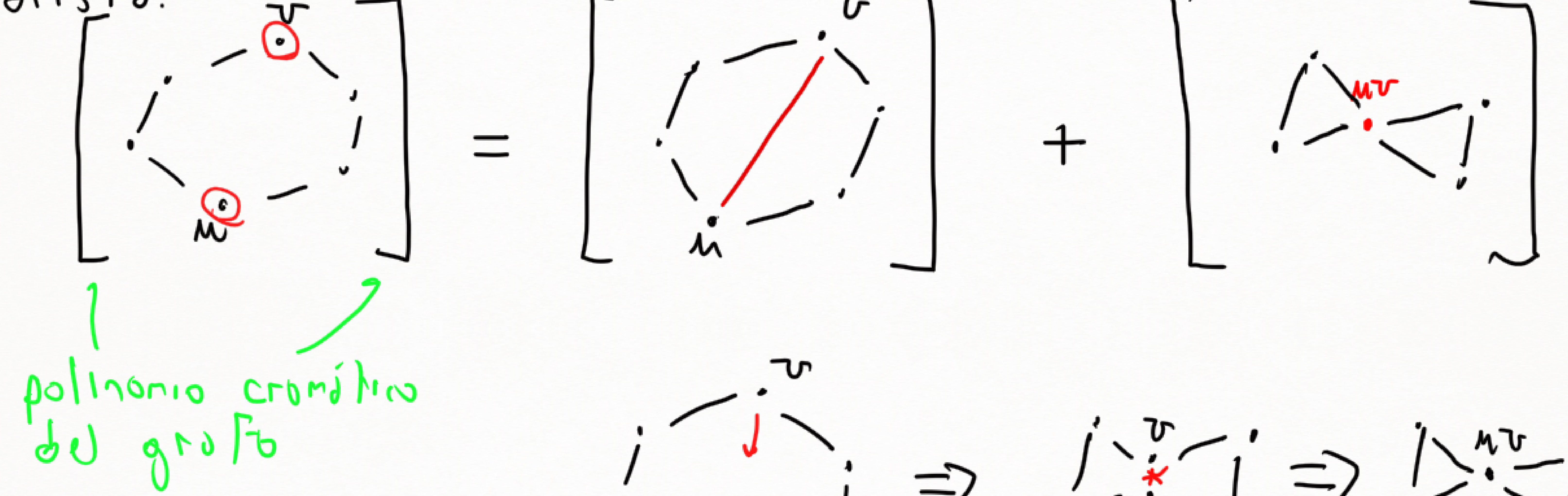
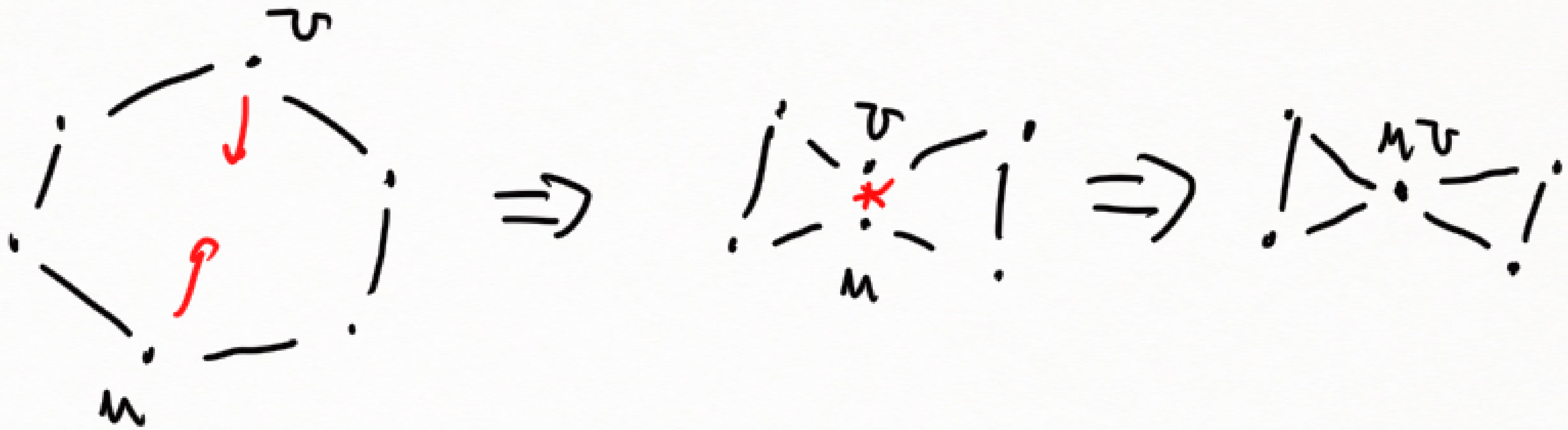
Agregado de arista y contracción

$G = (V,E)$ grafo $u, v \in V$ que no están conectados por una arista.





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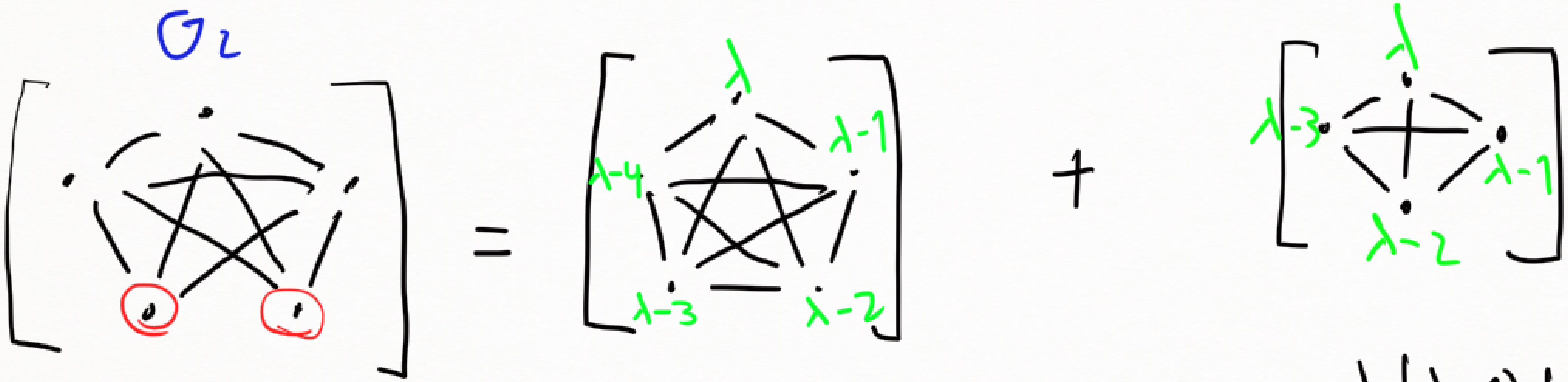
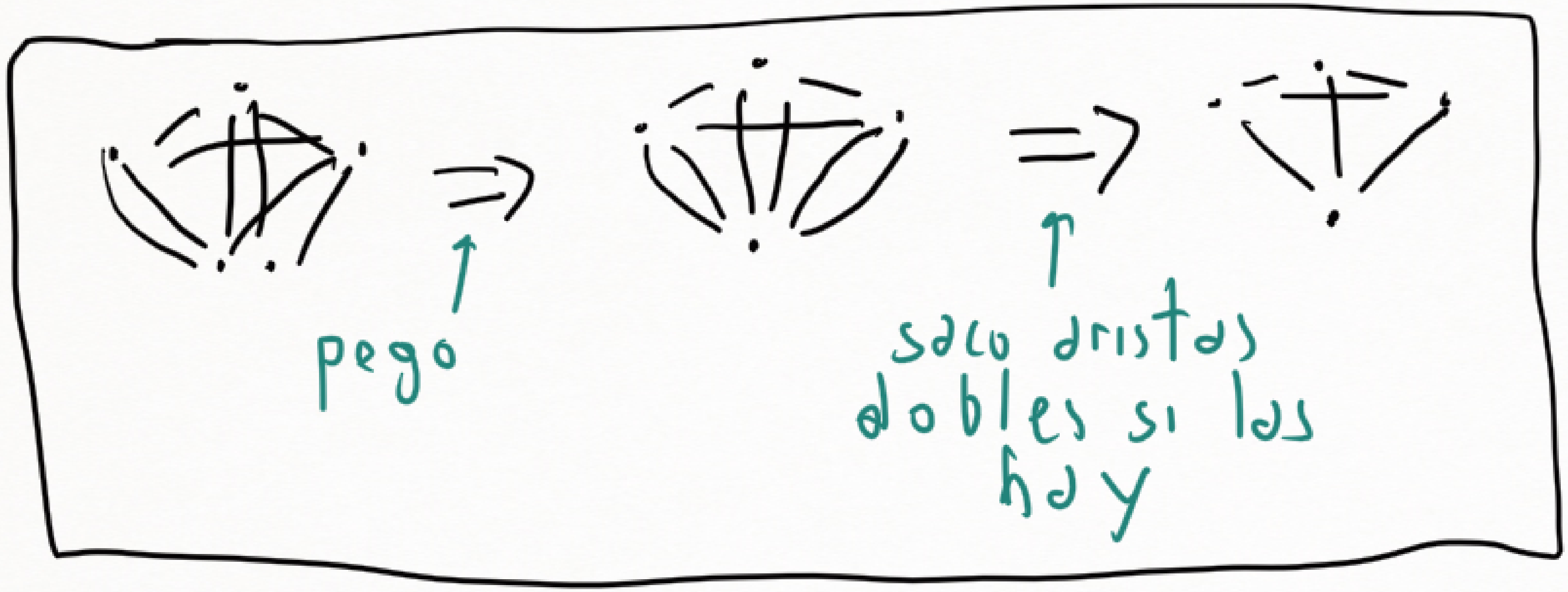




$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4) + \lambda(\lambda-1)(\lambda-2)(\lambda-3)\cdot 1$$

$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3)\,(\lambda-4)+1$$

$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3)^2$$

$P_{G_2}(\lambda)$: cantidad de formas de pintar G_2 disponiendo de λ colores distintos (no hay xq usar todos)

$\chi(G_2)$: mínima cantidad de colores necesarios para pintar G_2

$\chi(G_2) = \min\{\lambda > 0 \ \lambda \in \mathbb{N} \text{ tq } P_{G_2}(\lambda) > 0\}$ $\iff$ existe 1 o más coloraciones

$$P_{G_2}(\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)^2$$

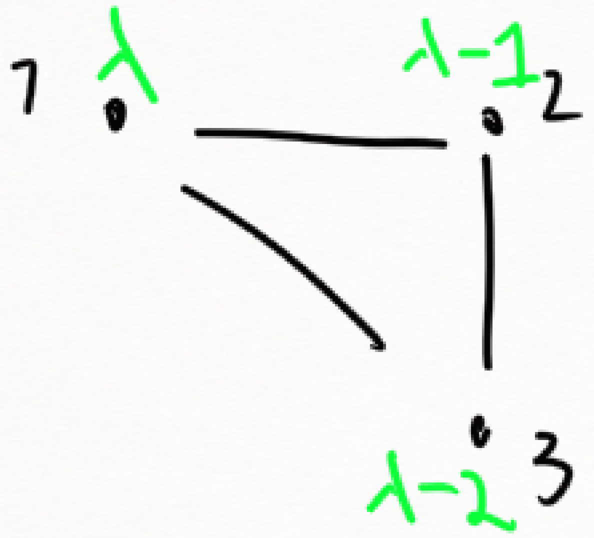
$P_{G_2}(1) = 0 \Rightarrow$ No se puede colorear con un color

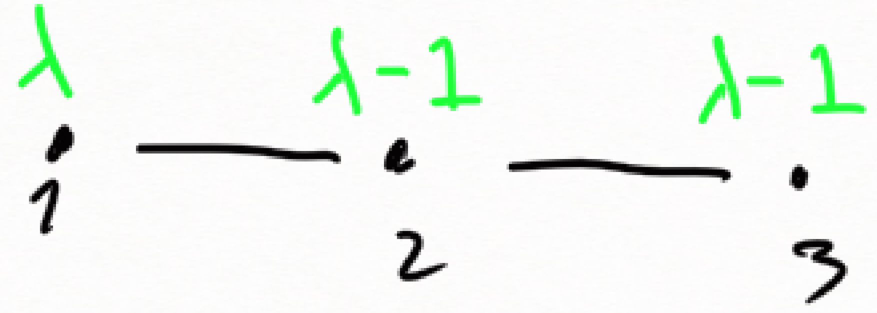
$P_{G_2}(2) = 0 \Rightarrow$ No se puede colorear con dos colores

$P_{G_2}(3) = 0 \Rightarrow$ No se puede colorear con 3 colores

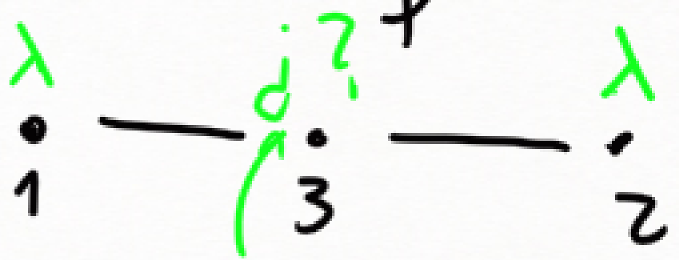
$P_{G_2}(4) = 4! > 0$

$\Rightarrow$ Con 4 colores sí se puede $\Rightarrow \chi(G_2) = 4$





Caso en que no funciona.

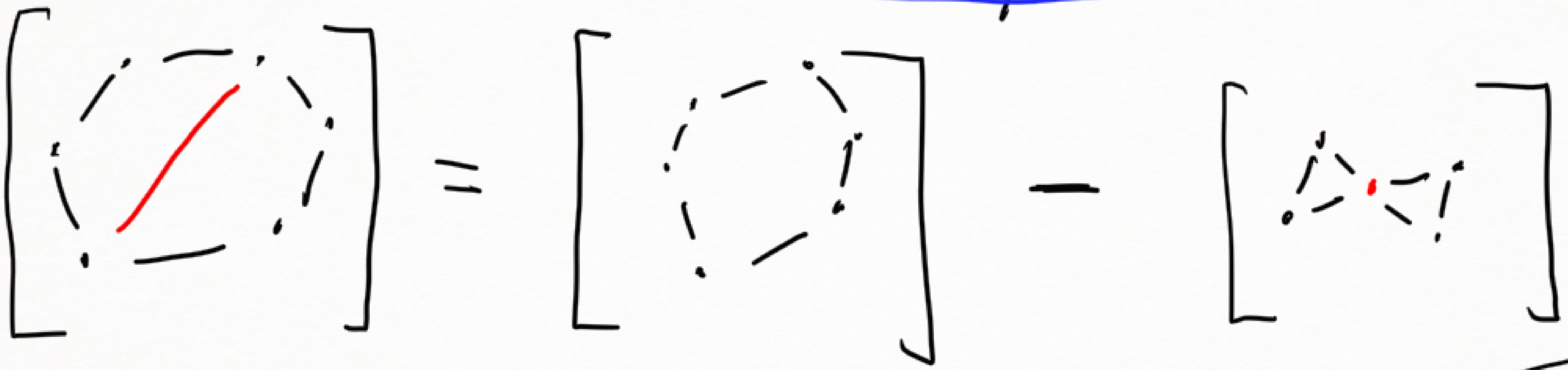


depende si el color del 1 y el 2 son = o ≠

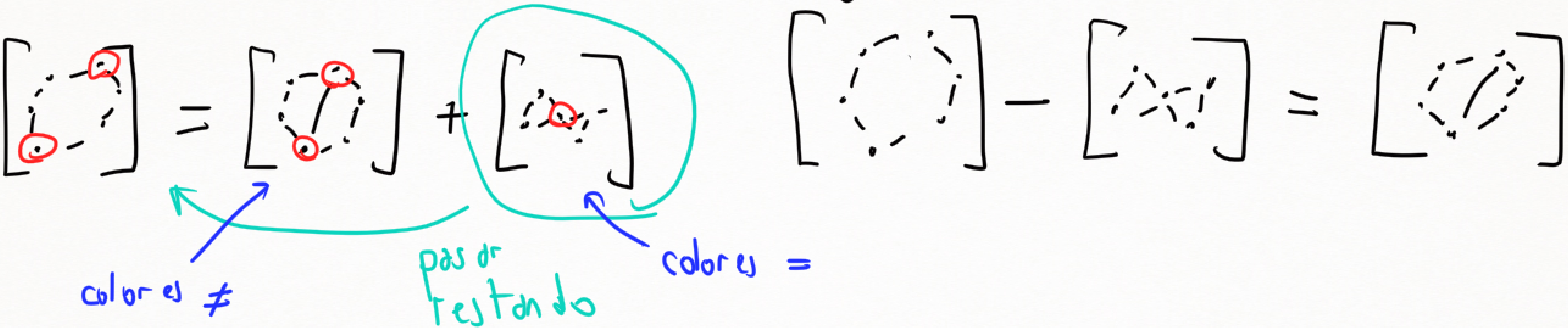
3 etapas:

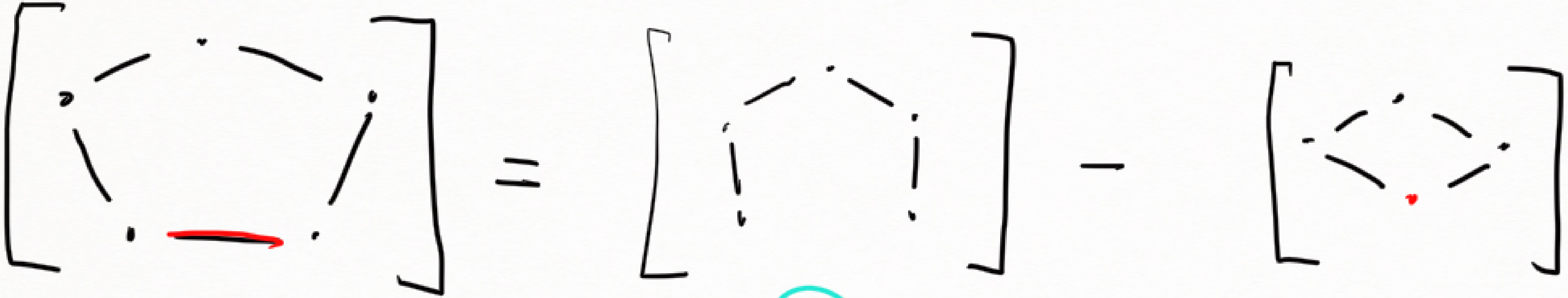
1. Colorear vértice 1
2. Colorear vértice 2 de color ≠ a sus vértices adyacentes que ya fueron pintados
3. Colorear vértice 3 de color ≠ ....

Quitado de aristas y contracción



Se despejo de la fórmula de agregado de arista.



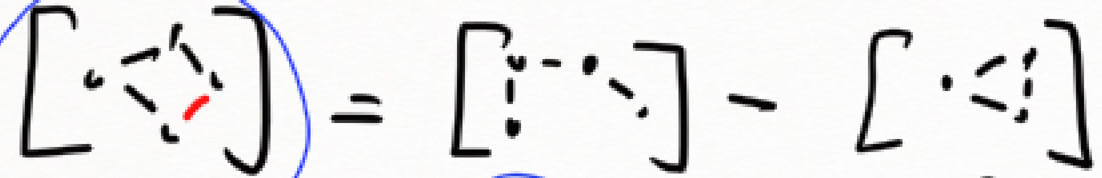


C_5

P_5

$= \lambda(\lambda-1)^4 -$ 

C_4



$= \lambda(\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2)$

$= \lambda(\lambda-1)^4 - \left(\lambda(\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2)\right)$

$= \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)$

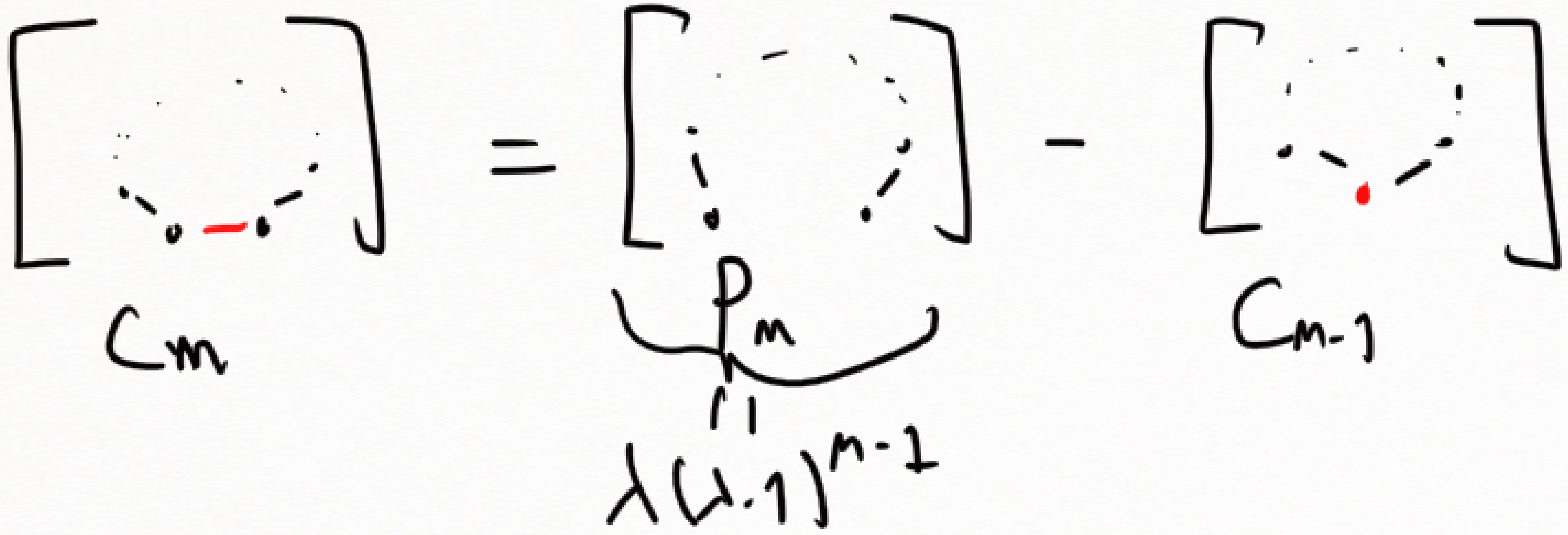
$\Rightarrow P_{C_5}(\lambda) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)$

$$P_{C_5}(\lambda) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)$$

$$P_{C_5}(1) = 0$$

$$P_{C_5}(2) = 2 - 2 + 0 = 0$$

$$P_{C_5}(3) = 3 \cdot 2^4 - 3 \cdot 2^3 + 3! = 30 \Rightarrow \chi_{C_5} = 3$$



$$\Rightarrow P_{C_m}(\lambda) = \lambda(\lambda-1)^{m-1} - P_{C_{m-1}}(\lambda)$$

Fijamos un λ genérico y lo tomamos como constante para el razonamiento.

$c_m := P_{C_m}(\lambda)$ sucesión

$$\Rightarrow c_m = \lambda(\lambda-1)^{m-1} - c_{m-1}$$

recurrencia

$$c_n + c_{n-1} = \lambda(\lambda-1)^{n-1}$$

Homogenea: $c_n + c_{n-1} = 0$ pol característico: $x+1=0 \Rightarrow x=-1$

$$\Rightarrow c_n^H = k(-1)^n$$

Particular: $c_n + c_{n-1} = \lambda(\lambda-1)^{n-1}$ λ constante $\Rightarrow$ tipo exponencial con base $\lambda-1$

$$c_n^P = K(\lambda-1)^n$$

sustituyendo: $\underbrace{K(\lambda-1)^n}_{(\lambda-1)(\lambda-1)^{n-1}} + K(\lambda-1)^{n-1} = \lambda(\lambda-1)^{n-1}$

$$\Rightarrow K(\lambda-1) + K = \lambda \Rightarrow K\lambda - K + K = \lambda \Rightarrow K\lambda = \lambda \Rightarrow K = 1$$

$$\Rightarrow c_n^P = (\lambda-1)^n$$

sol gral: $c_n = k(-1)^n + (\lambda-1)^n$

↑ hallarla con condiciones iniciales

$c_m = k(-1)^m + (\lambda-1)^m$ $c_3 = P_{C_3}(\lambda) = \lambda(\lambda-1)(\lambda-2)$

$\Rightarrow \lambda(\lambda-1)(\lambda-2) = k(-1)^3 + (\lambda-1)^3 = (\lambda-1)^3 - k$

$\Rightarrow k = (\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2) = (\lambda-1)\left((\lambda-1)^2 - \lambda(\lambda-2)\right)$

$= (\lambda-1)(\lambda^2 - 2\lambda + 1 - \lambda^2 + 2\lambda) = (\lambda-1)\times 1 \Rightarrow k = \lambda - 1$

$\Rightarrow c_m = (\lambda-1)(-1)^m + (\lambda-1)^m$

$$\Rightarrow \boxed{P_{C_m}(\lambda) = (\lambda-1)(-1)^m + (\lambda-1)^m}$$

$\lambda(\lambda-1)((\lambda-1)-1)$
$= \lambda(\lambda-1)^2 - \lambda(\lambda-1)$

Antes: $P_{C_5}(\lambda) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)^2$
$- \lambda(\lambda-1)$

$$-(\lambda-1)+(\lambda-1)^5$$

$$\lambda(\lambda-1)\left((\lambda-1)^3-(\lambda-1)^2+(\lambda-1)^1-1\right)$$

$$=-\lambda(\lambda-1)\left((-(\lambda-1))^3+(-(\lambda-1))^2+(-(\lambda-1))^1+(-(\lambda-1))^0\right)$$

$$=-\lambda(\lambda-1)\,\frac{1-(-(\lambda-1))^4}{1--(\lambda-1)}$$

$$\boxed{1+x+\dots+x^n=\frac{1-x^{n+1}}{1-x}}$$

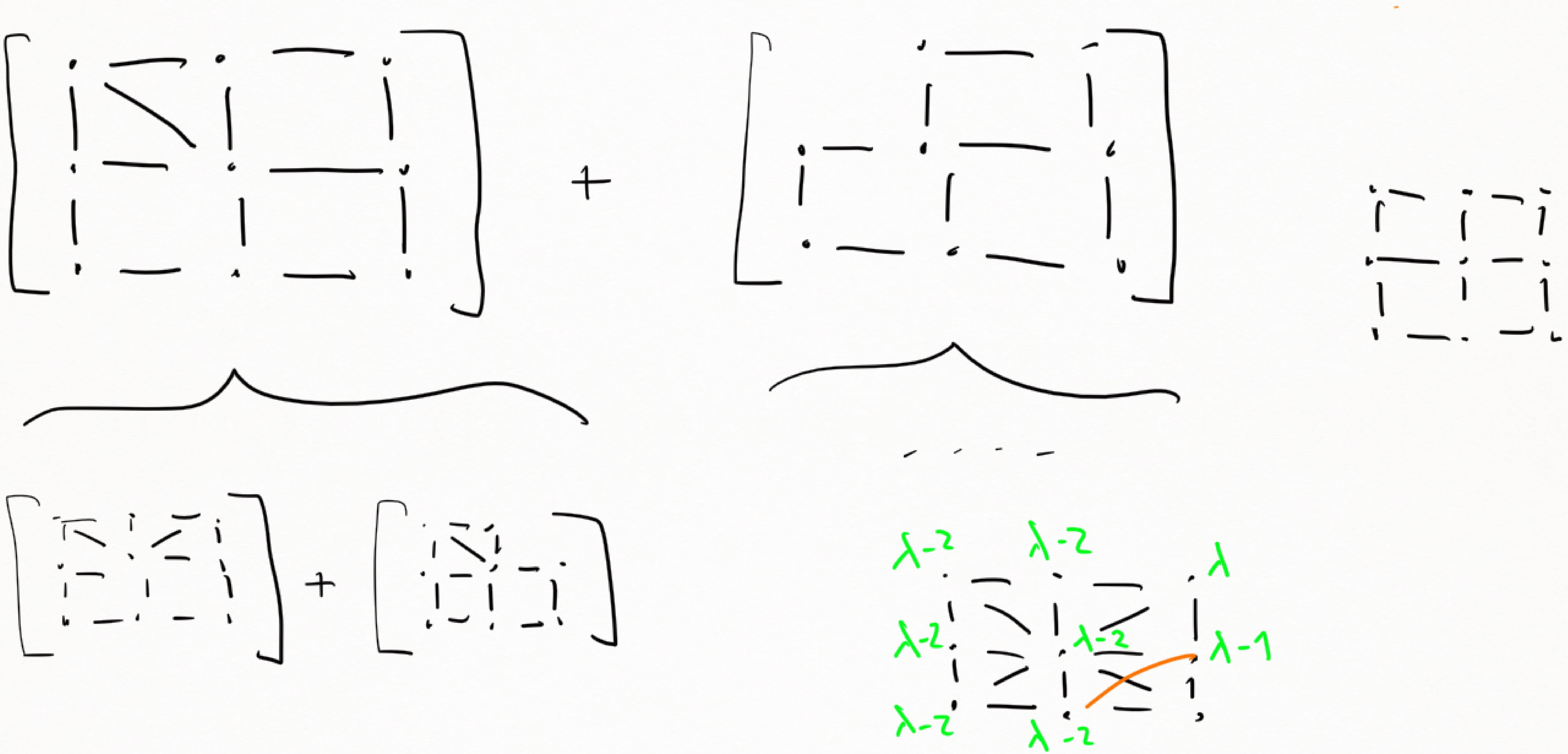
$$=-\lambda(\lambda-1)\,\frac{1-(\lambda-1)^4}{\lambda}=-(\lambda-1)(1-(\lambda-1)^4)$$

$$=-(\lambda-1)+(\lambda-1)^5$$

$$P_{C_n}(\lambda) = (\lambda-1)(-1)^n + (\lambda-1)^n$$

$$P_{C_n}(2) = 1 \times (-1)^n + 1^n = (-1)^n + 1 = \begin{cases} 2 & n \text{ par} \\ 0 & n \text{ impar} \end{cases}$$

$\lambda - 1 = 1$


λ-2
λ-2
λ
λ-2
λ-2
λ-1
1
λ-2
λ-2