5 cajas 10 pelotitas

$$\underset{\text{caja 1}}{x_1} + \underset{\text{caja 2}}{x_2} + x_3 + x_4 + \underset{\text{caja 5}}{x_5} \leq 10 \qquad 1 \leq x_i$$

$$\underbrace{x_1 - 1}_{y_1} + \underbrace{x_2 - 1}_{y_2} + \underbrace{x_3 - 1}_{y_3} + \underbrace{x_4 - 1}_{y_4} + \underbrace{x_5 - 1}_{y_5} \leq 10 - 5$$

$$y_1 + y_2 + y_3 + y_4 + y_5 \leq 5 \qquad 0 \leq y_i$$

6 cajas

$$\boxed{y_1 + y_2 + y_3 + y_4 + y_5 + z = 5}$$

pelotas

$$\boxed{0 \leq y_i \qquad 0 \leq z}$$

z es lo que falta para llegar a 5

$$y_1 + y_2 + y_3 + y_4 + y_5 = 3$$

$$\downarrow$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + \underset{z}{2} = 5$$

$$y_1 + y_2 + y_3 + y_4 + y_5 \leq 5$$

6 casos

$$y_1 + \cdots + y_5 = 0$$
$$y_1 + \cdots + y_5 = 1$$
$$y_1 + \cdots + y_5 = 2$$
$$\vdots$$
$$y_1 + \cdots + y_5 = 5$$

Sumar esos da el mismo resultado

$$\frac{7!}{2!\,2!} - \frac{6!}{2!} - \frac{6!}{2!} + 5! = \frac{7\times 6\times 5!}{4} - \frac{6\times 5!}{2} - \frac{6\times 5!}{2} + 1\times 5!$$

$$= 5!\left(\frac{7\times \cancel{6}^{\,3}}{\cancel{4}_{\,2}} - 3 - 3 + 1\right) = 5!\left(\frac{21}{2} - 5\right) = 5!\,\frac{(21-10)}{2}$$

Pide: $a_{14} + b_{14}$ $\qquad c_n := a_n + b_n$

$$\boxed{\begin{aligned} a_{n+1} &= b_n - a_n \\ b_{n+1} &= 3a_n + b_n \end{aligned}} \Bigg| + \Rightarrow \boxed{\underbrace{a_{n+1} + b_{n+1}}_{= c_{n+1}} = 2(\underbrace{a_n + b_n}_{= c_n})}$$

$$\Longrightarrow c_{n+1} = 2c_n \qquad c_{14} = 2 \cdot c_{13} = 4 \cdot c_{12} = \dots = 2^{14} \cdot c_0$$

$$c_0 = a_0 + b_0 = \frac{1}{2^{10}} + \frac{1}{2^{11}} = \frac{2}{2^{11}} + \frac{1}{2^{11}} = \frac{3}{2^{11}}$$

$$c_{14} = 2^{14} \cdot \frac{3}{2^{11}} = 2^3 \cdot 3 = 8 \cdot 3 = 24$$

$$\operatorname{dist}(n,m) = m - n$$
$$n < m$$

$\operatorname{dist}(n,n) = n - n = 0$ $\quad$ $\operatorname{dist}(n+1,n) = 1$ ..

$S = \{1, 2, \dots, 100\}$ subconj de 4 elms. $a, b, c, d \in S$ $\quad$ dist $\geq 3$

$a < b < c < d$ $\quad$ $\boxed{b - a \geq 3 \quad c - b \geq 3 \quad d - c \geq 3}$

ej $\quad c - a = \underbrace{(c - b)}_{\geq 3} + \underbrace{(b - a)}_{\geq 3} \geq 6$

$$
\begin{aligned}
a &= 1 + x_1 & x_1 &\geq 0 \\
b &= a + x_2 & \boxed{x_2 \geq 3} \\
c &= b + x_3 & x_3 &\geq 3 \\
d &= c + x_4 & x_4 &\geq 3 \\
100 &= d + x_5 & x_5 &\geq 0
\end{aligned}
$$

$(b - a = a + x_2 - a = x_2)$

$$100 = d + x_5 = c + x_4 + x_5$$
$$= b + x_3 + x_4 + x_5 = a + x_2 + x_3 + x_4 + x_5$$
$$= 1 + x_1 + x_2 + x_3 + x_4 + x_5$$

$$C^{k+m-1}_{m-1}$$
$$\shortparallel$$
$$CR^m_k = C^{k+m-1}_k$$

$$\Rightarrow 1 + x_1 + x_2 + x_3 + x_4 + x_5 = 100$$
$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 99$$
$$x_1 \geq 0 \quad x_2, x_3, x_4 \geq \boxed{3} \quad x_5 \geq 0$$

$$\boxed{C^m_k = C^m_{m-k}}$$

$$x_1 + (x_2 - 3) + (x_3 - 3) + (x_4 - 3) + x_5 = 99 - 9 = 90$$

$$x_1 + y_2 + y_3 + y_4 + x_5 = 90$$
$$x_1, y_2, y_3, y_4, x_5 \geq 0$$

$$\Rightarrow CR^5_{90} = C^{90+5-1}_{90} = \boxed{C^{94}_{90}}$$
$$= C^{90+5-1}_{5-1} = C^{94}_4$$

$$C^n_k = \frac{n!}{(n-k)!\,k!} \qquad C^n_{n-k} = \frac{n!}{\underbrace{(n-(n-k))}_{k}!\,(n-k)!} = \frac{n!}{k!\,(n-k)!}$$

$$\Longrightarrow \boxed{C^n_k = C^n_{n-k}}$$

$$CR^5_{90} = C^{90+5-1}_{\square} = C^{\overset{n}{94}}_{\underset{k}{90}} = C^{\overset{n}{94}}_{\underset{\substack{94-90\\ n-k}}{4}}$$

$90 \qquad 5-1$

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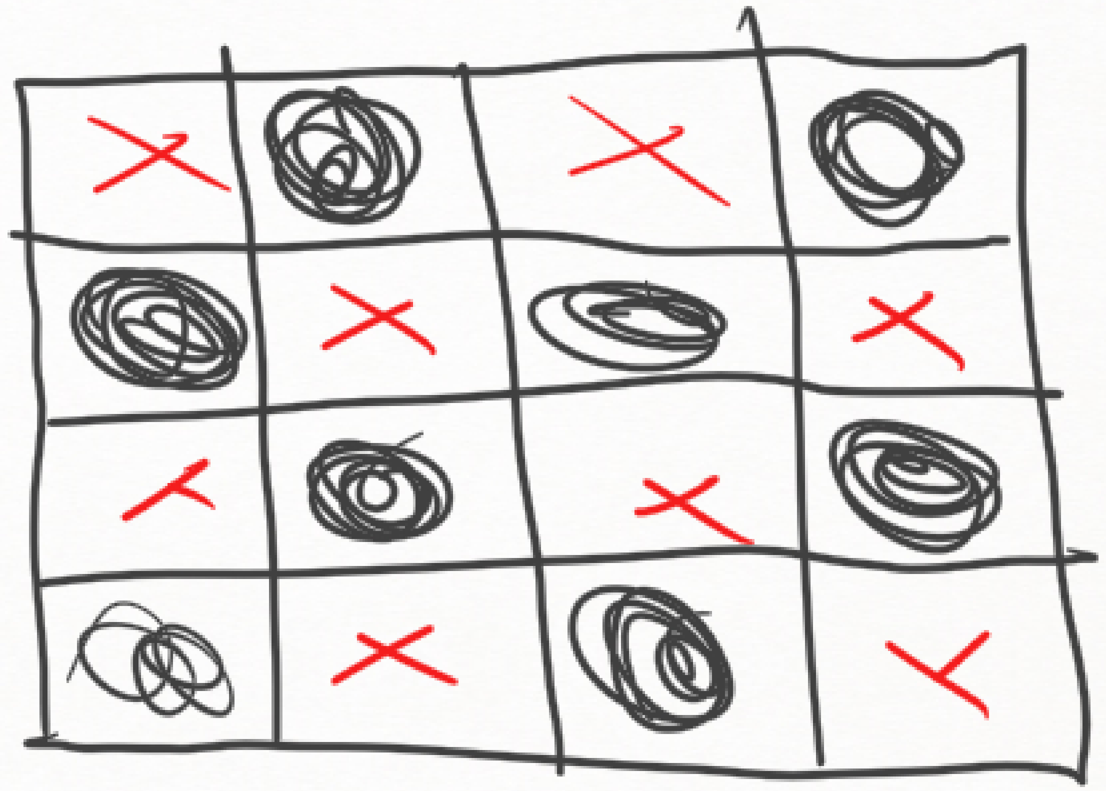
$$a_n + b_{n+1} =$$

al saltar cambia el color de la casilla



ninguno salta a la pos de otro

8 caballos


9 caballos
⇒ 2 en el mismo nido

$a_n = n^2, \quad b_n = 2^n \quad c_n = (a_n) * (b_n) \qquad b_n = 2^n$

$$c_n = \sum_{i=0}^{n} a_i b_{n-i} = a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \dots + a_n b_0$$

$n \quad n \quad n \quad n$

$$c_0 = a_0 b_0$$

$$\underline{c_1 = a_0 b_1 + a_1 b_0} = \overbrace{0^2 \cdot 2}^{0} + 1^2 \cdot 2^0 = 1$$

$$a_0 = 0^2 = 0$$

$$b_1 = 2^1 = 2$$

$$a_1 = 1^2 = 1$$

$$b_0 = 2^0 = 1$$

contar $f: \{1,2,\ldots,6\} \to \mathcal{P}\{1,2,\ldots,6\}$

tq $1 \notin f(1)$, $2 \notin f(2)$ ... etc.

$\mathcal{P}\{1,2,\ldots,6\} = \{A$ subconjuntos de $\{1,2,\ldots,6\}\}$

$\{1\}$, $\{1,3\}$, $\{1,4,6\}$, $\{4\}$, $\emptyset$, $\{1,2,\ldots,6\}$

$f(1) = \emptyset$

$f(2) = \{1\}$

$f(3) = \{1,2\}$

$f(4) = \{5,6\}$

$f(5) = \cancel{\{1\}} = \{6\}$

$f(6) = \{1,2,\ldots,6\}$

ejemplo

$$|\mathcal{P}\{1,2,3,\dots,6\}| = 2^6$$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \checkmark \times & \checkmark \times & \checkmark \times & \checkmark \times & \checkmark \times & \checkmark \times \end{array}$$

$$\begin{array}{cccccc} f(1) & f(2) & f(3) & f(4) & f(5) & f(6) \\ 32 & & & & & \end{array}$$

$f(1)$ subconjunto sin $1$ $\quad \underbrace{\mathcal{P}\{2,3,\dots,6\}}_{2^5}$

$f(2) \quad \overbrace{\mathcal{P}\{1,3,4,\dots,6\}}^{32}$

si $f(1)=\{3\} \Rightarrow \{3\} \in \mathcal{P}\{1,3,4,\dots,6\}$

si $f(1)=\{2\} \Rightarrow \{2\} \notin \mathcal{P}\{1,3,4,\dots,6\}$

Caso 1: $2 \notin f(1)$ $\quad$ letra $1 \notin f(1)$ $\quad 2 \notin f(2)$

| $f(1)$ | $f(2)$ | $f(3)$ | $f(4)$ | $f(5)$ | $f(6)$ | |
|---|---|---|---|---|---|---|
| 16 | 31 | $\underset{2^6}{64}-2$ | $64-3$ | $64-4$ | $64-5$ | $62 \cdot 61 \cdot 60 \cdot 59 \cdot 31 \cdot 16$ |

$f(1)$ está en $\mathcal{P}\{3,4,5,6\} \to 2^4$ $\qquad f(1) \in \mathcal{P}\{1,3,4,5,6\}$

$f(2)$ está en $\mathcal{P}\{1,3,4,5,6\} \to 2^5$ pero $f(1)$ está ahí

$\Rightarrow 32-1$ para no repetir $f(1)$

Caso 2: $2 \in f(1)$

| $f(1)$ | $f(2)$ | $f(3)$ | $f(4)$ | $f(5)$ | $f(6)$ |
|---|---|---|---|---|---|
| 16 | 32 | 62 | 61 | 60 | 59 |

$62 \cdot 61 \cdot 60 \cdot 59 \cdot 32 \cdot 16$

$f(1)$ $\quad \{2\}$ y $\overbrace{\mathcal{P}\{3,4,5,6\}}^{16}$

$f(2)$ $\quad \mathcal{P}(1,3,4,5,6)$

$2 \in f(1)$ $\quad f(1)$ no está en $\mathcal{P}\{1,3,4,5,6\}$

$$62\times 61\times 60\times 59\times 31\times 16+62\times 61\times 60\times 59\times 32\times 16$$

$$=62\times 61\times 60\times 59\times 16\,(\underbrace{31+32}_{63})=63\times 62\times 61\times 60\times 59\times 16$$