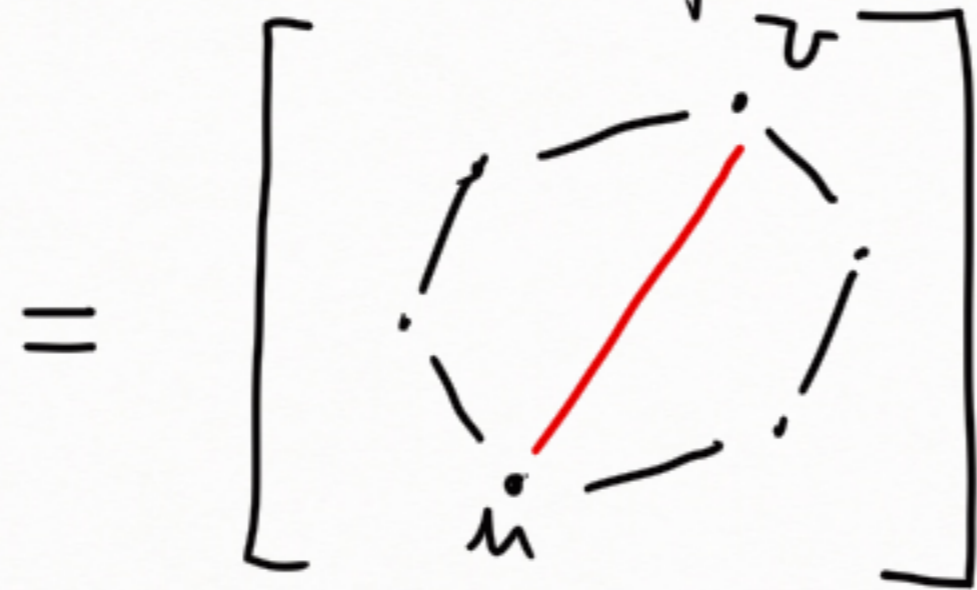
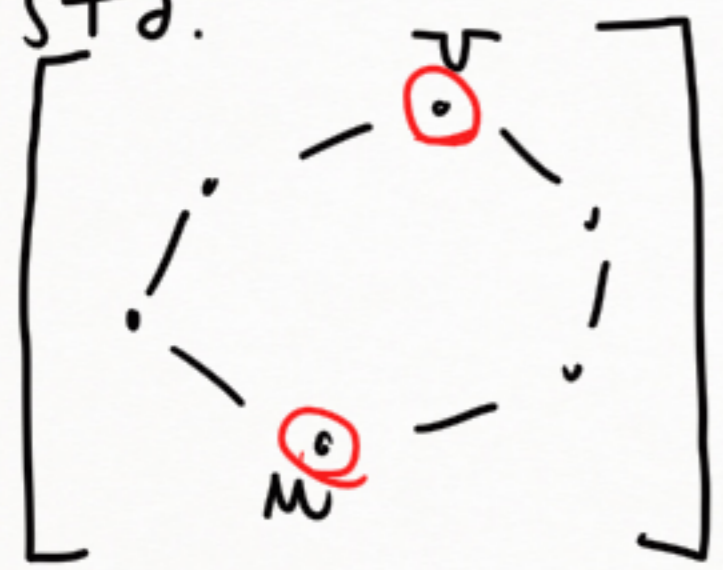


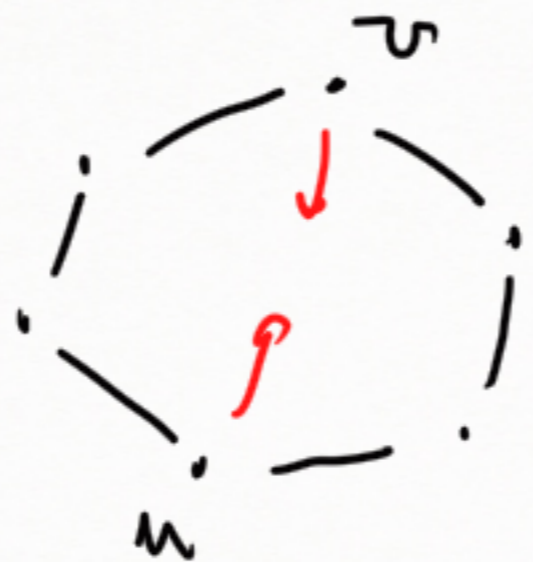
Agregado de arista y contracción

$G = (V, E)$  grafo  
arista.

$u, v \in V$  que no están conectados por una



polinomio cromático  
del grafo



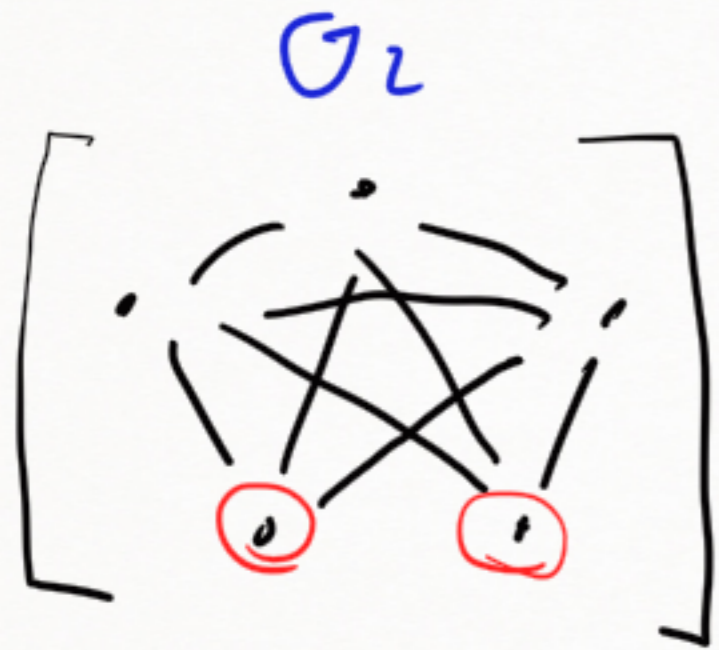
$\Rightarrow$



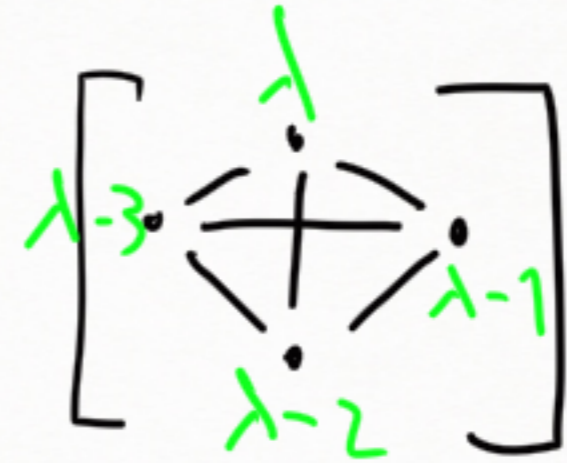
$\Rightarrow$



(12)



+



$$\begin{aligned}
 &= \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4) \\
 &\quad + \lambda(\lambda-1)(\lambda-2)(\lambda-3) \cdot 1 \\
 &= \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4) + 1 \\
 &= \lambda(\lambda-1)(\lambda-2)(\lambda-3)^2
 \end{aligned}$$

$\uparrow_{G_2}(\lambda)$ : cantidad de formas de pintar  $G_2$  disponiendo de  $\lambda$  colores distintos (no hay xq usar todos)

$\chi(G_2)$ : mínima cantidad de colores necesarios para pintar  $G_2$

$\chi(G_2) = \min \{ \lambda > 0 \lambda \in \mathbb{N} \text{ t.q. } \uparrow_{G_2}(\lambda) > 0 \} \Leftrightarrow$  existe 1 o más coloraciones

$$\uparrow_{G_2}(\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)^2$$

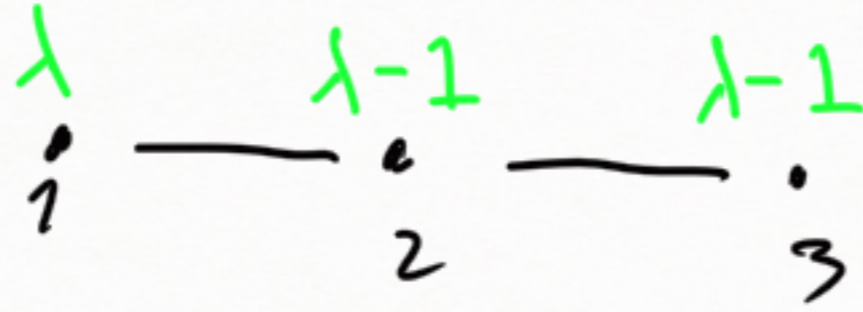
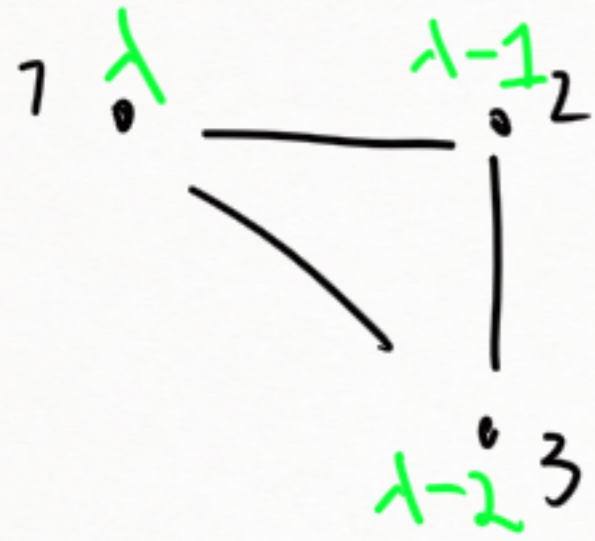
$\uparrow_{G_2}(1) = 0 \Rightarrow$  No se puede colorear con un color

$\uparrow_{G_2}(2) = 0 \Rightarrow$  No se puede colorear con dos colores

$\uparrow_{G_2}(3) = 0 \Rightarrow$  No se puede colorear con 3 colores

$$\uparrow_{G_2}(4) = 4! > 0$$

$\Rightarrow$  Con 4 colores sí se puede  $\Rightarrow \chi(G_2) = 4$



Caso en que no funciona.

depende si el color del 1 y el 2 son  $=$  o  $\neq$

3 etapas:

- ① Colorear vértice 1
- ② Colorear vértice 2 de color  $\neq$  a sus vértices adyacentes que ya fueron pintados
- ③ Colorear vértice 3 de color  $\neq$  ...

# Quitado de aristas y contracción

$$\left[ \begin{array}{c} \text{---} \\ \diagup \text{---} \\ \diagdown \text{---} \\ \text{---} \end{array} \right] = \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right] - \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right]$$

The first diagram shows a square with dashed edges and a solid red diagonal edge. The second diagram shows a square with all dashed edges. The third diagram shows a square with dashed edges and a small red dot at the center.

Se despeja de la fórmula de agregado de arista.

$$\left[ \begin{array}{c} \text{---} \\ \diagup \text{---} \\ \diagdown \text{---} \\ \text{---} \end{array} \right] = \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right] + \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right] - \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right] = \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right]$$

The first diagram has red circles around the top and bottom vertices. The second diagram has red circles around the top and bottom vertices. The third diagram has a red circle around the center. The fourth diagram is a square with dashed edges. The fifth diagram is a square with dashed edges and a red dot at the center. The sixth diagram is a square with dashed edges and a solid diagonal edge.

colores  $\neq$

pas de restando

colores =

$$\begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ \cdot & & & & \cdot \end{bmatrix} = \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix} - \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix}$$

$C_5$

$$= \lambda(\lambda-1)^4 - \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix}$$

$C_4$

$\lambda \quad \lambda-1 \quad \lambda-1 \quad \lambda-1 \quad \lambda-1$

$$\begin{aligned} \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix} &= \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix} - \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix} \\ &= \lambda(\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2) \end{aligned}$$

$$\begin{aligned} &= \lambda(\lambda-1)^4 - (\lambda(\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2)) \\ &= \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2) \\ \Rightarrow \chi_{C_5}(\lambda) &= \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2) \end{aligned}$$

$$\uparrow_{C_5}(\lambda) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)$$

$$\uparrow_{C_5}(1) = 0$$

$$\uparrow_{C_5}(2) = 2 - 2 + 0 = 0$$

$$\uparrow_{C_5}(3) = 3 \times 2^4 - 3 \times 2^3 + 3! = 30 \Rightarrow \chi_{C_5} = 3$$

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{C_m} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{C_{m-1}} - \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{C_{m-1}}$$

$\underbrace{\quad}_{P_m}$   
 $\lambda(\lambda-1)^{m-1}$

$$\Rightarrow \uparrow_{C_m}(\lambda) = \lambda(\lambda-1)^{m-1} - \uparrow_{C_{m-1}}(\lambda)$$

Fijamos un  $\lambda$  genérico y lo tomamos como constante para el razonamiento.

$c_m := \uparrow_{C_m}(\lambda)$  sucesión

$$\Rightarrow \boxed{c_m = \lambda(\lambda-1)^{m-1} - c_{m-1}}$$

recurrencia

$$r_m + r_{m-1} = \lambda (\lambda - 1)^{m-1}$$

Homogeneous:  $r_m + r_{m-1} = 0$  pol. característica:  $x + 1 = 0 \Rightarrow x = -1$   
 $\Rightarrow r_m^A = k(-1)^m$

Particular:  $r_m + r_{m-1} = \lambda (\lambda - 1)^{m-1}$   $\lambda$  constante  $\Rightarrow$  tipo exponencial con base  $\lambda - 1$

$$r_m^P = K(\lambda - 1)^m$$

substituyendo:  $K(\lambda - 1)^m + K(\lambda - 1)^{m-1} = \lambda (\lambda - 1)^{m-1}$   
 $(\lambda - 1)(\lambda - 1)^{m-2}$

$$\Rightarrow K(\lambda - 1) + K = \lambda \Rightarrow K\lambda - K + K = \lambda \Rightarrow K\lambda = \lambda \Rightarrow K = 1$$

$$\Rightarrow r_m^P = (\lambda - 1)^m \quad \text{sol. genl: } r_m = k(-1)^m + (\lambda - 1)^m$$

↑  
 hallarlo con condiciones iniciales



$$c_m = b(-1)^m + (\lambda-1)^m$$

$$c_3 = \uparrow_{C_3}(\lambda) = \lambda(\lambda-1)(\lambda-2)$$

$$\Rightarrow \lambda(\lambda-1)(\lambda-2) = b(-1)^3 + (\lambda-1)^3 = (\lambda-1)^3 - b$$

$$\Rightarrow b = (\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2) = (\lambda-1)[(\lambda-1)^2 - \lambda(\lambda-2)]$$

$$= (\lambda-1)(\lambda^2 - 2\lambda + 1 - \lambda^2 + 2\lambda) = (\lambda-1) \times 1 \quad \Rightarrow b = \lambda - 1$$

$$\Rightarrow c_m = (\lambda-1)(-1)^m + (\lambda-1)^m$$

$$\Rightarrow \uparrow_{C_m}(\lambda) = (\lambda-1)(-1)^m + (\lambda-1)^m$$

$$\begin{aligned} & \lambda(\lambda-1)(\lambda-1+1) \\ &= \lambda(\lambda-1)^2 - \lambda(\lambda-1) \end{aligned}$$

$$\text{Antes: } \uparrow_{C_5}(\lambda) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2) = \lambda(\lambda-1)^4 - \lambda(\lambda-1)^3 + \lambda(\lambda-1)^2 - \lambda(\lambda-1)$$

$$-(\lambda-1) + (\lambda-1)^5$$

$$\lambda(\lambda-1) \left( (\lambda-1)^3 - (\lambda-1)^2 + (\lambda-1)^1 - 1 \right)$$

$$= -\lambda(\lambda-1) \left( (-\lambda+1)^3 + (-\lambda+1)^2 + (-\lambda+1)^1 + (-\lambda+1)^0 \right)$$

$$= -\lambda(\lambda-1) \frac{1 - (-\lambda+1)^4}{1 - (-\lambda+1)}$$

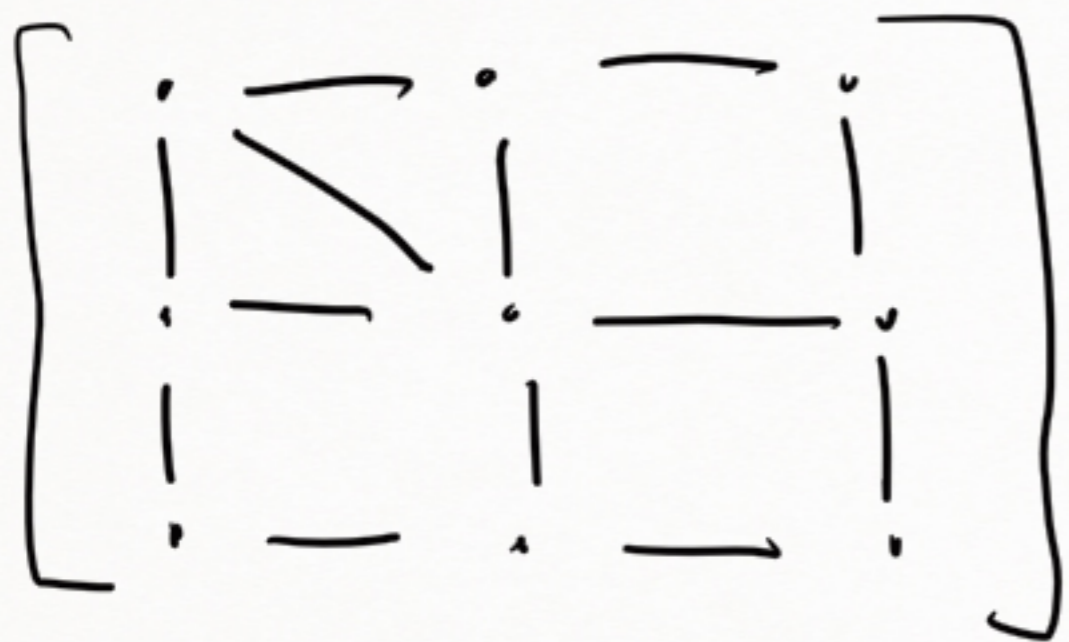
$$= -\lambda(\lambda-1) \frac{1 - (\lambda-1)^4}{1} = -(\lambda-1)(1 - (\lambda-1)^4)$$
$$= -(\lambda-1) + (\lambda-1)^5$$

$$1 + x + \dots + x^m = \frac{1 - x^{m+1}}{1 - x}$$

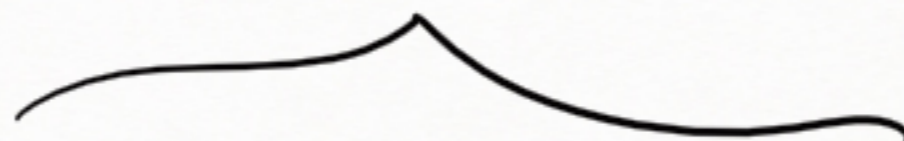
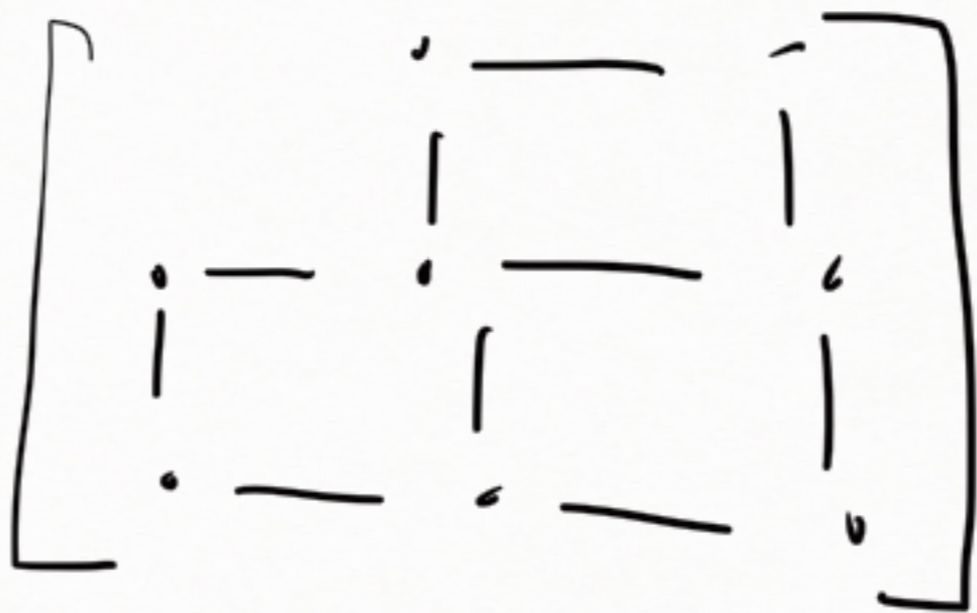
$$\uparrow_{C_n}(\lambda) = (\lambda-1)(-1)^m + (\lambda-1)^m$$

$$\uparrow_{C_n}(2) = 1 \times (-1)^m + 1^m = (-1)^m + 1 = \begin{cases} 2 & m \text{ par} \\ 0 & m \text{ impar} \end{cases}$$

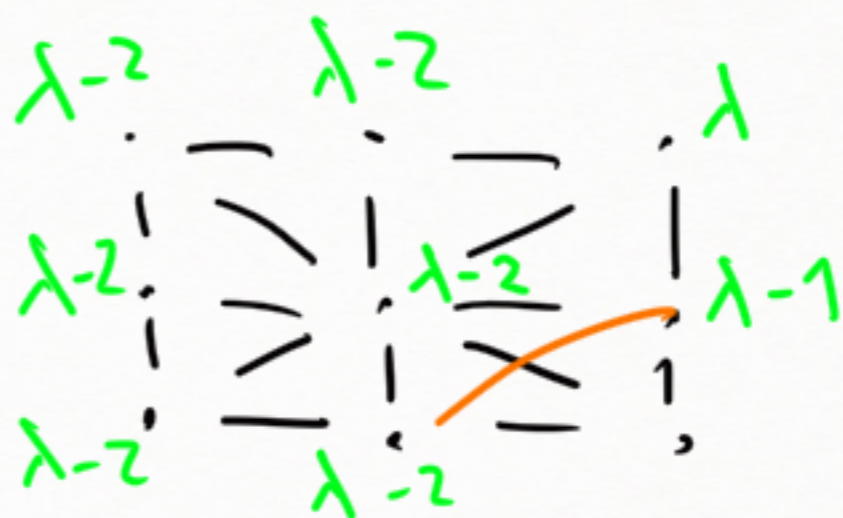
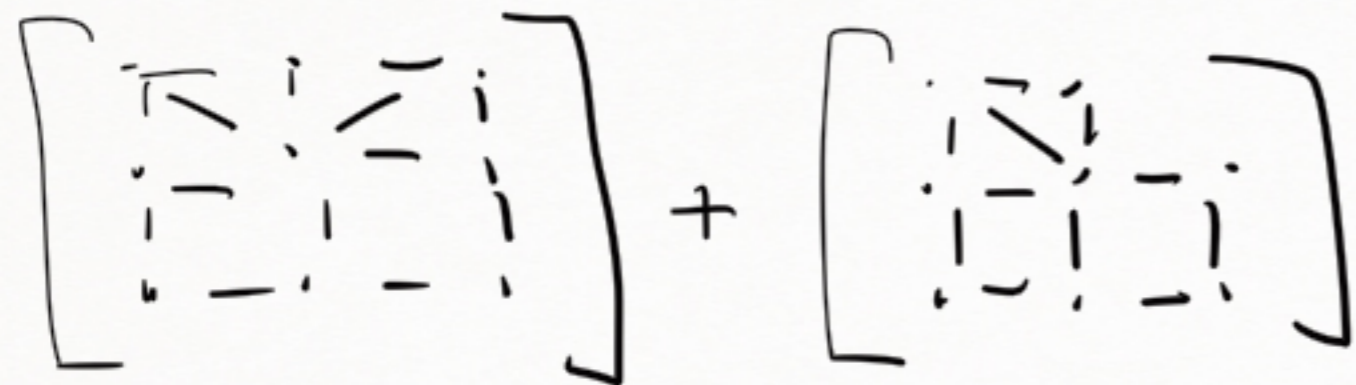
$$\lambda-1=1$$

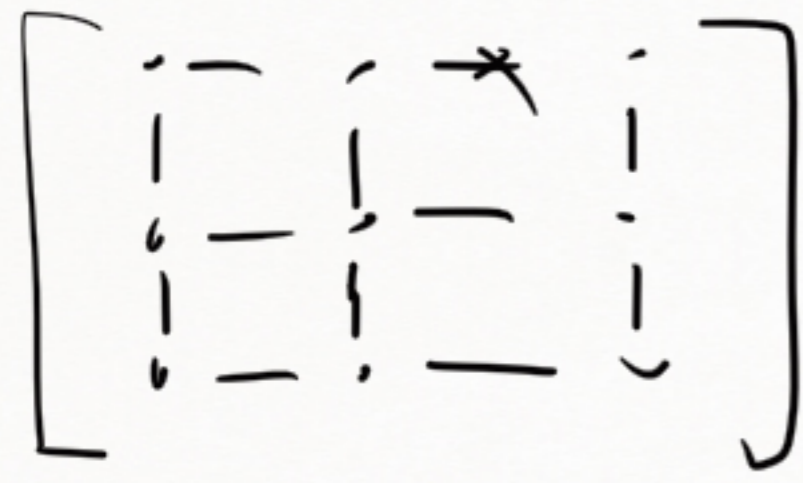


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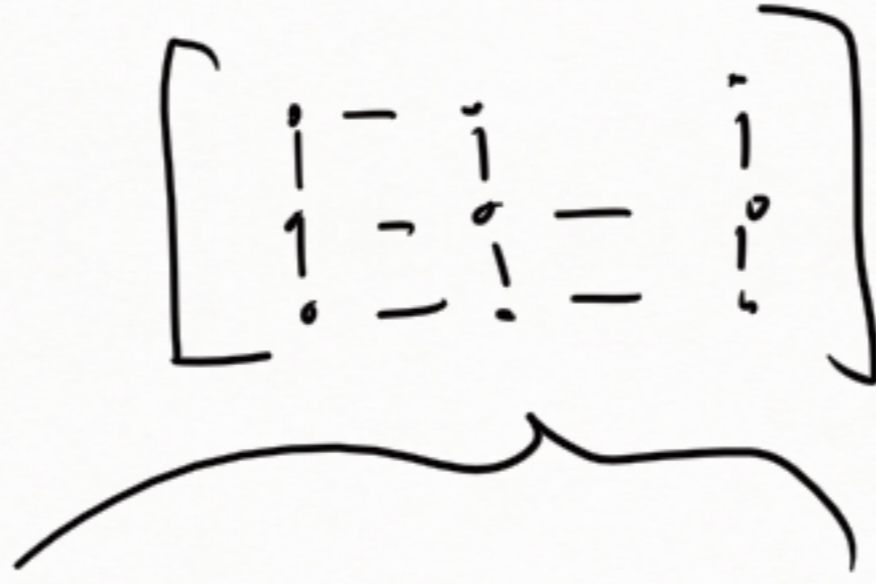


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