Curso

SISTEMAS Y CONTROL

Clase 23

Fotogramas de los pizarrones de clases filmadas

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Clase 23 –

- Root – Locus (3)
- uso en diseño

Usa las diapositivas “Diseño de un Plotter” y la animación “Lugar geométrico del plotter”

$$\begin{cases} p(s) + k\,q(s) = 0 \\ 1 + k\,H(s) = 0 \end{cases}$$

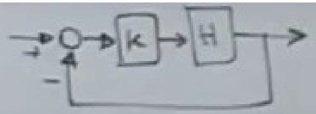
con $H(s) = \dfrac{q(s)}{p(s)} = \alpha \dfrac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$

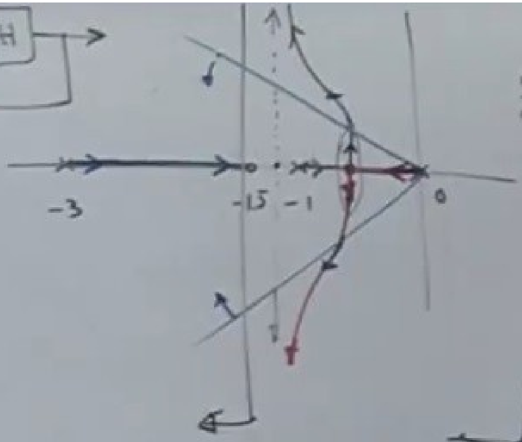
$m \leq n$

$\alpha > 0$

$$H(s) = \frac{(s+1.5)}{s(s+1)(s+3)}$$

$$c = \frac{0-1-3+1.5}{3-1} = \frac{-2.5}{2} = -1.25$$





$$\frac{dH}{ds} = 0$$

$$\begin{cases} p(s) + k\,q(s) = 0 \\ 1 + k\,H(s) = 0 \end{cases}$$

con $H(s) = \dfrac{q(s)}{p(s)} = \alpha \dfrac{\prod_{i=1}^{m}(s - z_i)}{\prod_{j=1}^{n}(s - p_j)}$

$m \leq n$

$\alpha > 0$

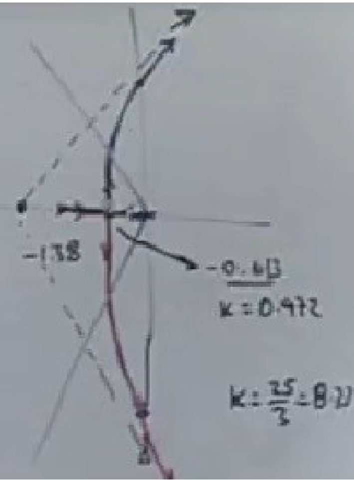
$p_1 = 0$

$$H(s) = \frac{3k}{s(s^2 + 5s + 5)}$$

$$\frac{-5 \pm \sqrt{25 - 20}}{2} = -2.5 \pm \frac{\sqrt{5}}{2} \begin{cases} -1.38 \\ -3.62 \end{cases}$$

$$c = \frac{-5}{3} \simeq -1.67$$

$$\frac{dk}{ds} = 0 \Rightarrow$$



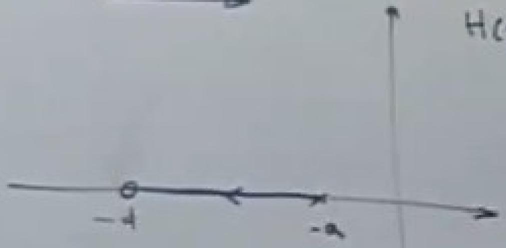
EFECTO sobre el LGP de agregar POLOS y CEROS

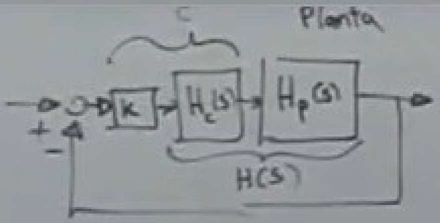
agr. ceros →

agrg polos ↓

$H(s) = \frac{1}{s+a}$

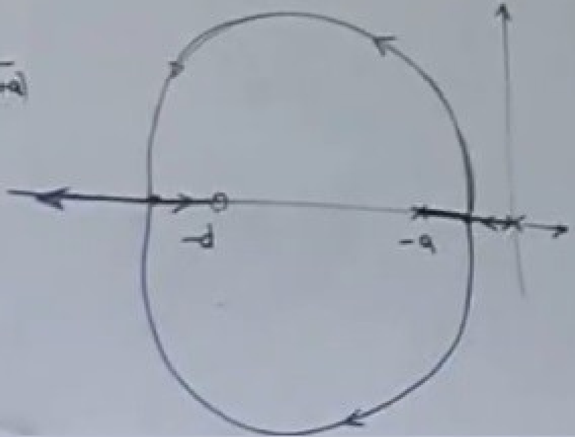
$H(s) = \frac{s+d}{s+a}$

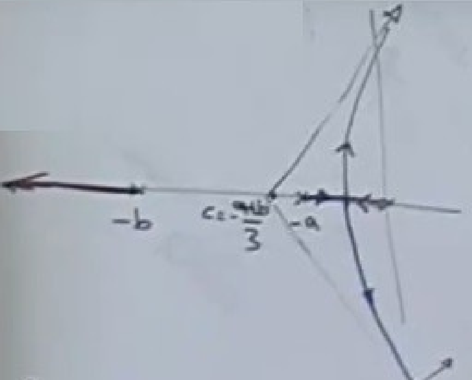


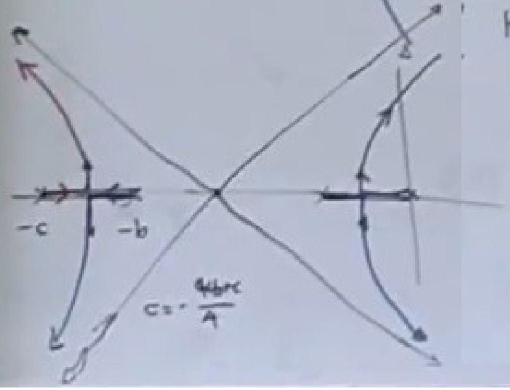


$H(s) = \frac{1}{s(s+a)}$

$H(s) = \frac{s+d}{s(s+a)}$

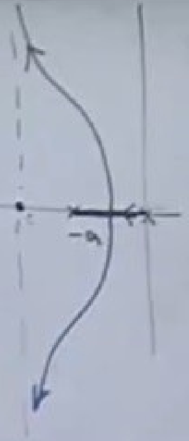






$$H(s) = \frac{1}{s(s+a)(s+b)}$$

$$H(s) = \frac{1}{s(s+a)(s+b)(s+c)}$$

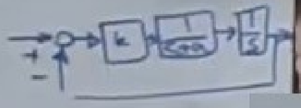


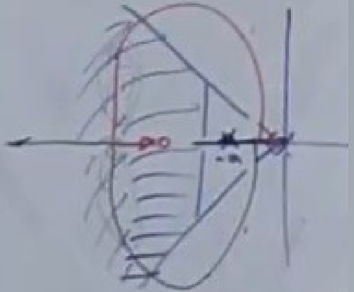
$$H(s) = \frac{s+d}{s(s+a)(s+b)}$$

$$c = -\frac{a+b-d}{2}$$

$$H_p(s) = \frac{1}{s+a}$$

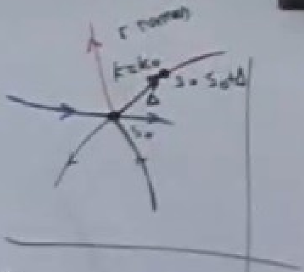
$$k H_c = \underbrace{k \cdot \frac{s+d}{s}}$$





comportamiento del LGP cerca de un pt. múltiple

$p(s)+K\,q(s)=0$ si en $K=K_0$ hay pt. de mult. r en $s=s_0$ $\Rightarrow$ $p(s)+K_0\,q(s) = (\underbrace{s-s_0}_{\Delta})^r f(s)$



$$p(s)+(\underbrace{K_0+k}_{K})\,q(s)=0 \Rightarrow p(s)+K_0 q(s)+k\,q(s)=0$$

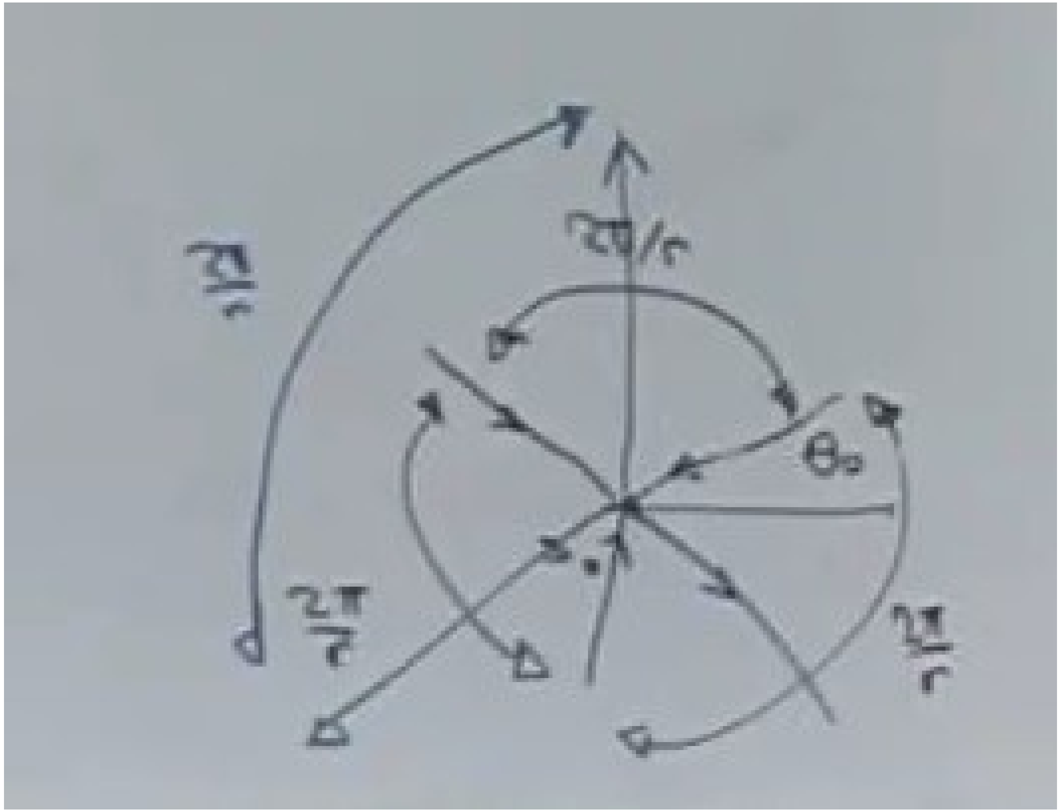
$$\Delta^r f(s)+k\,q(s)=0$$

$$\rightarrow \boxed{\Delta^r = -k\,\frac{q(s)}{f(s)}}$$

si $k\to 0$ $\quad \Delta^r \to -k\,\frac{q(s_0)}{f(s_0)} \quad \Delta \to \sqrt[r]{-k\,\frac{q(s_0)}{f(s_0)}}$ $\quad$ si $\frac{q(s_0)}{f(s_0)} = M\angle\varphi$

$$\Delta = \sqrt[r]{|-k|\cdot M}\ \angle\ \frac{1}{r}\left(\varphi+\arg(-k)\right)+\frac{2l\pi}{r}$$

$l=0,1,2\ldots r-1$


θ₀