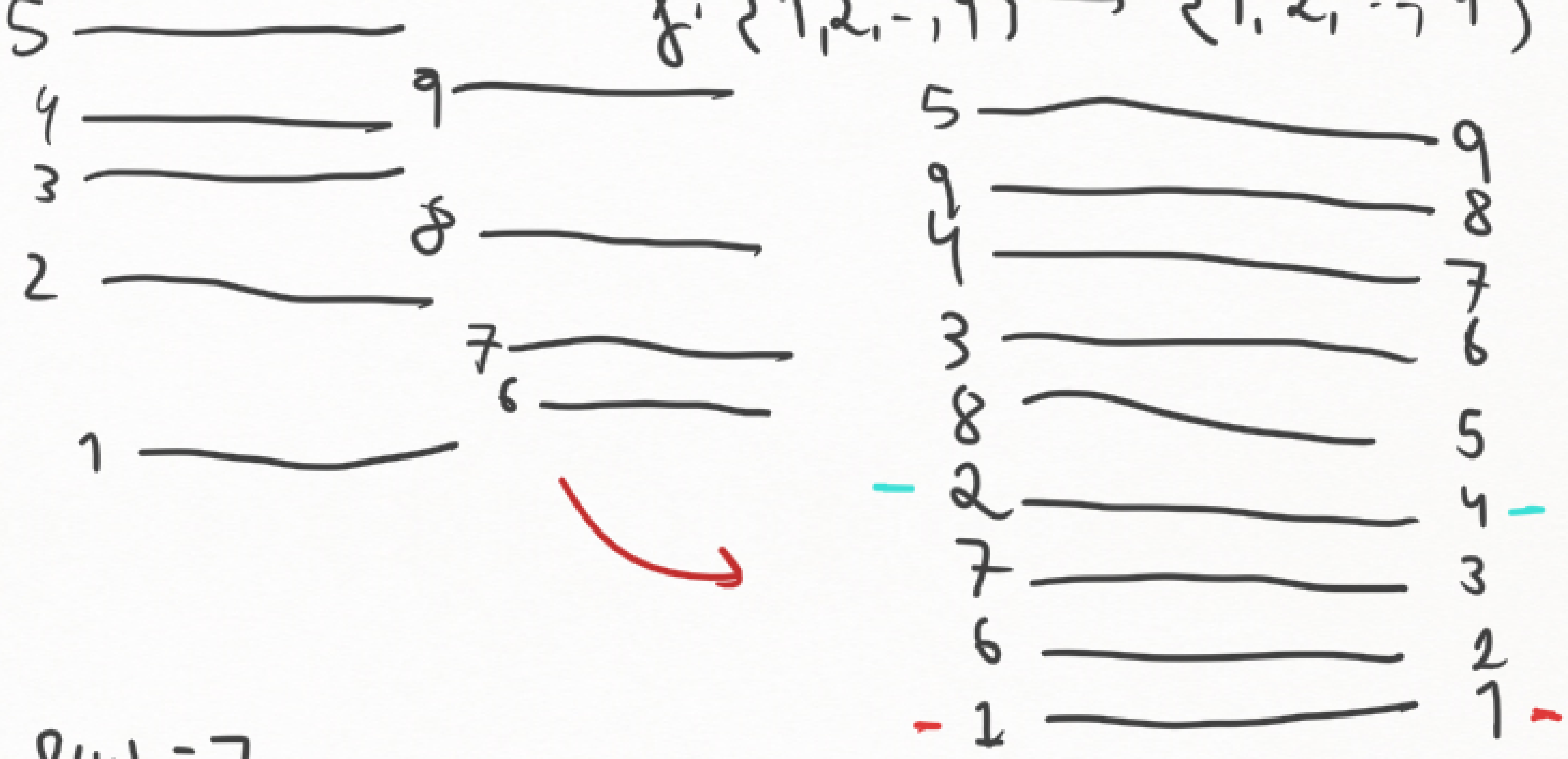
2do mazo

$I_1 = \{1, 2, 3, 4, 5\}$ $\qquad I_2 = \{6, 7, 8, 9\}$ $\qquad f: I_1 \cup I_2 \to I_1 \cup I_2$

1er mazo

$f: \{1, 2, \dots, 9\} \to \{1, 2, \dots, 9\}$





$f(1) = 1 \quad f(2) = 4 \quad f(3) = 6 \quad f(4) = 7$

$f(5) = 9 \quad f(6) = 2 \quad f(7) = 3 \quad f(8) = 5 \quad f(9) = 8$ (Un ejemplo)

Condiciones: $\forall i, j \in I_1$, $i < j$: $f(i) < f(j)$ $\qquad f(1) < f(2) \qquad f(2) < f(3) \dots$ (lo mismo se cumple para I_2)

se embaraja sin cambiar el orden de cada mazo

5 ____
4 ____
3 ____
2 ____
1 ____

9 ____
8 ____
7 ____
6 ____

Cada carta puede quedar a una de 9 posiciones

9	9
8	5
7	4
6	8
5	3
4	2
3	7
2	1
1	6

Elijo los 5 lugares a los que va el mazo 1

Si elegimos los 5 lugares a los que va el mazo 1, queda un solo barajamiento posible

Elegir 5 lugares de 9 posibles

$$C^9_5 = \frac{9\times8\times7\times6\times\cancel{5\times4\times3\times2}}{4!\ \cancel{5!}}$$

$$C^9_5 = \frac{9\times\cancel{8}^{2}\times7\times\cancel{6}}{\cancel{4}\times\cancel{3\times2}} = 9\times14 = 126$$

$$C^n_m = \frac{n!}{(n-m)!\ m!}$$

Ej 4

(a_n) $\quad A(x)$ Fg de a_n $\quad A(x) = \sum_{n=0}^{\infty} a_n x^n \quad A(x) = \frac{1+2x}{(1-x)^2}$

(b_n) $\quad B(x)$ Fg de b_n $\quad B(x) = \sum_{n=0}^{\infty} b_n x^n \quad B(x) = \frac{1}{(1+2x)(1-x)}$

$(c_n) = (a_n) * (b_n) \qquad c_n = \sum_{i=0}^{n} a_i b_{n-i}$

$$c_0 = \sum_{i=0}^{0} a_i b_{0-i} = a_0 b_0$$

$$c_{10} = a_{10} b_0 + a_9 b_1 + a_8 b_2 + \dots + a_1 b_9 + a_0 b_{10}$$

$$c_1 = \sum_{i=0}^{1} a_i b_{1-i} = a_0 b_1 + a_1 b_0$$

$$c_2 = \sum_{i=0}^{2} a_i b_{2-i} = \underline{a_2 b_0} + \underline{a_1 b_1} + \underline{a_0 b_2}$$

$$\begin{matrix} a_0 & a_1 & a_2 & a_3 & \dots \\ b_0 & b_1 & b_2 & b_3 & \dots \end{matrix}$$

Prop si (a_n) tiene Fg $A(x)$
(b_n) tiene Fg $B(x)$

$\Longrightarrow (c_n) = (a_n) * (b_n)$ tiene Fg $C(x) = A(x)B(x)$

(a_n) tiene Fg $\dfrac{1+2x}{(1-x)^2}$

(b_n) tiene Fg $\dfrac{1}{(1+2x)(1-x^2)}$

$(c_n) = (a_n) * (b_n)$

$\xRightarrow{\text{Prop}}$ $\underbrace{\text{La Fg de } (c_n)}_{C(x)}$ es $\dfrac{\cancel{(1+2x)}}{(1-x)^2} \; \dfrac{1}{\cancel{(1+2x)}(1-x)}$

$\boxed{a^{-n} = \dfrac{1}{a^n}}$

$C(x) = \dfrac{1}{(1-x)^3} = (1-x)^{-3}$

fórmula de combinaciones negativas

$= \displaystyle\sum_{n=0}^{\infty} CR^3_n \, x^n$

$c_{10} = CR^3_{10} = C^{12}_{10} = \dfrac{12 \cdot 11}{2} = 66$

$$\boxed{(1+x)^{-n} = \sum_{i=0}^{\infty} (-1)^i \, CR^n_i \, x^i}$$

$$(1+x)^{-n} = \sum_{i=0}^{\infty} (-1)^i CR^n_i \, x^i \qquad (-1)^i CR^n_i \to C^{-n}_i$$

$$(1-x)^{-3} = (1+y)^{-3} = \sum_{i=0}^{\infty} (-1)^i CR^3_i \, y^i = \sum_{i=0}^{\infty} (-1)^i CR^3_i (-x)^i$$

$y=-x$

Fórmula

$$= \sum_{i=0}^{\infty} (-1)^i CR^3_i (-1)^i x^i$$

$$= \sum_{i=0}^{\infty} \underbrace{\left[(-1)^i\right]^2}_{=1} CR^3_i \, x^i = \sum_{i=0}^{\infty} CR^3_i \, x^i$$

$$(1+x)^{-n} = \sum_{i=0}^{\infty} (-1)^i CR^n_i \, x^i$$

$$\boxed{(1-x)^{-n} = \sum_{i=0}^{\infty} CR^n_i \, x^i}$$

Ej 5

$$\begin{cases} a_{n+1} = b_n - a_n \\ b_{n+1} = 3a_n + b_n \end{cases}$$

Pregunta: $a_{14} + b_{14}$

sumar els +

$$a_{n+1} + b_{n+1} = 2a_n + 2b_n = 2(a_n + b_n)$$

$$\Rightarrow \underbrace{a_{n+1} + b_{n+1}}_{c_{n+1}} = 2\,\underbrace{(a_n + b_n)}_{c_n}$$

$$\boxed{c_n := a_n + b_n}$$

$$a_0 = \frac{1}{2^{10}} \quad b_0 = \frac{1}{2^{11}}$$

$$c_0 = \frac{1}{2^{10}} + \frac{1}{2^{11}} = \frac{2}{2^{11}} + \frac{1}{2^{11}} = \frac{3}{2^{11}}$$

$$\frac{2}{2^{11}}$$

$$\Rightarrow \begin{cases} c_{n+1} = 2c_n \\ c_0 = 3/2^{11} \end{cases}$$

$$c_n = 2^n \cdot \frac{3}{2^{11}}$$

$$60 = \underbrace{x_1 + x_1 + x_1}_{\text{1ra ciudad}} + \underbrace{x_2 + x_2 + x_2}_{\text{2da ciudad}} + \underbrace{2x_3 + x_3}_{\text{3ra ciudad}}$$

3 hospitales = vacunas

3 hosp = vacunas

2 hosp
2do doble de vacs

$$\boxed{C^m_k = C^m_{m-k}}$$

$$60 = 3x_1 + 3x_2 + 3x_3$$

$$\Rightarrow 20 = x_1 + x_2 + x_3 \qquad x_i \geq 1$$

$$\Rightarrow 17 = y_1 + y_2 + y_3 \qquad y_i \geq 0 \qquad CR^3_{17} = C^{17+3-1}_{17}$$

$$20 - 3 = (x_1 - 1) + (x_2 - 1) + (x_3 - 1) \qquad = C^{19}_{2}$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$

deriv

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots$$

$$\frac{x}{(1-x)^2} = 0 + 1x + 2x^2 + 3x^3 + \cdots$$

$\frac{x}{(1-x)^2}$ FG de $a_n = n$

$\frac{d}{dx}$

$$\frac{(1-x)^{\cancel{2}} + x2\cancel{(1-x)}}{(1-x)^{\cancel{4}\,3}} = 1 + 2^2 x + 3^2 x^2 + \cdots$$

$$\frac{1+x}{(1-x)^3} = 1 + 2^2 x + \cdots$$

$$\Rightarrow \frac{x(1+x)}{(1-x)^3} = 0 + 1x + 2^2 x^2 + 3^3 x^3 + \cdots$$

$\frac{x(1+x)}{(1-x)^3}$ Fg $a_n = n^2$

$f(x)$ FG $a_n = n^3$ $\quad b_n = 1$ FG $\frac{1}{1-x}$

$$c_n = \sum_{i=0}^{n} a_i \, 1 = \sum_{i=0}^{n} a_i \, b_{n-i} \qquad (c_n) = (a_n) * (b_n) \text{ conv}$$

FG de c_n es $\frac{f(x)}{1-x}$ $\leftarrow$ calcular c_n con combinaciones neg da la fórmula de la parte b

$$c_n = S_n$$