Curso

SISTEMAS Y CONTROL

Clase 33

Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
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ESTABILIDAD (Sist. en t. discreto)

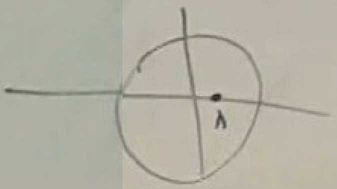
$$\begin{cases} x_{k+1} = \phi x_k + \Gamma u_k \\ y_k = C x_k + D u_k \end{cases} \qquad \left.\begin{matrix} u_k \in \mathbb{R}^r \\ x_k \in \mathbb{R}^n \\ y_k \in \mathbb{R}^m \end{matrix}\right\} \forall k \geq k_0$$

$\left.\begin{matrix} u_k \\ x_0 \end{matrix}\right\}$ conocidos

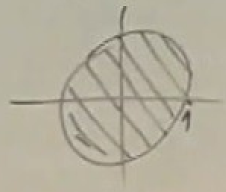
$$\frac{z}{z-\lambda} \rightarrow \lambda^k$$

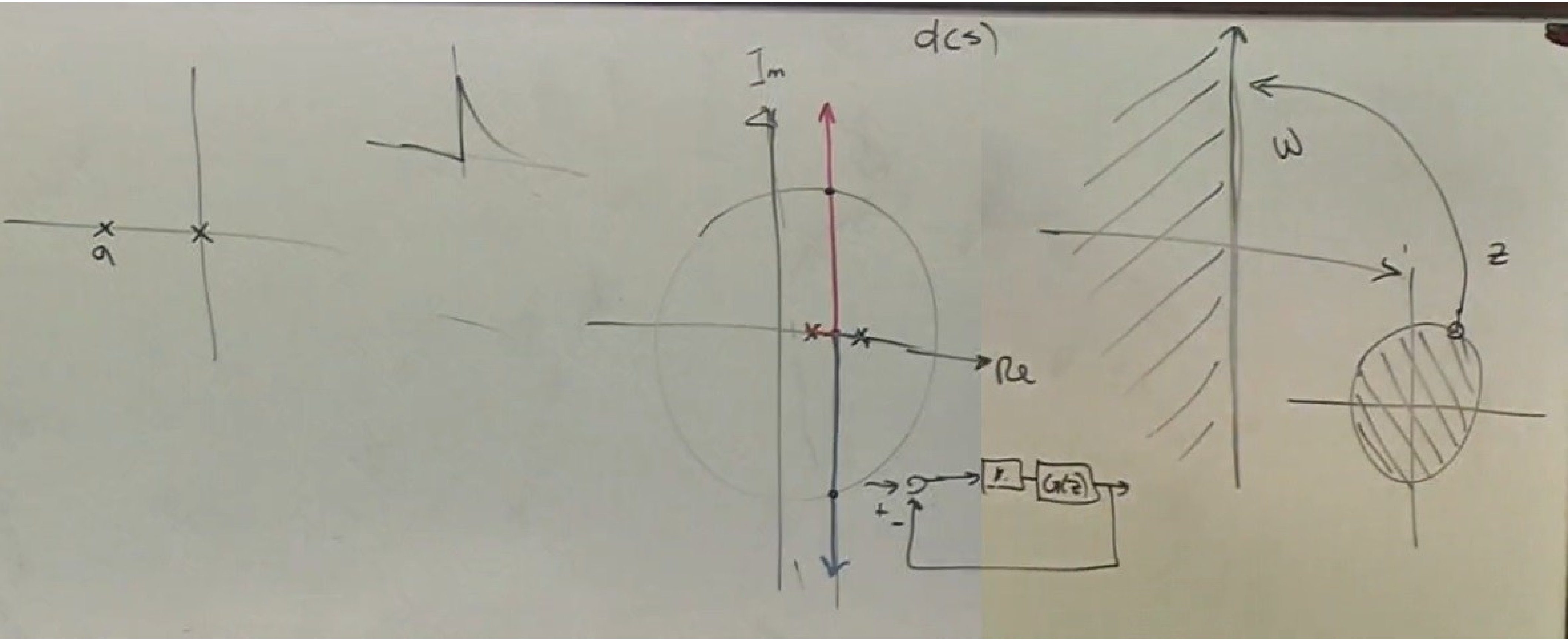
BIBO estable $\Longleftrightarrow$ todos los λ_i $(i = 1 \ldots n)$ / $|\lambda_i| < 1$

(λ_i : autovalores de ϕ)



$\det(zI - \phi) = 0$




a
Im
d(s)
Re
+
-
G(z)
ω
z

CRITERIOS de ESTABILIDAD — Criterio de Jury – Schur-Kohn

$$A(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots$$

	z^n	z^{n-1}	z^{n-2}	$\dots$	z	z^0
n $A(z)$	a_0	a_1	a_2	$\dots$	a_{n-1}	a_n
$z^n A(z^{-1})$	a_n	a_{n-1}	a_{n-2}	$\dots$	a_1	a_0
$n-1$	a_0^{n-1}	a_1^{n-1}	a_2^{n-1}	$\dots$	a_{n-1}^{n-1}	
	a_{n-1}^{n-1}	a_{n-2}^{n-1}	a_{n-3}^{n-1}	$\dots$	a_0^{n-1}	
$n-2$						

$$\alpha_n = \frac{a_n}{a_0}$$

$$a_i^{k-1} = a_i^k - \alpha_k a_{k-i}^k$$

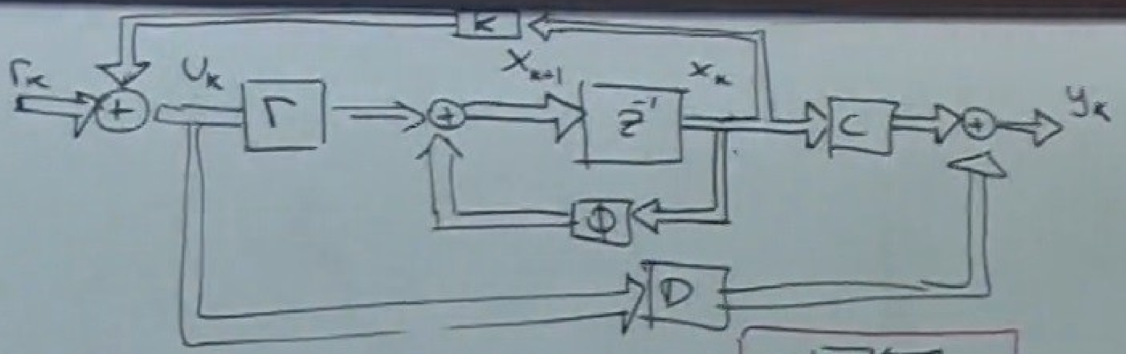
$$A(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n$$

$+\Gamma u_k$

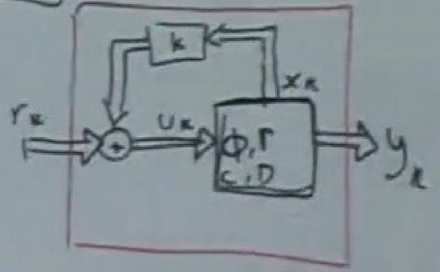
$+ D u_k$

REALIMENTACIÓN de ESTADOS

$$\left.\begin{array}{l} u_k \in \mathbb{R}^r \\ y_k \in \mathbb{R}^m \\ x_k \in \mathbb{R}^n \end{array}\right\} \forall k \geq k_0$$

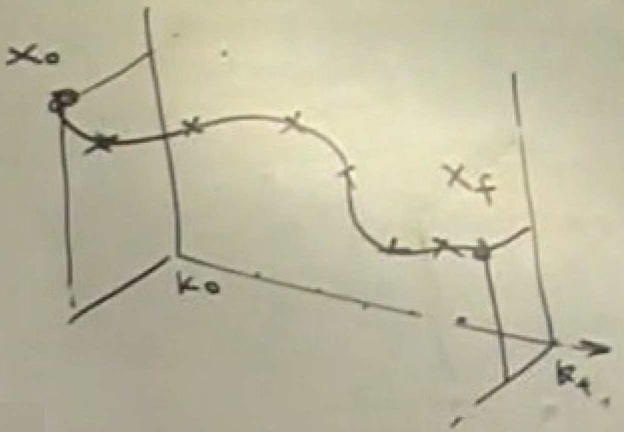


$$\underbrace{\left[\Phi + \Gamma K\right]}_{\Phi'}$$



$$\Rightarrow \quad \mathcal{C} = \left[\Gamma \;\vdots\; \Phi\Gamma \;\vdots\; \Phi^2\Gamma \;\vdots\; \cdots \;\vdots\; \Phi^{n-1}\Gamma\right]$$

es de rango completo



Controlabilidad.

$$\left[x_k - \underbrace{\Phi^k x_0} \right] = \sum_{i=0}^{\overset{n}{k-1}} \Phi^{k-i-1} \Gamma \underbrace{u_i}$$

u_0
u_1
u_2
$\vdots$
u_{k-1}

$$\mathcal{C} = \left[\Gamma \,\vdots\, \Phi\Gamma \,\vdots\, \Phi^2\Gamma \,\vdots\, \cdots \,\vdots\, \Phi^{\overset{n}{k-1}}\Gamma \right]$$