Curso

SISTEMAS Y CONTROL

Clase 20

Fotogramas de los pizarrones de clases filmadas

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Clase 20 –

- Estabilidad (2)

  - Criterios
  - Ejemplos

Esta clase hace uso de las transparencias, "estabilidad.pdf"

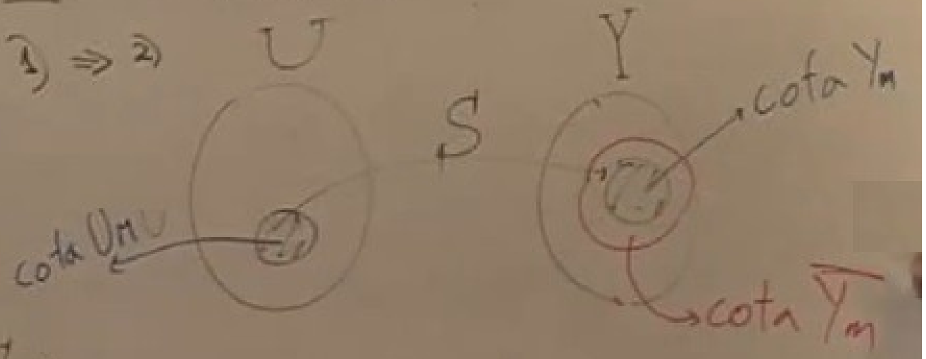
1) $U_n, \Rightarrow \exists\ Y_n$ / si $\|u(t)\| \leq U_n\ \forall t \Rightarrow \|y(t)\| \leq Y_n\ \forall t$

2) $\bar{Y}_n, \Rightarrow \exists\ \bar{U}_n$ / si $\|u(t)\| \leq \bar{U}_n\ \forall t \Rightarrow \|y(t)\| \bar{Y}_n\ \forall t$

$$\underbrace{\left[u \cdot \frac{\bar{Y}_M}{Y_n}\right]}_{u'} \longrightarrow \|y_1\| = \|y \frac{\bar{Y}_n}{Y_M}\| \leq \cancel{Y_n} \cdot \frac{\bar{Y}_M}{\cancel{Y_M}} = \bar{Y}_n$$

Prop Si S es lineal

1) $\Rightarrow$ 2)



S(

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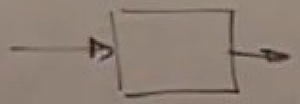
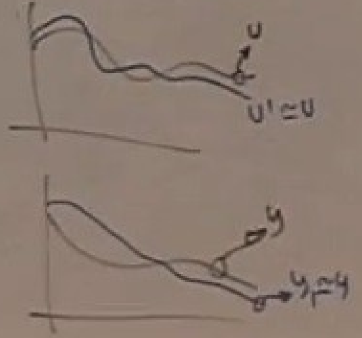
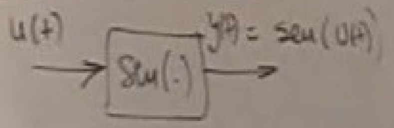
Si S lineal 1) $\Rightarrow$ 2)

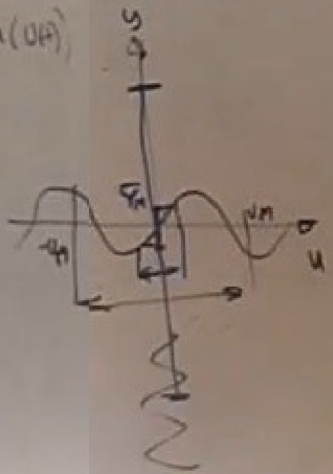
$y = S[u]$

$y_1 = S[u']$

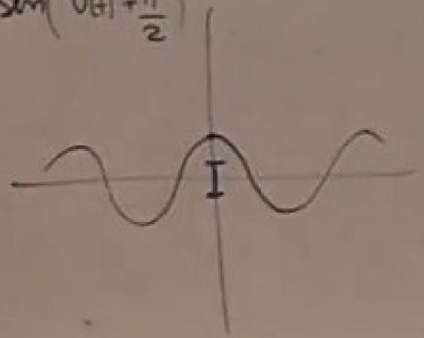


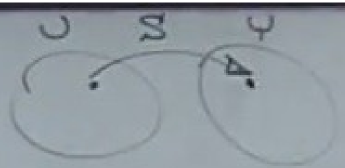






$y(t) = \text{sen}\left(u(t) + \frac{\pi}{2}\right)$





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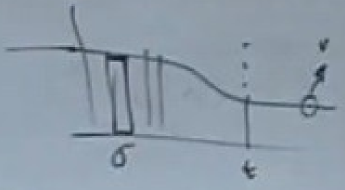
CRITERIOS de ESTABILIDAD para sistemas lineales

Si S cumple

$$y(t) = \int_{-\infty}^{t} \mathcal{H}(t,\sigma)\, u(\sigma)\, d\sigma$$

S es estable $\Rightarrow \exists \; L < \infty$ /

$$\int_{-\infty}^{t} \|\mathcal{H}(t,\sigma)\|\, d\sigma < L \quad \forall t$$



$$y_\sigma(t) = \int_{-\infty}^{t} \underbrace{\left[C(t)\, \phi(t,\sigma)\, B(\sigma) + \delta_{t-\sigma} D(\sigma) \right]}_{\mathcal{H}(t-\sigma)} u(\sigma)\, d\sigma$$

Demo

$\Leftarrow$

Si $\|u(t)\| \leq U_M \ \forall t$

$$\|y(t)\| = \left\| \int_{-\infty}^{t} \mathcal{H}(t,\sigma)\, u(\sigma)\, d\sigma \right\| \leq \int_{-\infty}^{t} \left\| \mathcal{H}(t,\sigma)\, u(\sigma) \right\| d\sigma \leq \int_{-\infty}^{t} \|\mathcal{H}(t,\sigma)\| \underbrace{\|u(\sigma)\|}_{\leq U_M} d\sigma \leq U_M \underbrace{\int_{-\infty}^{t} \|\mathcal{H}(t,\sigma)\| d\sigma}_{< L} \leq L\, U_M$$

Def Polinomio "Hurwitz"

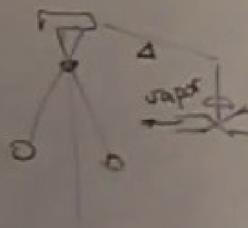
$$d(s) = d_0 (s-p_1)(s-p_2)(s-p_3)\dots(s-p_n)$$

$$(s^2 + 2\xi\omega_n s + \omega_n^2)$$

COND. NECESARIA

1) $d(s)$ tiene todos los coef $\neq 0$

2) $d(s)$ " " " " del mismo signo.



$$H(s) = \frac{n(s)}{d(s)}$$

CRITERIO de ESTABILIDAD de ROUTH-HURWITZ

Sea $d(s) = d_0 s^n + d_1 s^{n-1} + d_2 s^{n-2} + \dots + d_{n-1} s + d_n$

$$\begin{array}{l|l} d_0 s^n + d_2 s^{n-2} + d_4 s^{n-4} + \dots & d_1 s^{n-1} + d_3 s^{n-3} + d_5 s^{n-5} + \dots \\ -d_0 s^n - \frac{d_0 d_3}{d_1} s^{n-2} - \frac{d_0 d_5}{d_1} s^{n-4} & \frac{d_0}{d_1} s \\ \hline \frac{d_1 d_2 - d_0 d_3}{d_1} s^{n-2} + \frac{d_1 d_4 - d_0 d_5}{d_1} s^{n-4} + \dots & \end{array}$$

$\dots$

$$\begin{cases} f_n(s) = d_0 s^n + d_2 s^{n-2} + d_4 s^{n-4} + \dots \\ f_{n-1}(s) = d_1 s + d_3 s^{n-3} + d_5 s^{n-5} + \dots \end{cases}$$

$f_{n-2}(s) = 0 \quad 0 \quad 0 \quad 0$

$f_{n-3}(s) =$

$\vdots$

$f_1(s) =$

$f_0(s) =$

$f_{j-2}(s) = Re\left[\frac{f_j(s)}{f_{j-1}(s)}\right]$

Algoritmo de Euclides

$d(s) = f_n(s) + f_{n-1}(s)$

$f_j(s) = d_{j-2} s \; f_{j-1}(s) + f_{j-2}(s)$

$d(s) = s^3 + 5s^2 + 5s + 3k$

f_3	$1 \quad 5$
f_2	$5 \quad 3k$
f_1	$\frac{25-3k}{5} \quad -$
f_0	$3k$

$+ + + + + +$ k

$+ + + + + + + + +$

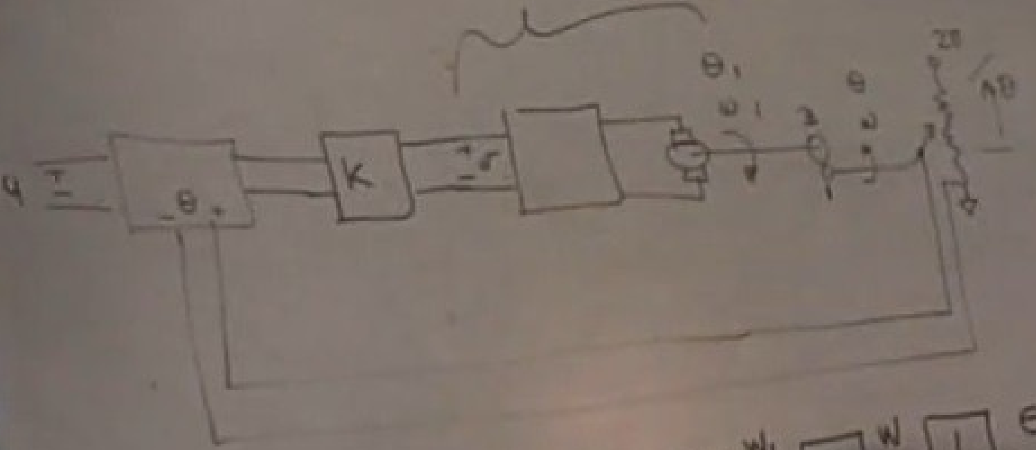
$+ + + + + \,|\, - - -$ k, $\frac{25}{3}$

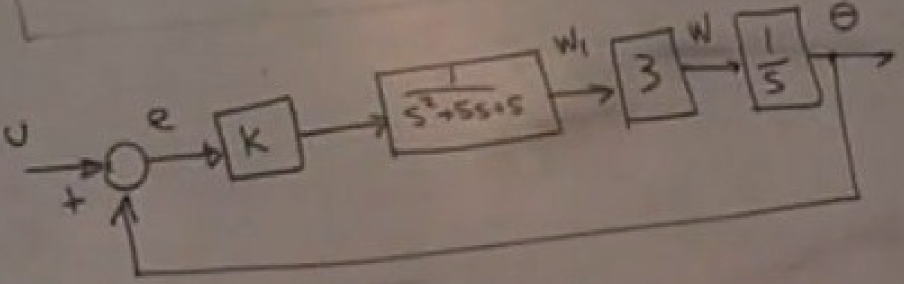
$- - - \,|\, + + + + + +$ k, 0

$1 \quad | \quad 0 \quad | \quad 2$ k, 0, $\frac{25}{3}$

Ejemplo:

Diseño de un plotter $\frac{1}{s^2+5s+5}$





e_∞ esc. $= 0$

e_∞ ramp $= \frac{1}{K_v} = \frac{5}{3k}$

$K_v = \lim_{s \to 0} s\,H(s) = \frac{3}{5}k$

$$\frac{\Theta(s)}{U(s)} = \frac{3K}{s(s^2+5s+5)+3k} = \frac{3K}{s^3+5s^2+5s+3k}$$