Curso

SISTEMAS Y CONTROL

Clase 23

Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
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Clase 24 –

- Respuesta en frecuencia (1)
  Definición
  Representaciones (Diags. De Bode, Nyquist, Nichols)
  Respuesta de sistemas de orden 1 y 2

Esta clase hace uso de las transparencias “Respuesta en frecuencia”

RESPUESTA EN FRECUENCIA

$\xrightarrow{u} \boxed{S} \xrightarrow{y}$ $\quad u: \mathbb{R} \to \mathbb{R}$, $y: \mathbb{R} \to \mathbb{R}$ $\quad S$: lineal e inv. en t y estable

$$u(t) = \sin(\omega t)\ \forall_t \Rightarrow y(t) \xrightarrow[t \to \infty]{} A \sin(\omega t + \varphi)$$

$G(j\omega)$: Respuesta en frecuencia

Ejemplo: S es din. lin, I.t, par. conc., 1e.-1s $\quad \xrightarrow{u} \boxed{G(s)} \xrightarrow{y}$

$$G(s) = \alpha \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{j=1}^{n}(s - p_j)}$$

hip G es estable.

$$u_1(t) = e^{j\omega t} \to \boxed{U_1(s) = \frac{1}{s - j\omega}} \quad \boxed{Y_1(s) = G(s)\,U_1(s)} \Rightarrow Y_1(s) = \frac{1}{s - j\omega} \cdot \underbrace{\alpha \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{j=1}^{n}(s - p_j)}}_{G(s)}$$

$$u_2(t) = e^{-j\omega t} \to \boxed{U_2(s) = \frac{1}{s + j\omega}} \quad \boxed{Y_2(s) = G(s)\,U_2(s)} \Rightarrow Y_2(s) = \frac{1}{s + j\omega}\, \alpha \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{j=1}^{n}(s - p_j)}$$

$$Y_1(s) = \frac{A_1}{s - j\omega} + \underbrace{\left(\frac{A_1'}{s - p_1}\right) + \left(\frac{A_2'}{s - p_2}\right) + \dots + \dots}_{\xrightarrow[t \to \infty]{} 0}$$

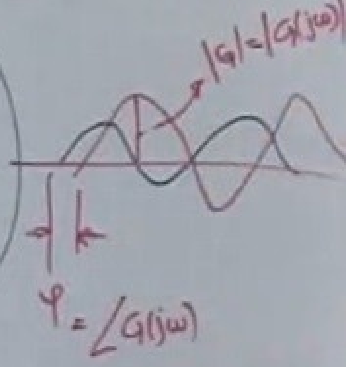
$$y_1(t) = A_1 e^{j\omega t} + \underbrace{\sum\sum t^j e^{\sigma_i t} \sin(\omega_i t + \varphi_{ij})}_{f_1(t)}$$

$$\begin{cases} y_1(t) = G(j\omega)\, e^{j\omega t} + f_1(t) \\ y_2(t) = G(-j\omega)\, e^{-j\omega t} + f_2(t) \end{cases}$$

$$\begin{cases} G(j\omega) = |G(j\omega)| \angle \overbrace{G(j\omega)}^{\varphi} \\ G(-j\omega) = |G(j\omega)| \angle - G(j\omega) \end{cases}$$

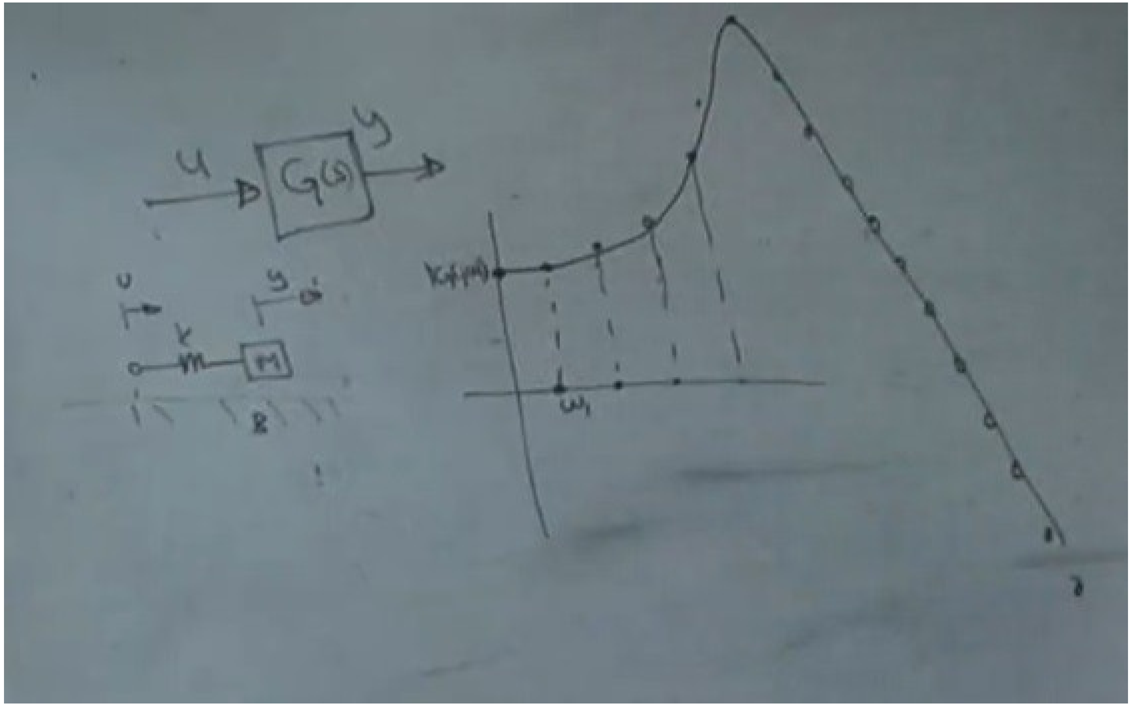
como $G(-j\omega) = \overline{G(j\omega)}$

$$u(t) = \frac{u_1(t) - u_2(t)}{2j} = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} = \operatorname{sen} \omega t \Rightarrow y(t) = \frac{y_1(t) - y_2(t)}{2j} = \frac{|G(j\omega)|\, e^{j(\omega t + \varphi)} - |G(j\omega)|\, e^{-j(\omega t + \varphi)}}{2j} + \overbrace{\frac{f_1(t) - f_2(t)}{2j}}^{= f(t)}$$



$$y(t) = |G(j\omega)| \underbrace{\left(\frac{e^{j(\omega t + \varphi)} - e^{-j(\omega t + \varphi)}}{2j} \right)}_{= \operatorname{sen}(\omega t + \varphi)} + f(t)$$

$$\boxed{y(t) \xrightarrow[t \to \infty]{} |G(j\omega)| \operatorname{sen}\left(\omega t + \angle G(j\omega)\right)}$$



u
G(s)
y
k
M
B
ω_1

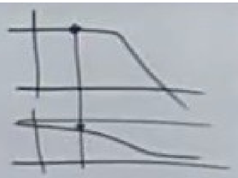
RESPUESTA EN FRECUENCIA

$\xrightarrow{u} \boxed{S} \xrightarrow{y}$ $\quad u:\mathbb{R}\to\mathbb{R}$, $\quad y:\mathbb{R}\to\mathbb{R}$

S: lineal e inv. en t y estable

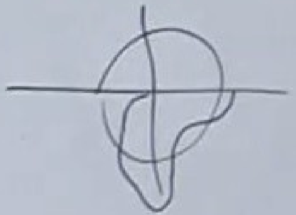
$u(t) = \text{sen}(\omega t)\ \forall t \Rightarrow y(t) \xrightarrow[t\to\infty]{} A\,\text{sen}(\omega t+\varphi)$

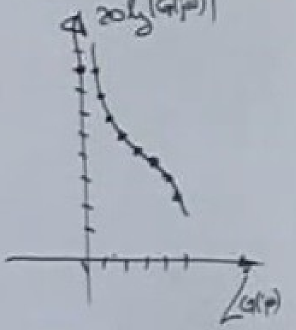
$G(j\omega)$: Respuesta en frecuencia

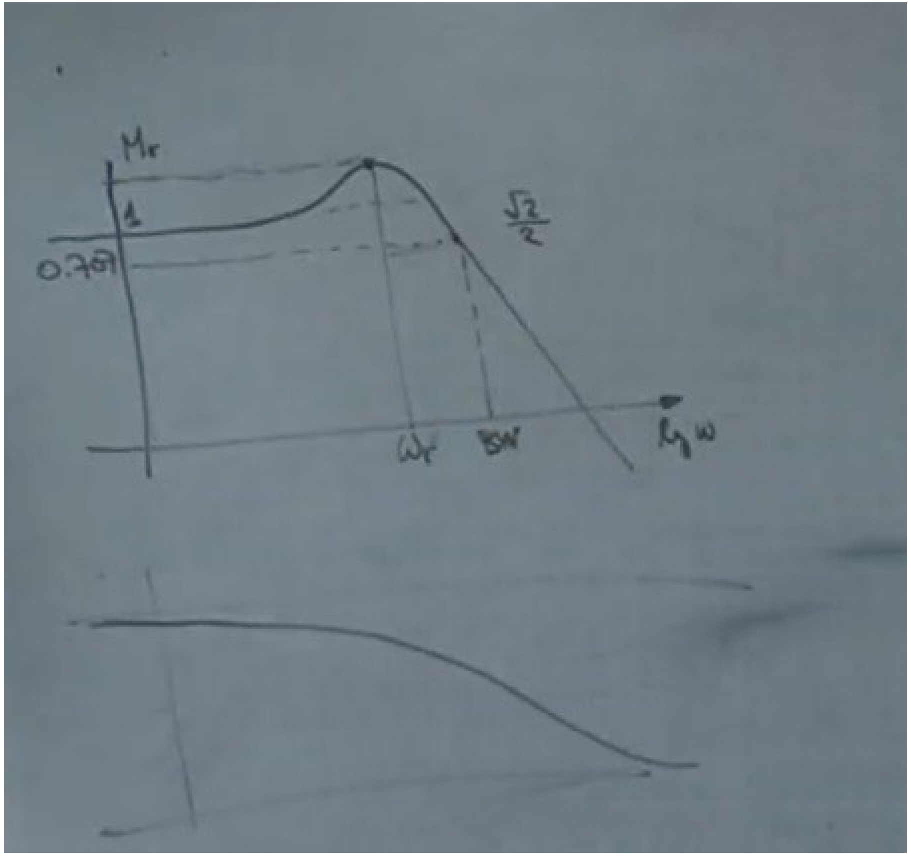


Representaciones:

- Diag. de Bode — $20\lg|G(j\omega)|$ vs $\lg\omega$; $\angle G(j\omega)$ vs. $\lg\omega$
- Diag. de Nyquist — $G(j\omega)$ en el plano complejo
- Diag. de Nichols — $20\lg|G(j\omega)|$ vs $\angle G(j\omega)$



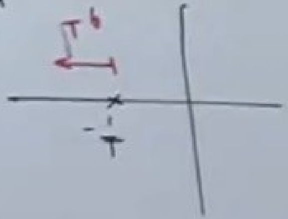


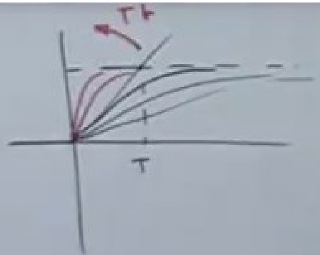


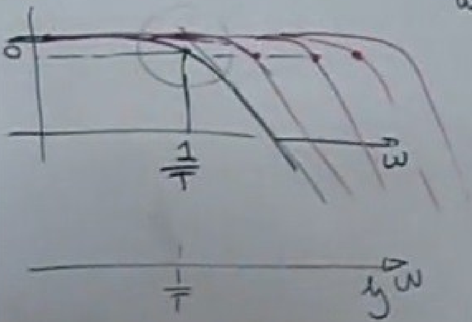
Mr
1
0.707
$\frac{\sqrt{2}}{2}$
ωr
BW
lg ω

Sistema 1^{er} orden

$$G(s) = \frac{1}{Ts+1}$$







$\omega = \frac{1}{T}$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$G = \frac{1}{1}$$

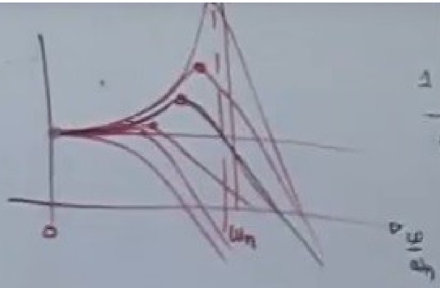
$$G = \frac{1}{j\omega T}$$

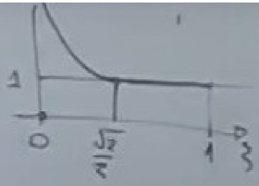
Sistema orden 2

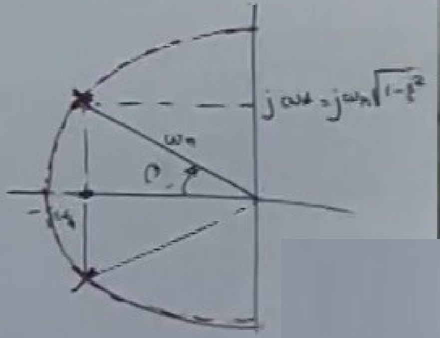
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

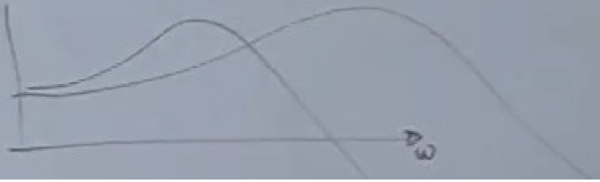
$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2} = \frac{1}{-\left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_n}\right) + 1}$$









Sistema orden 2

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2} = \frac{1}{-\left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_n}\right) + 1}$$

$$\boxed{\omega_r \leq \omega_d \leq \omega_n}$$

$$\boxed{BW = \omega_n\left[(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}\right]^{1/2}}$$