Curso

SISTEMAS Y CONTROL

Clase 31

Fotogramas de los pizarrones de clases filmadas

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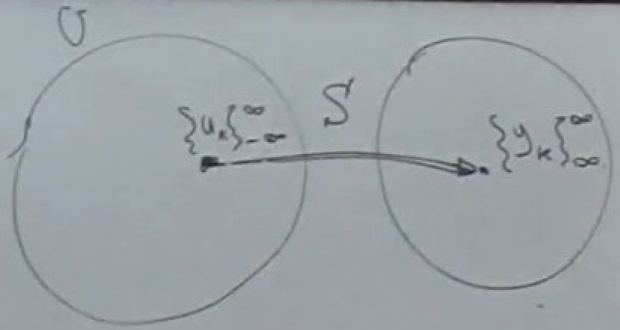
TEMA en TIEMPO DISCRETO



1 $\boxed{y_k = \sum_{i=0}^{n} \alpha_i u_{k-i} - \sum_{j=1}^{n} \beta_j y_{k-j}}$

$$Y(z) = \frac{\alpha_n + \alpha_{n-1} z^{-1} + \dots + \alpha_0 z^{-n}}{1 + \beta_{n-1} z^{-1} + \dots + \beta_0 z^{-n}} U(z)$$

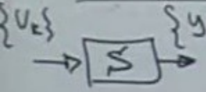
$$Y(z) = \frac{\alpha_n z^{n} + \alpha_{n-1} z^{n-1} + \dots + \alpha_0}{z^{n} + \beta_{n-1} z^{n-1} + \dots + \beta_0} U(z)$$

$$\boxed{H(z) = \frac{\alpha_n z^{n} + \alpha_{n-1} z^{n-1} + \dots + \alpha_0}{z^{n} + \beta_{n-1} z^{n-1} + \dots + \beta_0}}$$

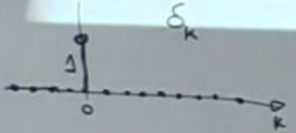
$$H(z) = \left[z^{n} + \beta_{n-1} z^{n-1} + \dots + \beta_0\right]^{-1} \left[\alpha_n z^{n} + \dots + \alpha_0\right]$$

2 $\Rightarrow \boxed{Y(z) = H(z)\,U(z)}$

SISTEMA en TIEMPO DISCRETO



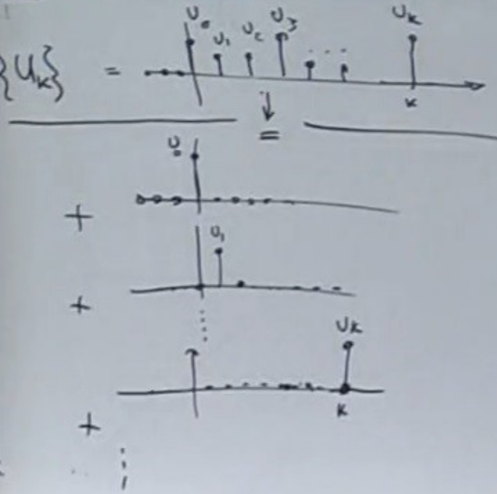
$\begin{cases} u_k \in \mathbb{R} \\ y_k \in \mathbb{R} \end{cases} \quad \forall k \geq k_0 \quad k \in \mathbb{Z}$



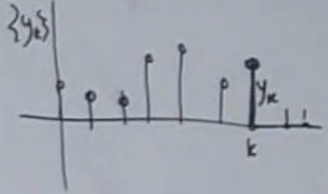
$\delta_k = \begin{cases} 1 & \text{si } k=0 \\ 0 & \text{si } k \neq 0 \end{cases}$

$\{h_k\} = S\left[\{\delta_k\}\right]$ $\begin{cases} \text{respuesta al pulso unitario.} \\ \text{sucesión de "peso" del sistema} \end{cases}$

Hip: S es causal, lineal e inv. en t.



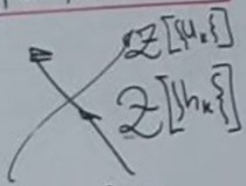
$= \{u_k\} = \sum_{n=0}^{\infty} u_{k-n}\{\delta_{k-n}\} \Rightarrow \boxed{\{y_k\} = \sum_{n=0}^{\infty} u_{k-n} \cdot \{h_{k-n}\}}$



$y_k = u_0 h_k + u_1 h_{k-1} + u_2 h_{k-2} + \ldots + u_n h_{k-n} + \ldots$

$\boxed{y_k = \sum_{n=0}^{\infty} u_n \underbrace{h_{k-n}}} = \sum_{n=0}^{k} u_n h_{k-n}$

$\boxed{Y(z) = H(z) \cdot U(z)}$



$\{y_k\} = \{u_k\} \otimes \{h_k\}$

$\boxed{y_k = \sum_{n=0}^{k} u_n h_{k-n}}$ causal lineal inv. ent.

SISTEMA din. LINEAL, de PARÁMETROS CONCENTRADOS

$u_k \to \boxed{x_k} \to y_k$

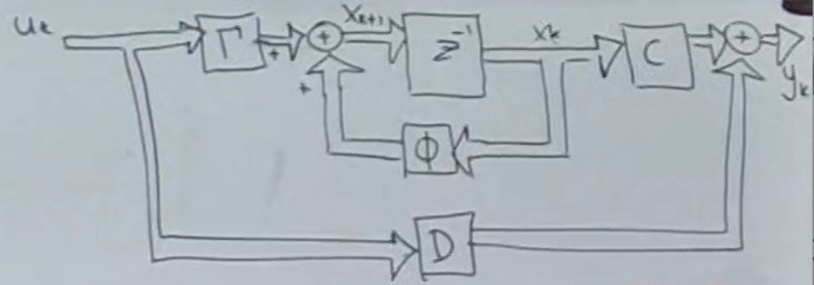
$$\begin{cases} u_k \in \mathbb{R}^r \\ y_k \in \mathbb{R}^m \\ x_k \in \mathbb{R}^n \end{cases} \forall\, k \geq k_0$$

$$\boxed{\begin{aligned} x_{k+1} &= \Phi_k x_k + \Gamma_k u_k \\ y_k &= C_k x_k + D_k u_k \end{aligned}}$$

var. en t

$$\boxed{\begin{aligned} x_{k+1} &= \Phi x_k + \Gamma u_k \\ y_k &= C x_k + D u_k \end{aligned}}$$

S. din lineal par. conc inv. en t



$$\left.\begin{cases} u_k & (k \geq k_0) \\ x_{k_0} = x_0 \end{cases}\right\} \text{conocidos}$$

¿Como calcular $\{y_k\}$?

$k_0 = 0$

① Tr. Z

$$\begin{cases} z\left[X(z) - x_0\right] = \Phi X(z) + \Gamma U(z) \Rightarrow \left[zI - \Phi\right] X(z) = z x_0 + \Gamma U(z) \Rightarrow \boxed{X(z) = z\left[zI - \Phi\right]^{-1} x_0 + \left[zI - \Phi\right]^{-1} \Gamma U(z)} \\ Y(z) = C X(z) + D U(z) \end{cases}$$

$$\boxed{Y_\sigma(z) = H(z) U(z)}$$

con $\boxed{H(z) = C(zI - \Phi)^{-1}\Gamma + D}$

$$\boxed{Y(z) = C z\left[zI - \Phi\right]^{-1} x_0 + \underbrace{\underbrace{\left[C(zI - \Phi)^{-1}\Gamma + D\right]}_{H} U(z)}_{Y_\sigma(z)}}$$