

Curso

SISTEMAS Y CONTROL

Clase 20

Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
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Este material fue elaborado como material de apoyo para ser utilizado por los estudiantes de este curso de Ingeniería Eléctrica de la Facultad de Ingeniería, Universidad de la República (UdelaR).

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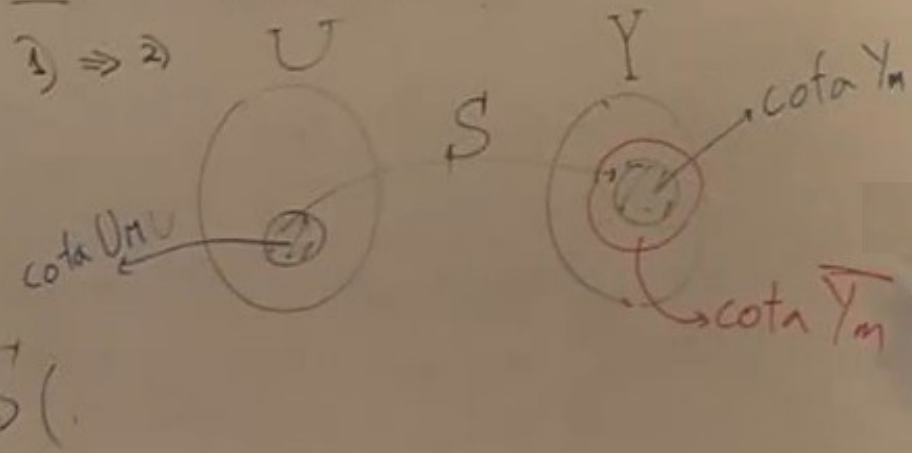
Clase 20 –

- Estabilidad (2)
 - Criterios
 - Ejemplos

Esta clase hace uso de las transparencias, “estabilidad.pdf”

1) $U_n \Rightarrow \exists Y_n$ / $\forall \epsilon > 0 \exists \delta > 0 \forall t \Rightarrow \|y(t)\| \leq Y_n + \epsilon$
 2) $\bar{Y}_n \Leftarrow \exists \bar{U}_n$ / $\forall \epsilon > 0 \exists \delta > 0 \forall t \Rightarrow \|y(t)\| \leq \bar{U}_n + \epsilon$

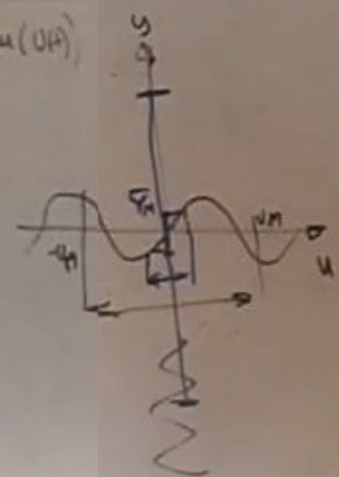
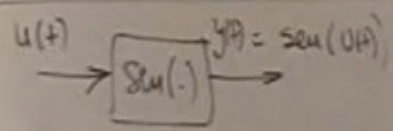
Prop Si S es lineal



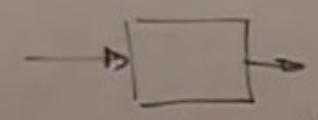
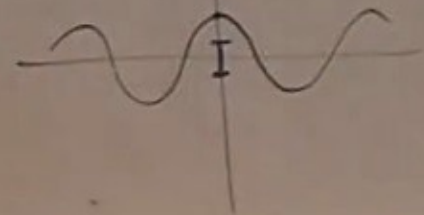
$$\left[\underbrace{\bar{U}}_u \cdot \frac{\bar{Y}_M}{Y_n} \right] \rightarrow \|y\| \leq \|y\| \frac{\bar{Y}_M}{Y_n} \leq \frac{Y_n}{Y_n} \cdot \frac{\bar{U}}{Y_n} = \bar{U}_n$$

1) $U_n \Rightarrow \exists Y_n$ / $\times \quad \|u(t)\| \leq U_n \forall t \Rightarrow \|y(t)\| \leq Y_n \forall t$
 2) $\bar{Y}_n \Rightarrow \exists \bar{U}_n$ / $\times \quad \|u(t)\| \leq \bar{U}_n \forall t \Rightarrow \|y(t)\| \leq \bar{Y}_n \forall t$

Si S lineal 1) \Rightarrow 2)

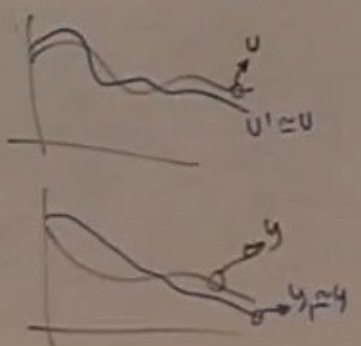


$$y(t) = \sin\left(u(t) + \frac{\pi}{2}\right)$$



$$y = S[u]$$

$$y_1 = S[u_1]$$

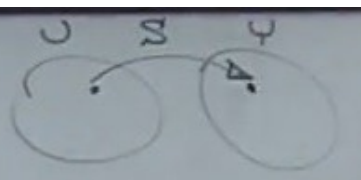
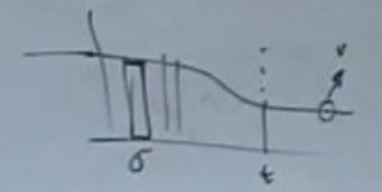


- 1) $U_n, \Rightarrow \exists Y_n$ / si $\|u(t)\| \leq U_n \forall t \Rightarrow \|y(t)\| \leq Y_n \forall t$
- 2) $\bar{Y}_n, \Rightarrow \exists \bar{U}_n$ / si $\|y(t)\| \leq \bar{Y}_n \forall t \Rightarrow \|u(t)\| \leq \bar{U}_n \forall t$

CRITERIOS de ESTABILIDAD para sistemas lineales

Si S cumple S es estable $\Rightarrow \exists L < \infty$

$$y(t) = \int_{-\infty}^t \mathcal{L}(t, \sigma) u(\sigma) d\sigma$$



$$y_\sigma(t) = \int_{-\infty}^t \left[C(t) \phi(t, \sigma) B(\sigma) + \int_{t-\sigma}^t D(\sigma) \right] u(\sigma) d\sigma$$

$\mathcal{L}(t, \sigma)$

$$\int_{-\infty}^t \|\mathcal{L}(t, \sigma)\| d\sigma < L \quad \forall t$$

Demo

$$\|y(t)\| = \left\| \int_{-\infty}^t \mathcal{A}(t, \sigma) u(\sigma) d\sigma \right\| \leq \int_{-\infty}^t \|\mathcal{A}(t, \sigma) u(\sigma)\| d\sigma \leq \int_{-\infty}^t \underbrace{\|\mathcal{A}(t, \sigma)\|}_{\leq U_M} \|u(\sigma)\| d\sigma \leq U_M \underbrace{\int_{-\infty}^t \|\mathcal{A}(t, \sigma)\| d\sigma}_{< L} \leq L U_M$$

$\& \|u(t)\| \leq U_M \forall t$

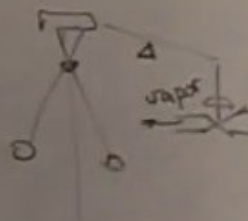
Def Polinomio "Hurwitz"

$$d(s) = d_0 \cdot (s-p_1)(s-p_2)(s-p_3)\dots(s-p_n)$$

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

COND. NECESARIA

- (1) $d(s)$ tiene todos los coef $\neq 0$
- (2) $d(s)$ " " " " del mismo signo.



$$H(s) = \frac{n(s)}{d(s)}$$

CRITERIO de ESTABILIDAD de ROUTH - HURWITZ

Sea $d(s) = d_0 s^n + d_1 s^{n-1} + d_2 s^{n-2} + \dots + d_{n-1} s + d_n$

$$\begin{array}{r} d_0 s^n + d_2 s^{n-2} + d_4 s^{n-4} + \dots \\ \underline{d_0 s^n - \frac{d_0 d_2}{d_1} s^{n-2} - \frac{d_0 d_4}{d_1} s^{n-4} + \dots} \\ \frac{d_0 d_2}{d_1} s^{n-2} + \dots \end{array}$$

$$\frac{d_1 d_2 - d_0 d_3}{d_1} s^{n-3} + \dots$$

$$\begin{cases} f_n(s) = d_0 s^n + d_1 s^{n-1} + d_2 s^{n-2} + \dots \\ f_{n-1}(s) = d_1 s^{n-1} + d_2 s^{n-2} + d_3 s^{n-3} + \dots \end{cases}$$

$$\begin{array}{r} f_{n-2}(s) = \dots \\ f_{n-3}(s) = \dots \\ \vdots \\ f_1(s) = \dots \\ f_0(s) = \dots \end{array}$$

$$d(s) = s^3 + 5s^2 + 5s + 3k$$

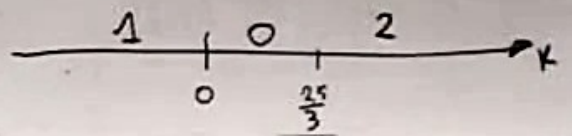
$f_j(s) = \text{Re} \left[\frac{f_j(s)}{f_{j-1}(s)} \right]$ Algoritmo de Euclides

$d(s) = f_n(s) + f_{n-1}(s)$

$f_j(s) = \alpha \frac{f_j(s)}{f_{j-1}(s)} + f_{j-2}(s)$

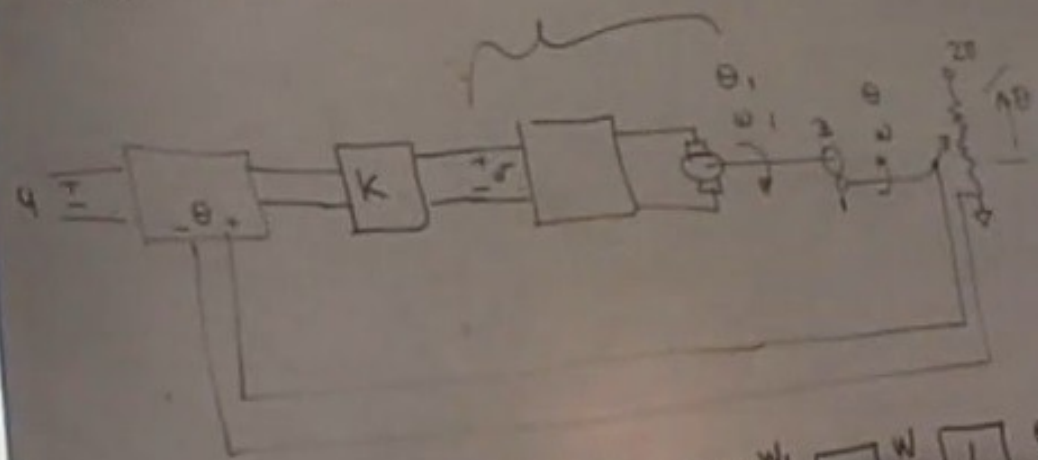
f_3	1			
f_2	5			
f_1	$\frac{25-3k}{5}$	-		
f_0	$3k$			

Annotations: A vertical line is drawn through the first column. A horizontal line is drawn through the second row. An arrow points from the '5' in the second row, first column to the '5' in the second row, second column. Another arrow points from the '5' in the second row, first column to the '3k' in the fourth row, first column.



Ejemplo:

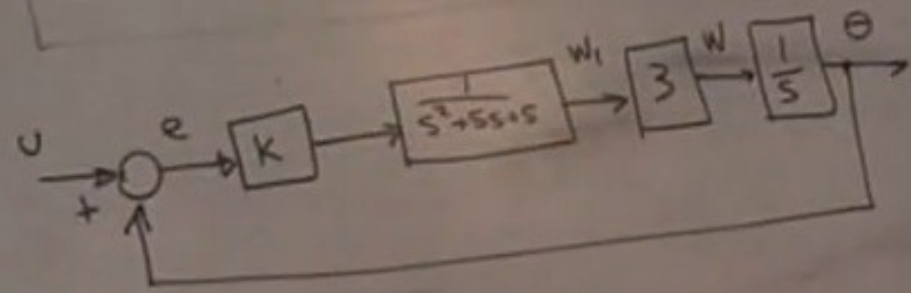
Diseño de un plotter ³⁺⁵⁺⁵⁺⁵



$$e_{\infty ac} = 0$$

$$e_{\infty ramp} = \frac{1}{K_V} = \frac{5}{3K}$$

$$K_V = \lim_{s \rightarrow 0} s H(s) = \frac{3}{5} K$$



$$\frac{\Theta(s)}{U(s)} = \frac{3K}{s(s^2 + 5s + 5) + 3K} = \frac{3K}{s^3 + 5s^2 + 5s + 3K}$$