Variable aleatoria discreta

$$X : \Omega \longrightarrow \mathbb{R}$$

X toma valores en un conjunto discreto

- fpp

$$P_X : \mathbb{R} \to [0,1]$$

$$P_X(k) = P(X = k)$$

- fda

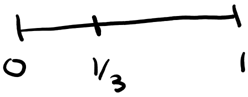
$$F_X : \mathbb{R} \to [0,1]$$

$$F_X(k) = P(X \leq k) = \sum_{x \leq k} P_X(x)$$

Variable aleatoria (absolutamente) continua

$$X : \Omega \longrightarrow \mathbb{R}$$

X toma valores en un conjunto no discreto (intervalo)



$$P(X = 1/3) = 0$$

$$P(X = x) = 0$$

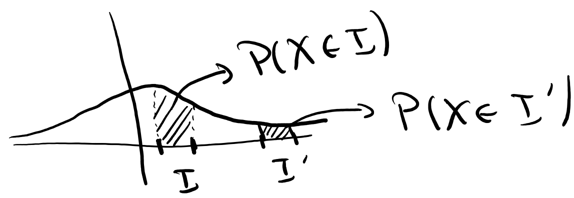
- función de densidad de probabilidad

$$f : \mathbb{R} \to \mathbb{R}$$

tal que:
- integrable
- positiva : $f(x) \geq 0 \ \forall x$
- integra 1 : $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

- X es absolutamente continua si existe una función densidad f_X tal que

$$P(X \in I) = \int_I f_X(x)\,dx$$



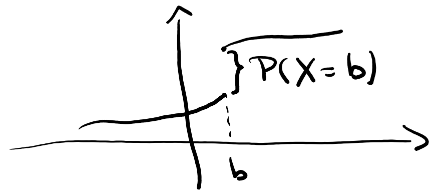
- función de distribución acumulada

$$F_X : \mathbb{R} \to [0,1]$$

$$F_X(y) = P(X \leq y) = \int_{-\infty}^{y} f_X(x)\,dx$$

$\rightarrow F_X' = f_X$

$\rightarrow F_X$ es continua



$\rightarrow P(a \leq X \leq b) = P(a < X \leq b)$

$= P(a \leq X < b)$

$= P(a < X < b)$

$= F_X(b) - F_X(a)$

Ejercicio 2
Se considera la variable aleatoria X absolutamente continua con densidad:

$$f_X(x) = \begin{cases} 0 & \text{si } x < 0 \\ bx & \text{si } x \in (0,1] \\ ae^{-x} & \text{si } x > 1 \end{cases}$$

Hallar a y b sabiendo que $\mathbf{P}(X \in [0,2]) = 2\mathbf{P}(X \in [2,4])$.

- $\underbrace{\int_{-\infty}^{+\infty} f_X(x)}_{\parallel} = 1$

$$\cancel{\int_{-\infty}^{0} f_X(x)\,dx} + \int_{0}^{1} f_X(x)\,dx + \int_{1}^{+\infty} f_X(x)\,dx$$

$$1 = \int_{0}^{1} f_X(x)\,dx + \int_{1}^{+\infty} f_X(x)\,dx$$

$$= \int_{0}^{1} bx\,dx + \int_{1}^{+\infty} ae^{-x}\,dx$$

$$= b\frac{x^2}{2}\Big|_0^1 \overbrace{- ae^{-x}\Big|_1^{+\infty}} = -ae^{-\infty} - (-ae^{-1})$$

$$= \frac{bx}{2}\Big|_0 - ae \Big|_1$$

$$= \frac{b}{2} + ae^{-1} \Rightarrow \boxed{1 = \frac{b}{2} + \frac{a}{e}}$$

- $P(X \in [0,2]) = 2P(X \in [2,4])$

$$P(X \in [0,2]) = \int_0^2 f_X(x)\,dx = \int_0^1 f_X(x)\,dx + \int_1^2 f_X(x)\,dx$$

$$= \int_0^1 bx\,dx + \int_1^2 ae^{-x}\,dx$$

$$= b\frac{x^2}{2}\Big|_0^1 - ae^{-x}\Big|_1^2$$

$$= \frac{b}{2} - ae^{-2} + ae^{-1}$$

$$= \frac{b}{2} - \frac{a}{e^2} + \frac{a}{e}$$

$$P(X \in [2,4]) = \int_2^4 f_X(x)\,dx$$

$$= \int_2^4 ae^{-x}\,dx = -ae^{-x}\Big|_2^4 = -ae^{-4} + ae^{-2} = \frac{a}{e^2} - \frac{a}{e^4}$$

$$P(X \in [0,2]) = 2P(X \in [2,4])$$

$$\boxed{\frac{b}{2} - \frac{a}{e^2} + \frac{a}{e} = 2\left(\frac{a}{e^2} - \frac{a}{e^4}\right)} \quad (1)$$

$$\boxed{1 = \frac{b}{2} + \frac{a}{e}} \quad (2)$$

$$(1): \overbrace{\frac{b}{2} + \frac{a}{e}}^{=1} - \frac{a}{e^2} = \frac{2a}{e^2} - \frac{2a}{e^4}$$

$$1 - \frac{a}{e^2} = \frac{2a}{e^2} - \frac{2a}{e^4}$$

$$1 = \frac{3a}{e^2} - \frac{2a}{e^4} = a\left(\frac{3}{e^2} - \frac{2}{e^4}\right) = a\left(\frac{3e^2 - 2}{e^4}\right)$$

$$\Rightarrow \boxed{a = \frac{e^4}{3e^2 - 2}}$$

Ejercicio 3
Se consideran las siguientes funciones reales:

$$f_1(x) = \begin{cases} c_1\sqrt{x} & \text{si } x \in (0,1) \\ 0 & \text{si } x \notin (0,1) \end{cases}$$

• buscamos c_1 para que f_1 sea una función densidad

$$1 = \int_{-\infty}^{+\infty} f_1(x) = \cancel{\int_{-\infty}^{0} f_1(x)\,dx} + \int_{0}^{1} f_1(x)\,dx + \cancel{\int_{1}^{+\infty} f_1(x)\,dx}$$

$$= \int_0^1 c_1 x^{1/2}\,dx$$

$$= c_1 \left.\frac{x^{3/2}}{3/2}\right|_0^1$$

$$= c_1 \frac{1}{3/2}$$

$$\Rightarrow \boxed{c_1 = 3/2}$$

$$f_1(x) = \begin{cases} \frac{3}{2} x^{1/2} & \text{si } x \in (0,1) \\ 0 & \text{si no} \end{cases}$$

• buscamos $F_X(x)$

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_1(t)\,dt$$

• si $x \leq 0$: $F_X(x) = \int_{-\infty}^{x} f_1(t)\,dt = 0$

• si $x \in (0,1)$: $F_X(x) = \int_{-\infty}^{x} f_1(t)\,dt = \cancel{\int_{-\infty}^{0} f_1(t)\,dt} + \int_{0}^{x} f_1(t)\,dt$

$$= \int_0^x \frac{3}{2} t^{1/2}\,dt$$

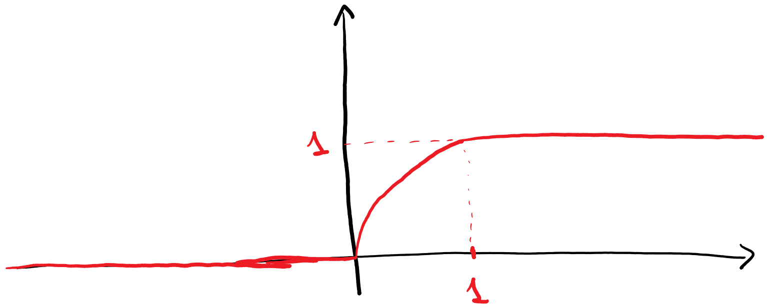
$$= \frac{3}{2} \left. \frac{t^{3/2}}{3/2} \right|_0^x = x^{3/2}$$

• si $x \geq 1$: $F_X(x) = \int_{-\infty}^{x} f_1(x)\,dx$

$$= \underbrace{\int_{-\infty}^{0} f_1(x)\,dx}_{=0} + \underbrace{\int_{0}^{1} f_1(x)\,dx}_{} + \underbrace{\int_{1}^{x} f_1(x)\,dx}_{=0}$$

$$= \int_0^1 \frac{3}{2} t^{1/2}\,dt = \frac{3}{2} \left. \frac{t^{3/2}}{3/2} \right|_0^1 = 1$$

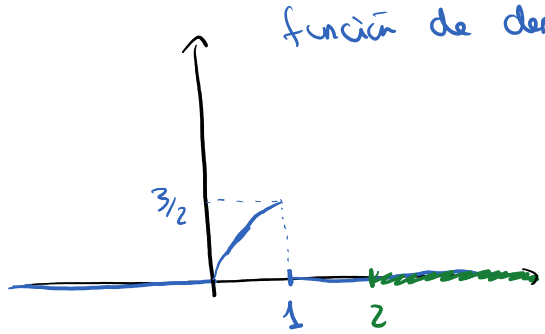
$$F_X(x) = \begin{cases} 0 & \text{si } x \leq 0 \\ x^{3/2} & \text{si } x \in (0,1) \\ 1 & \text{si } x \geq 1 \end{cases}$$



a) Calcular $\mathbf{P}(0{,}3 < X < 0{,}6)$, $\mathbf{P}(X > 2)$ y $\mathbf{P}\left(\frac{1}{2} < X < \frac{3}{2}\right)$.

$$\begin{aligned} P(0{,}3 < X < 0{,}6) &= P(X < 0{,}6) - P(X \leq 0{,}3) \\ &= P(X \leq 0{,}6) - P(X \leq 0{,}3) \\ &= F_X(0{,}6) - F_X(0{,}3) \\ &= (0{,}6)^{3/2} - (0{,}3)^{3/2} \end{aligned}$$

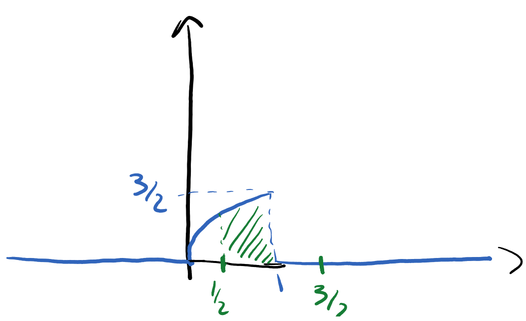
función de densidad



$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - F_X(2) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

$$P(X > 2) = \int_2^{+\infty} \underbrace{f_1(x)}_{=0} \, dx = 0$$

$$\begin{aligned} P\left(\tfrac{1}{2} < X < \tfrac{3}{2}\right) &= F_X\left(\tfrac{3}{2}\right) - F_X\left(\tfrac{1}{2}\right) \\ &= 1 - \left(\tfrac{1}{2}\right)^{3/2} \end{aligned}$$



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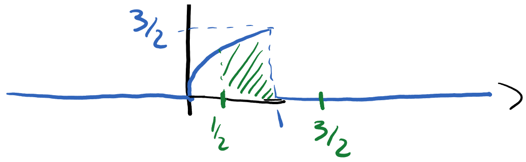
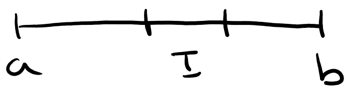


Ejercicio 4
En pruebas de medición de distancia de frenado de automóviles, los vehículos que viajan a determinada velocidad tienden a recorrer distancias de frenado que están distribuidas uniformemente entre dos puntos a y b. Calcular la probabilidad de que uno de estos automóviles:

1. se detenga más cerca de a que de b.
2. se detenga de tal modo que la distancia a a sea por lo menos 3 veces mayor que la distancia a b.

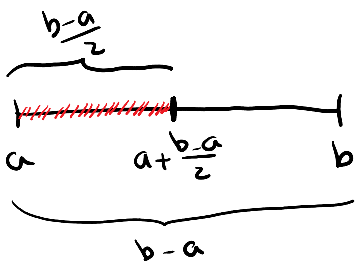
X = punto donde frena el auto



$$P(X \in I) = \frac{\text{long}(I)}{\text{long}([a,b])}$$

① P(el auto se detiene más cerca de a que de b) $= \dfrac{\frac{b-a}{2}}{b-a}$

$= \dfrac{1}{2}$



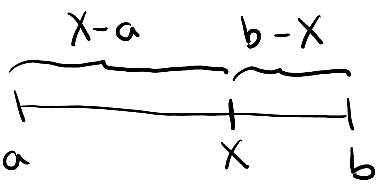
② P(la distancia a a es por lo menos 3 veces mayor que la distancia a b) $= P(X-a \geq 3(b-X))$

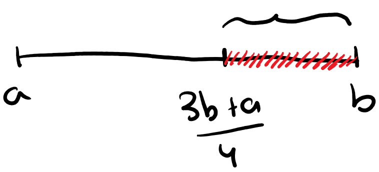
$= P(X-a \geq 3b-3X)$

$= P(4X \geq 3b+a)$

$= P\left(X \geq \dfrac{3b+a}{4}\right)$

$= \dfrac{b-\frac{3b+a}{4}}{b-a}$





$$= \frac{\frac{4b-3b-a}{4}}{b-a}$$

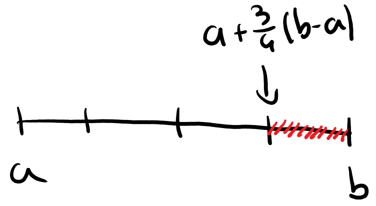
$$= \frac{\frac{b-a}{4}}{b-a}$$

$$= \frac{1}{4}$$

$$X \geq \frac{3b+a}{4}$$

$$X \geq \frac{3b+a-4a}{4} + a$$

$$X \geq a + \frac{3b-3a}{4}$$



$$X \geq a + \frac{3}{4}(b-a)$$