Curso

# SISTEMAS Y CONTROL

# Clase 32

## Fotogramas de los pizarrones de clases filmadas

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$$\begin{cases} x_{k+1} = \phi x_k + \Gamma u_k & u_k \in \mathbb{R}^r \\ y_k = C x_k + D u_k & y_k \in \mathbb{R}^n \\ & x_k \in \mathbb{R}^n \end{cases} \quad \forall k \geq k_0$$

$$\left.\begin{array}{l} x_{k_0} = x_0 \\ u_k,\ k \geq k_0 \end{array}\right\} \text{conocidas}$$

si $x_0 = \underline{0}$ $\quad Y(z) = H(z)\,U(z)$ $\quad \boxed{H(z) = C(zI-\phi)^{-1}\Gamma + D}$

polos del sistema

$z \,/\, \det(zI-\phi) = 0$

$$x_0 = x_0$$

$$x_1 = \phi x_0 + \Gamma u_0$$

$$x_2 = \phi^2 x_0 + \phi\Gamma u_0 + \Gamma u_1$$

$$x_3 = \phi^3 x_0 + \phi^2\Gamma u_0 + \phi\Gamma u_1 + \Gamma u_2$$

$$\vdots$$

$$x_k = \underbrace{\phi^k x_0}_{x_k^h} + \underbrace{\phi^{k-1}\Gamma u_0 + \phi^{k-2}\Gamma u_1 + \phi^{k-3}\Gamma u_2 + \dots + \Gamma u_{k-1}}_{x_k^o} = \underbrace{\phi^k x_0} + \underbrace{\sum_{i=0}^{k-1}\phi^{k-i-1}\Gamma u_i}$$

$$y_k = \underbrace{C\phi^k x_0}_{y_k^h} + \underbrace{C\sum_{i=0}^{k-1}\phi^{k-i-1}\Gamma u_i + D u_k}_{y_k^o}$$

$$x_k = \sum_{n=0}^{\infty} f_n g_{k-n} \qquad \{x_k\} = \{f_k\} \circledast \{g_k\}$$

$$Z[\{x_k\}] = \sum_{k=0}^{\infty} x_k z^{-k} = \sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} f_n g_{k-n} \right) z^{-k} = \sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} f_n g_{k-n} z^{-(k-n)} z^{-n} \right).$$

$$\boxed{Z[\{x_k\}] = Z[\{f_k\}] \cdot Z[\{g_k\}]}$$

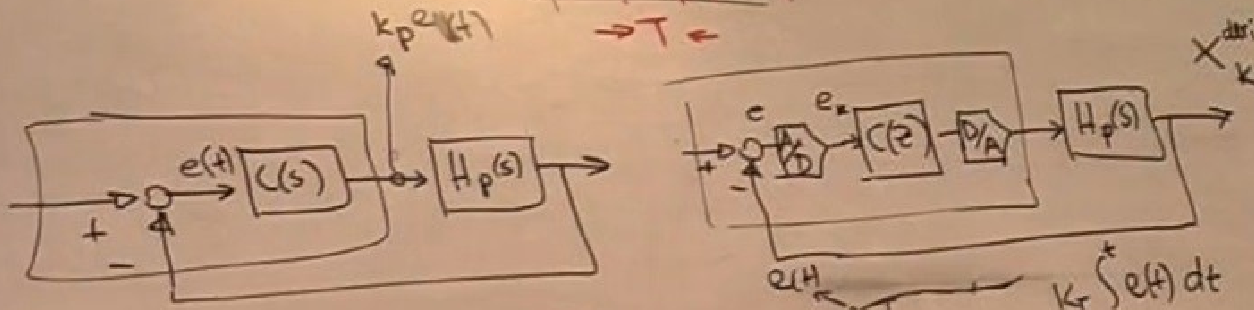
$$\sum_{n=0}^{\infty} \left( \sum_{k=0}^{\infty} f_n g_{k-n} z^{-(k-n)} z^{-n} \right)$$

$$\sum_{n=0}^{\infty} f_n z^{-n} \cdot \sum_{k=0}^{\infty} g_{k-n} z^{-(k-n)}$$

$$\downarrow k-n=j$$

$$\sum_{n=0}^{\infty} f_n z^{-n} \cdot \sum_{j=-n}^{\infty} g_j z^{-j} = \underbrace{\sum_{n=0}^{\infty} f_n z^{-n}}_{Z[\{f_k\}]} \cdot \underbrace{\sum_{j=0}^{\infty} g_j z^{-j}}_{Z[\{g_k\}]}$$

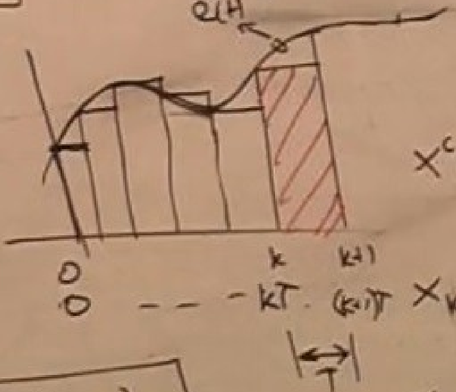
$$X^{deri}_{k+1} = \frac{e_{k+1}-e_k}{T}$$




$C(s) = k_p$

$C(z) = k_p$

$C(s) = \frac{k_I}{s}$




$$k_I \int_{t_0}^{t} e(t)\,dt$$

$$x^c(t) = k_I \int_{t_0}^{t} e(t)\,dt$$

$$x_k = k_I \sum_{i=0}^{k-1} e_i T$$

$$x_{k+1} = k_I \sum_{i=0}^{k} e_i T = \underbrace{k_I \sum_{i=0}^{k-1} e_i T}_{x_k} + k_I e_k T$$

$$\boxed{x_{k+1} = x_k + (k_I T) e_k}$$

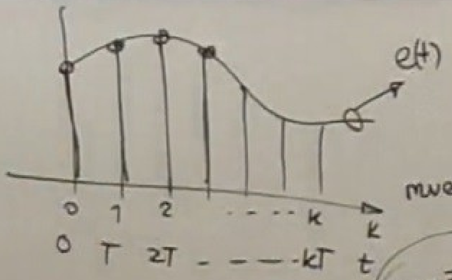
RELACIÓN entre SISTEMAS de TIEMPO CONTINUO y SIST. de TIEMPO DISCRETO.

1) Aproximación en t. disc. de sistemas en t. continuo.

2) Transf. de señales de t. discreto $\rightleftarrows$ t. continuo

3) Muestreo (discretización) de sistemas de t. continuo.

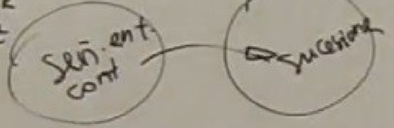
$$\bar{e}_k = e(kT)$$




$e: \mathbb{R} \to \mathbb{R}$

$e: \mathbb{R} \to \mathbb{R}^r$

muestreo




$\bar{e}: \mathbb{Z} \to \mathbb{R}$

$\bar{e}: \mathbb{Z} \to \mathbb{R}^r$

RELACIÓN entre SISTEMAS de TIEMPO CONTINUO y SÍST. de TIEMPO DISCRETO.

1) Aproximación en t. disc. de sistemas en t. continuo.

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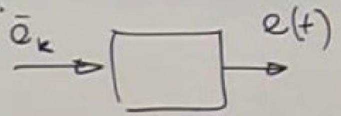
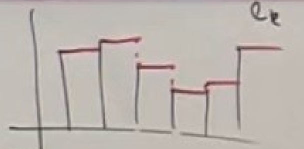
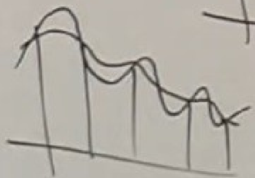
3) Muestreo (discretización) de sistemas de t. continuo.



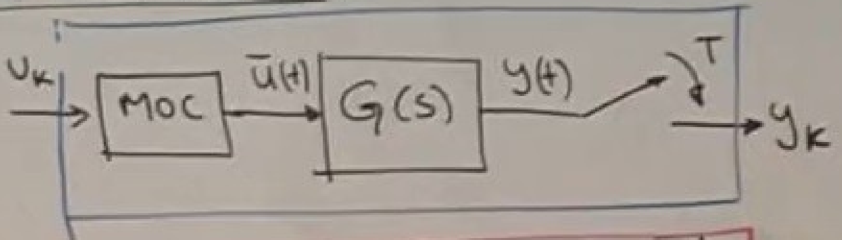

$$e(t) = \bar{e}_k \quad \text{para } kT \leq t < (k+1)T$$



## RELACIÓN entre SISTEMAS de TIEMPO CONTINUO y SIST. de TIEMPO DISCRETO.

### TEOREMA de la TRANSMITANCIA MUESTREADA




**Hip:**

- $G(s)$ sist. din. causal, lineal, inv. en t
- Se construye un sist. en t. discreto como la fig.

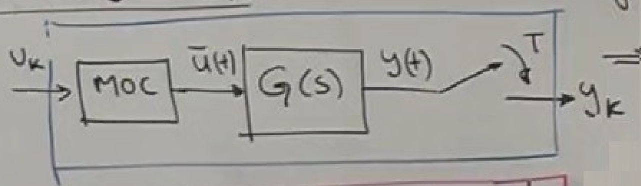
**T.**

- $\{y_k\} = S[\{u_k\}]$
- $S$ es din., causal, lineal e inv. en t (discreto)
- Su f. de transf. $G(z)$:

$$\boxed{G(z) = \frac{z-1}{z}\, \mathcal{Z}\left[ \mathcal{L}\left\{\frac{G(s)}{s}\right\}\bigg|_{t=kT} \right]}$$

RELACIÓN entre SISTEMAS de TIEMPO CONTINUO y SIST. de TIEMPO DISCRETO.

## TEOREMA de la TRANSMITANCIA MUESTREADA




Hip • $G(s)$ sist. din. causal, lineal, inv. en t

• Se construye un sist. en t. discreto como la fig.

T. • $\{y_k\} = S[\{u_k\}]$

• $S$ es din., causal, lineal e inv. en t (discreto)

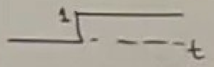
• Su f. de transf. $G(z)$:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{z-1}{z}\, \mathcal{Z}\left[ \mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]\Big|_{t=kT} \right]$$

$$\boxed{G(z) = \frac{z-1}{z}\, \mathcal{Z}\left[ \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\}\Big|_{t=kT} \right]}$$

elijo $u_k: 1 \ \forall k \geq 0$

$\Longrightarrow \bar{u}(t) = Y_t$



$$Y(s) = \frac{G(s)}{s}$$

$$\rightarrow y(t) = \underbrace{\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]}$$

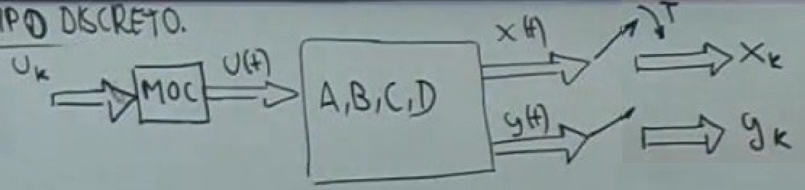
$$y_k = \mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]\Big|_{t=kT}$$

$$\downarrow$$

$$Y(z) = \mathcal{Z}\left[\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]_{t=kT}\right]$$

## RELACIÓN entre SISTEMAS de TIEMPO CONTÍNUO y SIST. de TIEMPO DISCRETO.

### MUESTREO de MODELOS en Variables de Estado.



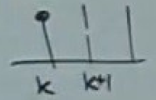

$$\begin{cases} x_{k+1} = \phi x_k + \Gamma U_k \\ y_k = C x_k + D u_k \end{cases}$$

$$\begin{cases} \dot{x}(t) = A\, x(t) + B u(t) \\ y(t) = C\, x(t) + D u(t) \end{cases}$$

con $x(t) \in \mathbb{R}^n$
$u(t) \in \mathbb{R}^r$
$y(t) \in \mathbb{R}^m$

$\left.\begin{array}{l} x(t_0) = x_0 \\ u(t) \end{array}\right\}$ conocidas

$t_0 = kT$
$t = (k+1)T$




$$x(t) = e^{A(t-t_0)} x_0 + \int_{t_0}^{t} e^{A(t-\sigma)} B\, u(\sigma)\, d\sigma$$

$$z = (k+1)T - \sigma$$

$$x_{k+1} = e^{AT} x_k + \left[ \int_{kT}^{(k+1)T} e^{A[(k+1)T-\sigma]} B\, d\sigma \right] U_k = \underbrace{e^{AT}}_{\phi} x_k + \underbrace{\boxed{\int_0^T e^{Az}\, dz\, B}}_{\Gamma} U_k$$