Curso

SISTEMAS Y CONTROL

Clase 22

Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
Año 2020

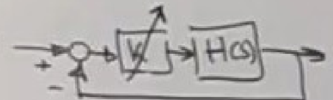
Clase 22 –

- Root – Locus (2)

LUGAR de las Raíces

p.ej.: $H(s) = \frac{q(s)}{p(s)} = \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$ $\quad (\alpha>0)$

G(s):



$G(s) = \frac{kH(s)}{1+kH(s)} = \frac{k\,q(s)}{p(s)+k\,q(s)}$

polos de G(s): sols. de $\begin{cases} 1+kH(s)=0 \\ p(s)+k\,q(s)=0 \end{cases}$ $\rightarrow H(s) = -\frac{1}{k}$

Hip: $\begin{cases} 1)\ m \leq n \\ 2)\ \alpha > 0 \end{cases}$

Si $k \in \mathbb{R}$ variando k: $-\infty < k < \infty$

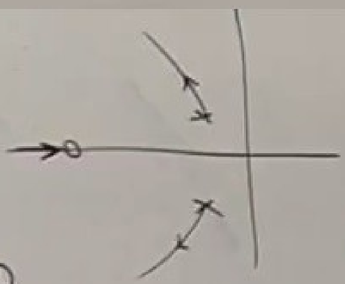
¿como varían los polos del lazo cerrado?

Rama: $\mathcal{C} = \{ s / s \in \mathbb{C},\ \forall k_0 \in \mathbb{R}\ \exists s_0 \in \mathcal{C} : p(s_0)+k_0 q(s_0)=0;\ \forall s_0 \in \mathcal{C}\ \exists k_0 \in \mathbb{R} : p(s_0)+k_0 q(s_0)=0 \}$

LG: $\{ s_0 / \operatorname{Im}\{H(s_0)\}=0 \}$

LGP $= \{ s_0 / \operatorname{Im}\{H(s_0)\}=0,\ \operatorname{Re}\{H(s_0)\}<0 \}$

LGN $= \{ s_0 /$ " $,\ \operatorname{Re}\{H(s_0)\}>0 \}$



Reglas de Construcción del LGP

① El LGP está const por $\max(n,m)=n$ ramas indeps.

– comienzan en los polos del lazo abierto. $(k \to 0)$

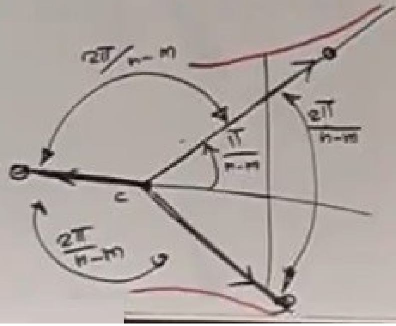
– m de ellos terminan $(k \to \infty)$ en los ceros del lazo abierto.

② Asíntotas

hay n-m soluciones que tienden asintóticamente a las sols. de

$(s-c)^{n-m} + k = 0$ donde $c = \frac{\sum_{j=1}^{n} p_j - \sum_{i=1}^{m} z_i}{n-m}$ (centroide)

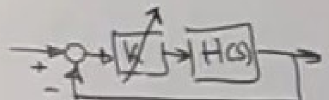
$\sqrt[n-m]{|k|}$



LUGAR de las Raices

p.ej.: $H(s)=\dfrac{q(s)}{p(s)} = \alpha\,\dfrac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$ $\quad(\alpha>0)$

G(s):



$G(s)=\dfrac{k\,H(s)}{1+k\,H(s)} = \dfrac{k\,q(s)}{p(s)+k\,q(s)}$

polos de G(s): sols. de $\begin{cases} 1+kH(s)=0 \longrightarrow H(s)=-\frac{1}{k} \\ p(s)+k\,q(s)=0 \end{cases}$

Hip: $\begin{cases} 1)\ m\leq n \\ 2)\ \alpha>0 \end{cases}$

Si $k\in\mathbb{R}$ variamos k: $-\infty<k<\infty$

¿como varían los polos del lazo cerrado?

Rama $\mathcal{C}=\{s/s\in\mathbb{C},\ \forall k_0\in\mathbb{R}\ \exists s_0\in\mathcal{C}: p(s_0)+k_0q(s_0)=0;\ \forall s_0\in\mathcal{C}\ \exists k_0\in\mathbb{R}: p(s_0)+k_0q(s_0)=0\}$

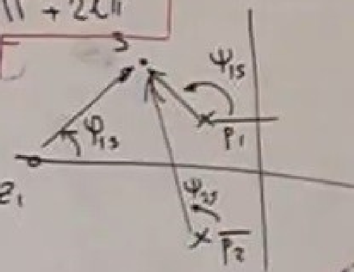
LG: $\{s_0 / \operatorname{Im}\{H(s_0)\}=0\}$

LGP $=\{s_0/\operatorname{Im}\{H(s_0)\}=0,\ \operatorname{Re}\{H(s_0)\}<0\}$

LGN $=\{s_0/\quad "\quad ,\ \operatorname{Re}\{H(s_0)\}>0\}$

$H(s) = |\alpha|\cdot\dfrac{\prod_{i=1}^{m}|s-z_i|}{\prod_{j=1}^{n}|s-p_j|}\ \angle\ \sum_{i=1}^{m}\varphi_{is}-\sum_{j=1}^{n}\psi_{js}$

$$\sum_{i=1}^{m}\varphi_{is}-\sum_{j=1}^{n}\psi_{js} = \pi + 2l\pi$$



Reglas de construcción del L.G.P.

1) El LGP está const por max(n,m) = n ramas indeps.

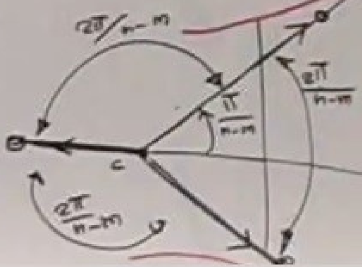
— comienzan en los polos del lazo abierto. $(k\to 0)$

— m de ellos terminan $(k\to\infty)$ en los ceros del lazo abierto.

2) Asintotas

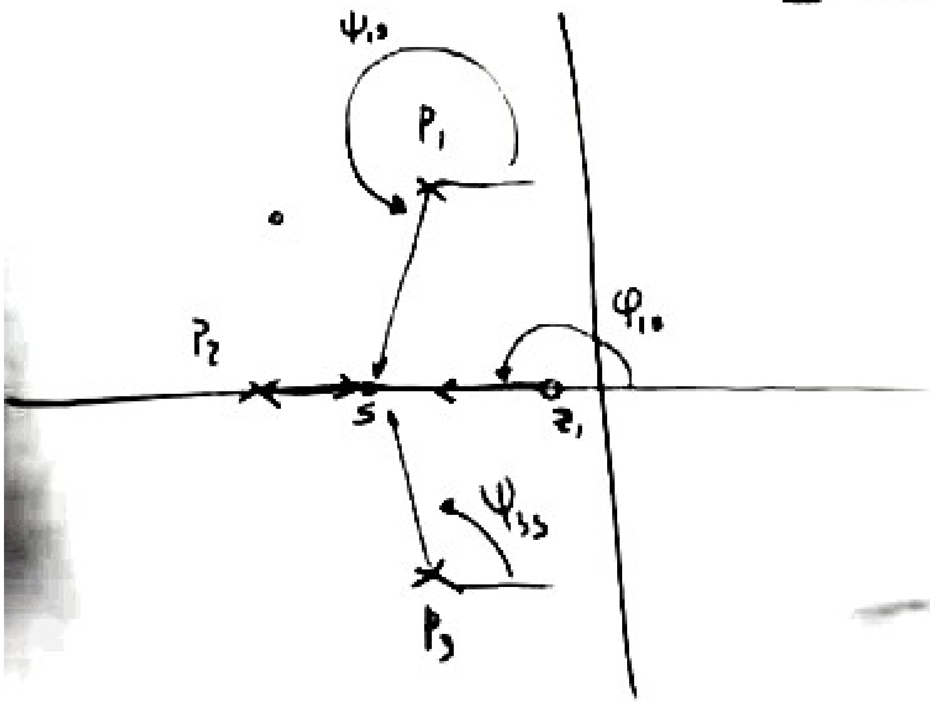
hay n−m soluciones que tienden asintóticamente a las sols. de

$(s-c)^{n-m}+k=0$ donde $c=\dfrac{\sum_{j=1}^{n}p_j-\sum_{i=1}^{m}z_i}{n-m}$ (centroide)



Regla 3   Demo

$$\angle H(s) = \sum_{i=1}^{m} \varphi_{is} - \sum_{j=1}^{n} \psi_{js}$$



Regla 4 Demo.

$$H(s) = \frac{q(s)}{p(s)}$$

$$p(s) + k\, q(s) = 0$$

si $\underline{s_o} \in$ L.G.P. $\rightarrow \exists\, k_o \in \mathbb{R}^+ / \left[p(s) + k_o q(s)\right]\Big|_{s=s_o} = 0$

$$k_o = -\frac{p(s)}{q(s)}\Big|_{s=s_o}$$

si s_o = pt. múltiple

$$p(s) + k_o\, q(s) = (s-s_o)^r f(s)$$

$$\rightarrow \left[p'(s) + \underline{k_o}\, q'(s)\right]\Big|_{s=s_o} = \left[r(s-s_o)^{r-1} f(s) + (s-s_o)^r f'(s)\right]\Big|_{s=s_o} = 0$$

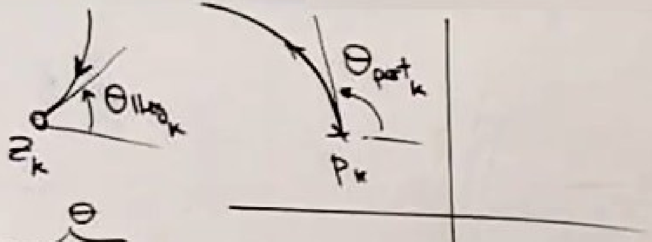
$$\overline{\left[p'(s) - \frac{p(s_o)}{q(s_o)}\, q'(s)\right]}\Bigg|_{s=s_o} = \left[\frac{p'(s)\, q(s_o) - p(s_o)\, q'(s)}{q(s_o)}\right]\Bigg|_{s=s_o} = 0$$

$$\frac{dH}{ds} = \frac{q'(s)\,p(s) - q(s)\,p'(s)}{p^2(s)}\Bigg|_{s=s_o} = 0$$

③ $\exists$ L.G.P. los pts. del eje real que tienen a su derecha un # impar de sing. de $H(s)$ (polos+ceros)

④ Pts. múlt.

s_0 es pt. multiple del L.G.P $\iff \begin{cases} 1)\ s_0 \in \text{L.G.P.} \\ 2)\ \left.\dfrac{dH(s)}{ds}\right|_{s=s_0} = 0 \end{cases}$



⑤ Ángulos de partida, Ángulos de llegada

$$\theta_{part_k} = \pi + \sum_{i=1}^{m} \varphi_{ik} - \sum_{\substack{j=1 \\ j\neq k}}^{n} \psi_{jk}$$

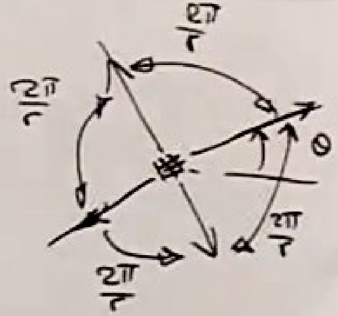
si mult.=r

$$\theta_{part_k} = \frac{\overbrace{\left(\pi + \sum_{i=1}^{m} \varphi_{ik} - \sum_{\substack{j=1 \\ j\neq k, k_1, \dots}}^{n} \psi_j\right)}^{\theta}}{r} + \frac{2l\pi}{r} \quad (l = 0, 1, \dots (r-1))$$

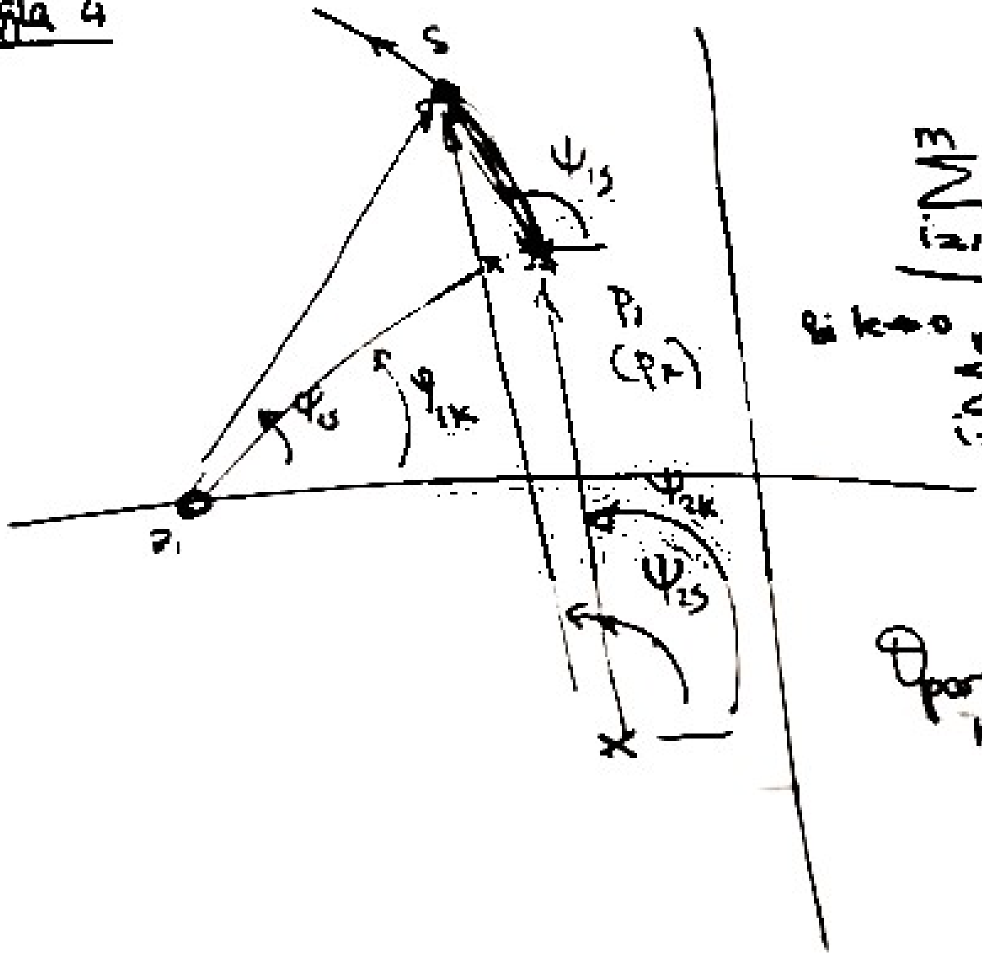
$$\theta_{lleg_k} = \pi - \sum_{\substack{i=1 \\ i\neq k}}^{m} \varphi_{ik} + \sum_{j=1}^{n} \psi_{jk}$$

"

$$\theta_{lleg_k} = \frac{\left(\pi - \sum_{\substack{i=1 \\ i\neq k}}^{m} \varphi_{ik} + \sum_{j=1}^{n} \psi_{jk}\right)}{r} + \frac{2l\pi}{r} \quad l = 0 \dots (r-1)$$



Regla 4



$$\sum_{i=1}^{m}\varphi_{is} - \sum_{j=1}^{n}\psi_{js} = \pi + 2l\pi$$

si k=0

$$\sum_{i=1}^{m}\varphi_{ik} - \sum_{\substack{j=1\\ j\neq k}}^{n}\psi_{jk} - \theta_{part_k} = \pi + 2l\pi$$

$$\theta_{part_k} = \pi + \sum_{i=1}^{m}\varphi_{ik} - \sum_{\substack{j=1\\ j\neq k}}^{n}\psi_{jk}$$

5) pts. de corte con el eje imaginario

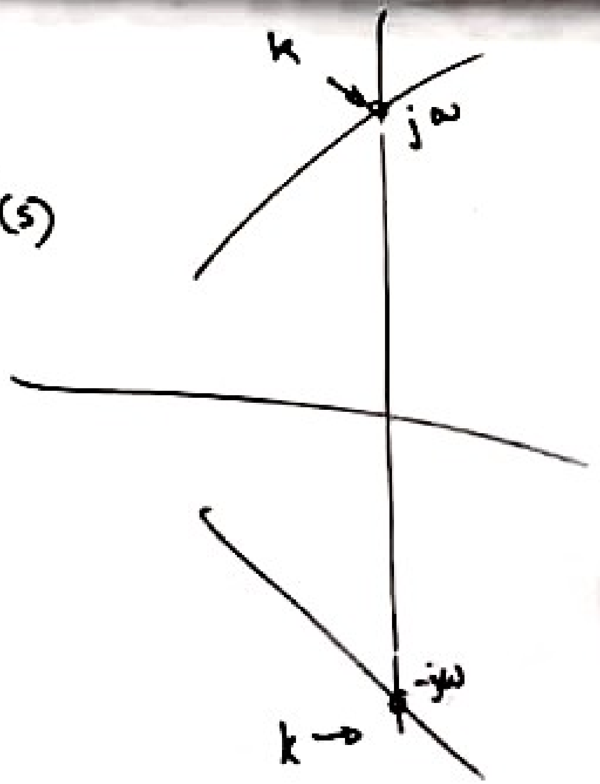
$$p(s)+k\,q(s)=0$$

Se determinan los pts. de corte con

a) Routh Hurwitz

b) Coefs. indeterminados.

$$p(s)+k\,q(s)=(s^2+\omega^2)\,f(s)$$

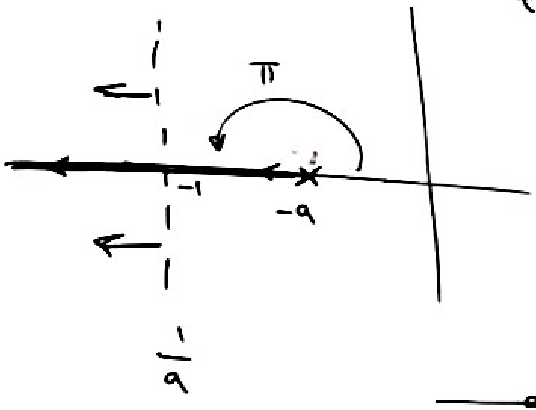


ej. 1

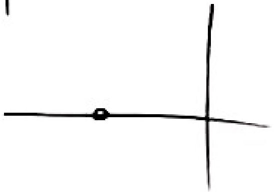
$H(s) = \frac{1}{s+a}$

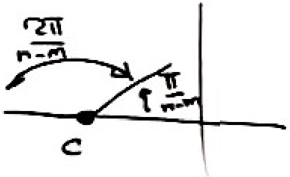
Nº : 1 rama

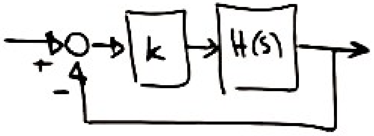
parte en: $-a$

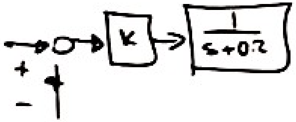


$t_s \leq 3s$









[redacted] ps. mit

ej. 2 $H(s) = \frac{1}{s(s+a)}$

$s = -\frac{a}{2}$

$N = 2$

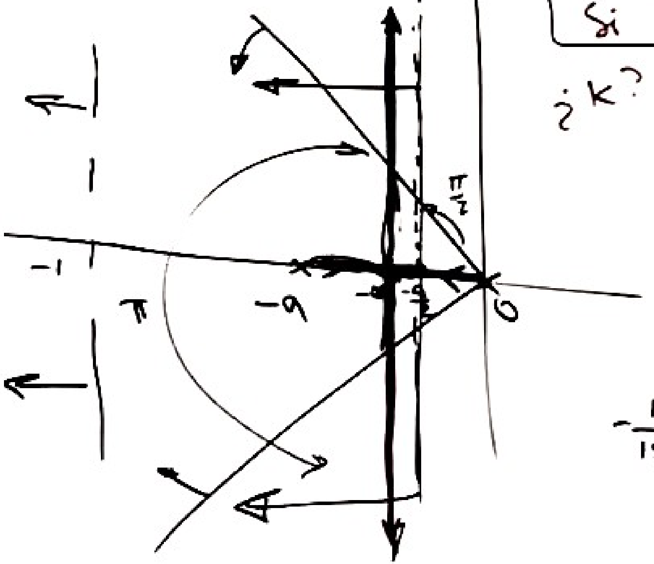
parten de $0, -a$

llegan a infinit.

$c = \frac{-a-0}{2-0} = -\frac{a}{2}$

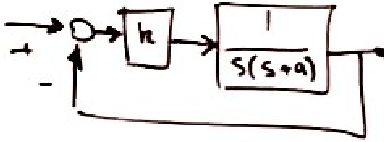
$\frac{dH}{ds} = 0 \Rightarrow s = -\frac{a}{2}$

$$\frac{dH}{ds} = -\frac{(s+a)+s}{(\quad)^2} = -\frac{2s+a}{(\quad)^2} = 0$$



Si $a = 0.2$

¿K? $t_s \leq 3$ seg



$t_s \leq 15$ seg

$-\frac{1}{15} = -0.067$