$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Combinaciones negativas

recordar $\quad C_r^n = \dfrac{n!}{(n-r)!\,r!} \qquad CR_r^n = C_r^{n+r-1}$

$$\boxed{C_r^{-n} = (-1)^r CR_r^n = (-1)^r C_r^{n+r-1}}$$

$$\boxed{(1+x)^{-n} = \sum_{i=0}^{\infty} C_i^{-n} x^i}$$

Recordar:

$$(1+x)^n = \sum_{i=0}^{n} C_i^n x^i$$ Fórmula binomio

Ejemplo

$$\frac{1}{1-x} = (1-x)^{-1} = (1+y)^{-1} = \sum_{i=0}^{\infty} C^{-1}_{i}\, y^{i}$$

$y=-x$

$$= \sum_{i=0}^{\infty} (-1)^{i}\, CR^{1}_{i}\, y^{i}$$

$$= \sum_{i=0}^{\infty} (-1)^{i}\, \underbrace{C^{i}_{i}}_{=1}\, y^{i}$$

$$= \sum_{i=0}^{\infty} (-1)^{i} y^{i} = \sum_{i=0}^{\infty} (-1)^{i} (-x)^{i}$$

$$= \sum_{i=0}^{\infty} \underbrace{(-1)^{i}(-1)^{i}}_{((-1)^{i})^{2} = 1}\, x^{i} = \sum_{i=0}^{\infty} x^{i}$$

Llegamos a

$$\frac{1}{1-x} = \sum_{i=0}^{\infty} x^{i} \quad \checkmark$$

③ a) coef x^8 en $(1+x+x^2+\dots)^{10}$

$$(1+x+x^2+\dots)^{10} = \left(\frac{1}{1-x}\right)^{10} = \left((1-x)^{-1}\right)^{10} = (1-x)^{-10}$$

deshago CV

$$(1-x)^{-10} = (1+y)^{-10} \overset{\text{Form}}{=} \sum_{i=0}^{\infty} C_i^{-10} y^i = \sum_{i=0}^{\infty} (-1)^i CR_i^{10} y^i = \sum_{i=0}^{\infty} (-1)^i CR_i^{10} (-x)^i$$

$y=-x$

$$= \sum_{i=0}^{\infty} (-1)^i CR_i^{10} (-1)^i x^i = \sum_{i=0}^{\infty} \underbrace{(-1)^i (-1)^i}_{((-1)^i)^2 = 1} CR_i^{10} x^i$$

$$= \sum_{i=0}^{\infty} CR_i^{10} x^i = CR_0^{10} + CR_1^{10} x + CR_2^{10} x^2 + \dots + CR_8^{10} x^8 + \dots + CR_{20}^{10} x^{20} + \dots$$

coef de x^8 es CR_8^{10}

$$\frac{x^3-5x}{(1-x)^3}$$ coef de x^{15}

la cuenta que acabamos de hacer

$$(x^3-5x)\times(1-x)^{-3} = (x^3-5x)\sum_{i=0}^{\infty} CR^3_i x^i$$

$$= x^3\sum_{i=0}^{\infty} CR^3_i x^i - 5x\sum_{i=0}^{\infty} CR^3_i x^i$$

$$= \sum_{i=0}^{\infty} CR^3_i x^3 x^i + \sum_{i=0}^{\infty} -5\, CR^3_i\, x\, x^i$$

$$= \underbrace{\sum_{i=0}^{\infty} CR^3_i x^{i+3}}_{\text{coef de } x^{15}} + \underbrace{\sum_{i=0}^{\infty} -5\, CR^3_i\, x^{i+1}}_{\text{coef de } x^{15}}$$

$i=12$

$CR^3_{12}\, x^{15}$

$i=14$

$-5\,CR^3_{14}\, x^{15}$

$\Rightarrow$ Coef de x^{15} es

$CR^3_{12} - 5\,CR^3_{14} = a_{15}$

En general

$a_m = CR^3_{m-3} - 5\,CR^3_{m-1}$ si $m \geq 3$

la FG de la cantidad de soluciones de $x_1 + x_2 = n$

$f(x) = \sum_{n=0}^{\infty} a_n x^n$ $\quad a_n =$ cantidad de soluciones de $x_1 + x_2 = n$

La Función generatriz de un problema de conteo que depende de n es la que tiene como coeficientes las soluciones al problema

---

$d_1 + d_2 + d_3 = 5$

$0 \leq d_i \leq 3$

$d_1 = 3 \quad d_2 = 1 \quad d_3 = 1$

$d_1 = 0 \quad d_2 = 2 \quad d_3 = 3$

$f(x) = \underbrace{(1 + x + x^2 + x^3)}_{d_1} \underbrace{(1 + x + x^2 + x^3)}_{d_2} \underbrace{(1 + x + x^2 + x^3)}_{d_3}$

La respuesta al problema es el coef de x^5

$x^3 x x = x^5 \Rightarrow$ aporta 1 al coef x^5

$1 \times x^2 \times x^3 = x^5 \Rightarrow$ aporta 1 al coef x^5

$d_1 + d_2 = 5$

$0 \leq d_i \leq 4$

- $d_1 = 1 \quad d_2 = 4$
- $d_1 = 2 \quad d_2 = 3$
- $d_1 = 3 \quad d_2 = 2$
- $d_1 = 4 \quad d_2 = 1$

$(1 + x^1 + x^2 + x^3 + x^4)(1 + x^1 + x^2 + x^3 + x^4)$

$x^5 + x^5 + x^5 + x^5 = 4x^5$

$x_1 + x_2 + x_3 = 10$

$0 \leq x_i$

$$f(x) = (1 + x + x^2 + x^3 + \cdots)(1 + x + x^2 + \cdots)(1 + x + x^2 + \cdots)$$

$$= \frac{1}{1-x}\,\frac{1}{1-x}\,\frac{1}{1-x} = (1-x)^{-3}$$

$$= \sum_{n=0}^{\infty} CR^3_n x^n$$

cuenta que ya hicimos

coef x^{10}: CR^3_{10}

a_n: Formas en que el cajero puede dar n pesos.
Dado $100, 200, 500, 1000$

$$d_1 + d_2 + d_3 + d_4 = n$$
$$0 \leq d_i$$

d_1 es múltiplo de 100
d_2 es múltiplo de 200
d_3 múltiplo de 500
d_4 múltiplo de 1000

d_1: plata que da en billetes de 100
d_2: plata que da en billetes de 200
d_3: " " " " " 500
d_4: " " " " " 1000

$$f(x) = \underbrace{\left(1 + x^{100} + x^{200} + x^{300} + \cdots\right)}_{d_1}\underbrace{\left(1 + x^{200} + x^{400} + x^{600} + \cdots\right)}_{d_2}\underbrace{\left(1 + x^{500} + x^{1000} + x^{1500} + \cdots\right)}_{d_3}\underbrace{\left(1 + x^{1000} + x^{2000} + \cdots\right)}_{d_4}$$

x^0 $\quad x^{1600}$

$$1 + \underbrace{x^{100}}_{=y} + (x^{100})^2 + (x^{100})^3 + \cdots = 1 + y + y^2 + \cdots$$
$$= \frac{1}{1-y} = \frac{1}{1-x^{100}}$$

$$\boxed{\begin{matrix} d_1 = 200 & d_3 = 1000 \\ d_2 = 400 & d_4 = 0 \end{matrix}}$$

$$\boxed{\begin{matrix} d_1 = d_3 = d_4 = 0 \\ d_2 = 1600 \end{matrix}}$$

$$\boxed{f(x) = \frac{1}{1-x^{100}} \frac{1}{1-x^{200}} \frac{1}{1-x^{500}} \frac{1}{1-x^{1000}}}$$

$$\underbrace{(1 + x^{100} + \cdots + x^{2000})(1 + x^{200} + \cdots + x^{200})(1 + x^{500} + \cdots + x^{2000})(1 + x^{1000} + x^{2000})}$$

$$9! - C^4_1 8! + C^4_2 7!$$

$$b_{n+2} - 5b_{n+1} + 6b_n = 7n \qquad a_0 = 1, \quad a_1 = 1$$

1. Hallar sol general de la homogenea

$$b_{n+2} - 5b_{n+1} + 6b_n = 0$$

$$\lambda^2 - 5\lambda + 6 = 0 \qquad \frac{5 \pm \sqrt{1}}{2} \begin{cases} \lambda_1 = 2 \\ \lambda_2 = 3 \end{cases}$$

$$\boxed{b_n^H = c_1 2^n + c_2 3^n}$$ Solución general de la homogenea asociada.

grado 1 $\quad g(n) = 7n$

2. Hallar una solución particular de $b_{n+2} - 5b_{n+1} + 6b_n = 7n \, 1^n = r^n$

$$b_n^P = \alpha n + \beta$$ ← polinomio genérico de grado 1 $\quad$ 1 no es raiz pol car

$$= n(\alpha n + \beta)$$ → si 1 fuera raiz

$$= n^2(\alpha n + \beta)$$ → si 1 raiz doble

$$b_{n+2} - 5b_{n+1} + 6b_n = 7n$$

$$\underline{b_n^p = \alpha n + \beta}$$

$$\underbrace{\alpha(n+2)+\beta}_{b_{n+2}} - 5\underbrace{(\alpha(n+1)+\beta)}_{b_{n+1}} + 6\underbrace{(\alpha n+\beta)}_{b_n} = 7n$$

$$\cancel{\alpha n} + \cancel{2\alpha} + \cancel{\beta} \cancel{-5\alpha n} - \cancel{5\alpha} - \cancel{5\beta} + \cancel{6\alpha n} + \cancel{6\beta} = 7n$$

$$2\alpha n - 3\alpha + 2\beta = 7n$$

$$2\alpha n \quad -3\alpha + 2\beta = 7n + 0$$

$$\begin{cases} 2\alpha = 7 \Rightarrow \alpha = 7/2 \\ -3\alpha + 2\beta = 0 \end{cases}$$

$$-3 \times \frac{7}{2} + 2\beta = 0 \Rightarrow 2\beta = \frac{3 \times 7}{2}$$

$$\Rightarrow \beta = \frac{3 \times 7}{4} = \frac{21}{4}$$

$$\Rightarrow \boxed{b_n^p = \frac{7}{2}n + \frac{21}{4}}$$

3) sumar la general homogenea con la particular y determinar las ctes

$$b_n = b_n^h + b_n^p = \boxed{c_1 2^n + c_2 3^n + \frac{7}{2}n + \frac{21}{4}}$$

$$b_0 = 1 \qquad c_1 2^0 + c_2 3^0 + \frac{21}{4} = 1 \Rightarrow c_1 + c_2 = 1 - \frac{21}{4} = \frac{-17}{4}$$

$$b_1 = 1 \qquad c_1 2^1 + c_2 3^1 + \frac{7}{2} + \frac{21}{4} = 1 \Rightarrow 2c_1 + 3c_2 = 1 - \frac{7}{2} - \frac{21}{4}$$

$$= \frac{4 - 14 - 21}{4} = \frac{-31}{4}$$

$$\begin{cases} c_1 + c_2 = -17/4 \\ 2c_1 + 3c_2 = -31/4 \end{cases}$$

Con los valores que se obtienen, tenemos la solución