Curso

SISTEMAS Y CONTROL

Clase 18

Fotogramas de los pizarrones de clases filmadas

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Clase 18 –

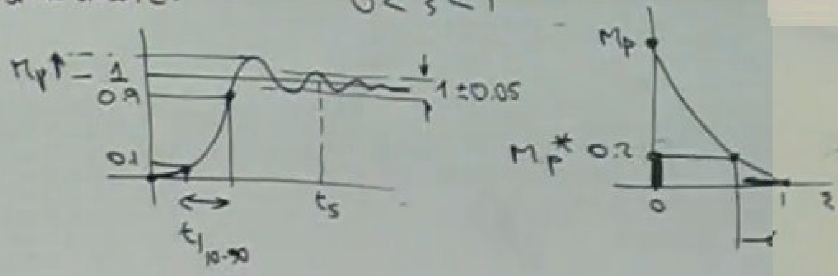
- Respuesta Temporal 5 (Respuesta temporal del Sistema de Parámetros Concentrados a entradas específicas)

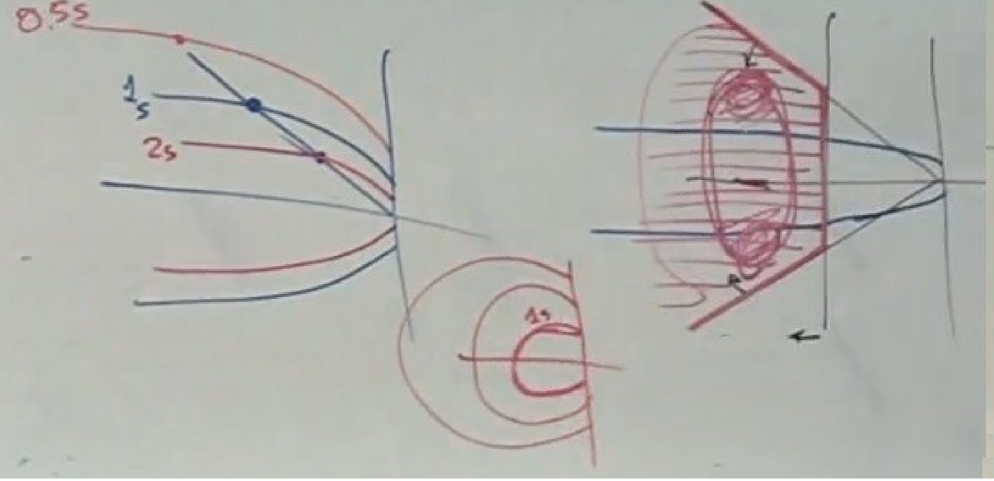
    - Sistema de orden 2, criterios de diseño
    - Comportamiento inicial (en 0+) de la respuesta
    - Evaluación gráfica de residuos
    - Polos dominantes
    - Comportamiento asintótico, (diseño de servomecanismos) Kp, Kv, Ka

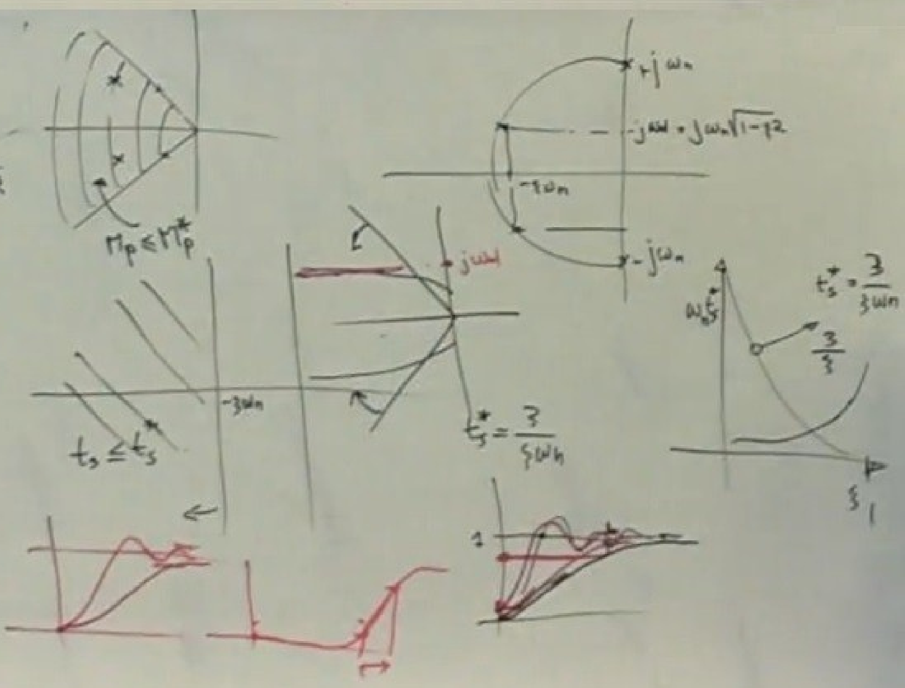
Esta clase hace uso de las transparencias, “respuesta_orden2_tl_ts.pdf”

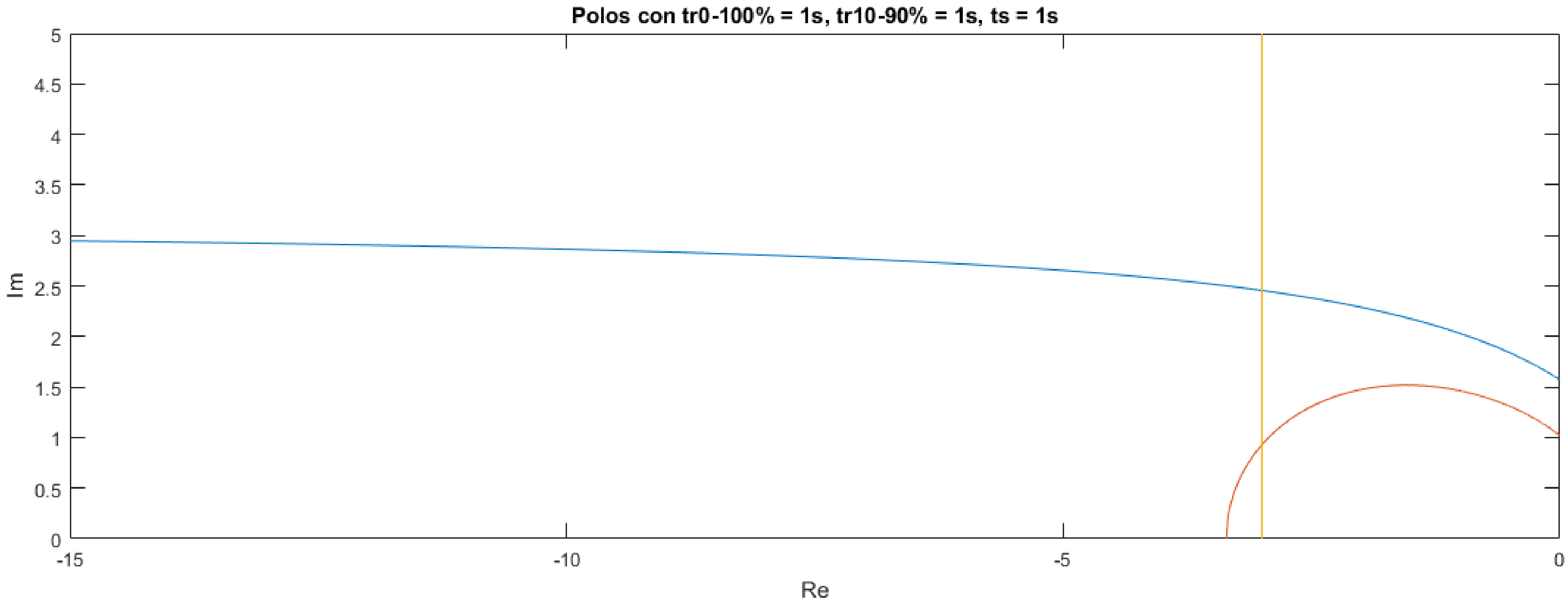
ESP. TEMPORAL Sist. de orden 2. $0 < \xi < 1$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$





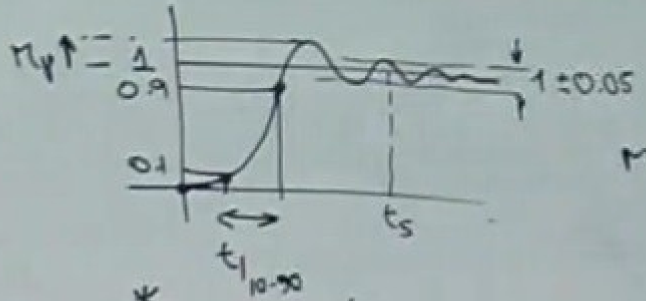


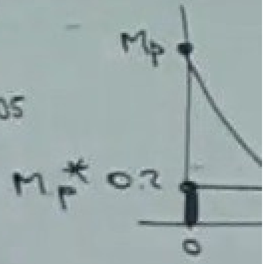


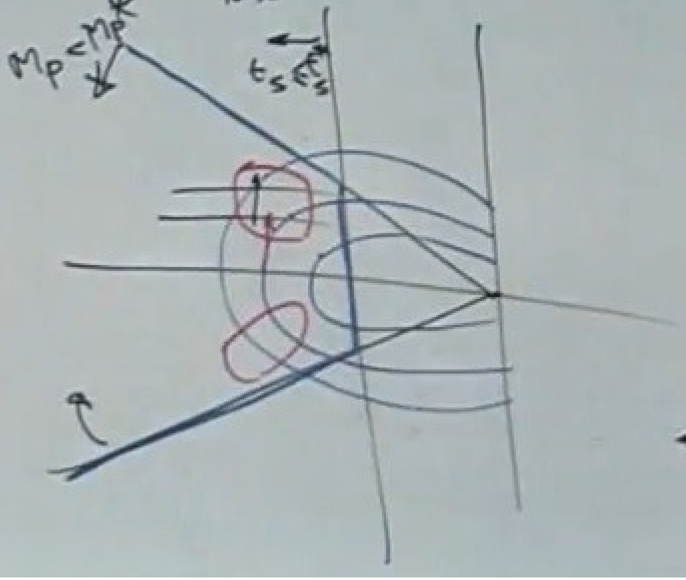
Polos con tr0-100% = 1s, tr10-90% = 1s, ts = 1s
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Im
-15
-10
-5
0
Re

RESP. TEMPORAL   Sist. de orden 2. $0 < \xi < 1$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$







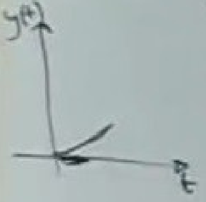
RESP. TEMPORAL

comportamiento en 0^+

$\lim\limits_{t \to 0^+} y(t)$

teo. valor inicial

$$\boxed{y(0^+) = \lim_{s \to \infty} s\,Y(s)}$$

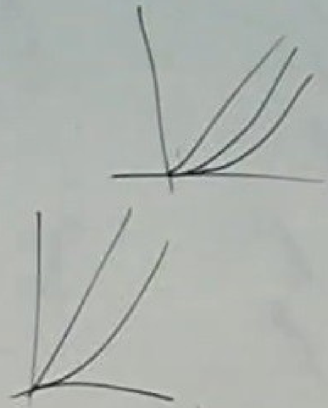


$$y'(0^+) = \lim_{s \to \infty} s\left(sY(s)\right) = \lim_{s \to \infty} s^2 Y(s)$$

$$y''(0^+) = \lim_{s \to \infty} s\left(s^2 Y(s)\right) = \lim_{s \to \infty} s^3 Y(s)$$

Si $Y(s) = \alpha \dfrac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$

$n-m$ = exceso de polos-ceros



p. ej.

$$\boxed{H(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

$$Y(s) = H(s)\frac{1}{s}$$

la resp. a escalón

$s\dfrac{1}{s}H(s) \; - \; 0$ $\qquad y(0^+) = 0$

$s^2\dfrac{1}{s}H(s) \; - \; 0$ $\qquad y'(0^+) = 0$

$s^3\dfrac{1}{s}\cdot H(s) \; -$ $\qquad y''(0^+) \neq 0$

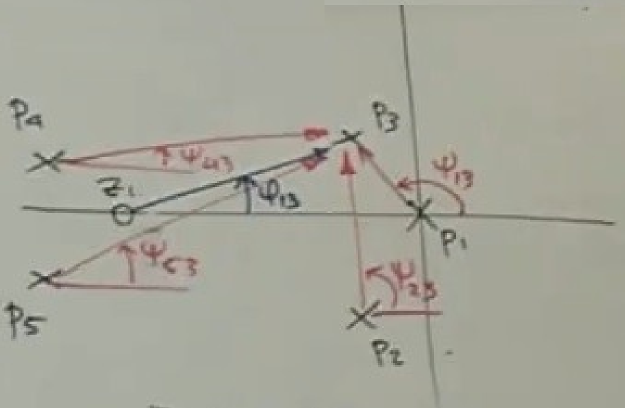
RESP. TEMPORAL

Evaluación gráfica de residuos

$$\boxed{Y(s) = \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}(s-p_j)}}$$

$m<n$

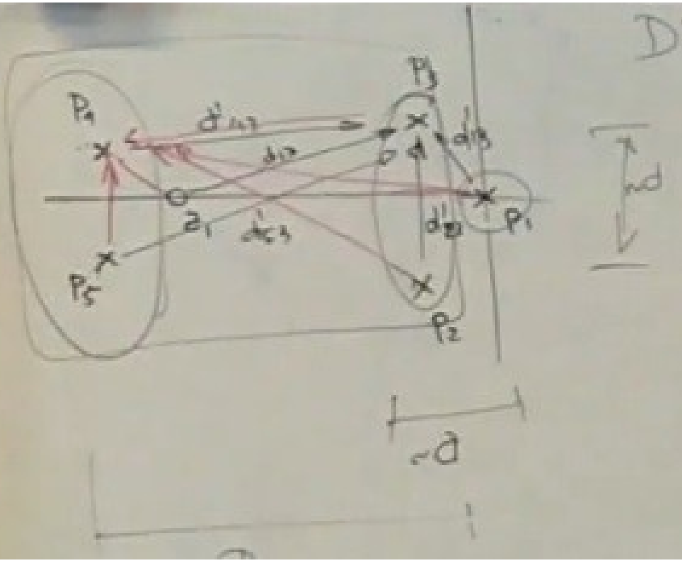
$$A_k = \lim_{s\to p_k} Y(s)(s-p_k) = \alpha \frac{\prod_{i=1}^{m}(p_k-z_i)}{\prod_{\substack{j=1\\ j\neq k}}^{n}(p_k-p_j)}$$



¿ $y(t)$? si los polos son $\neq_s$

$$Y(s) = A_0 + \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \dots + \frac{A_n}{s-p_n} \Rightarrow y(t) = \underline{A_0}\,\delta_t + \underline{A_1}\, e^{p_1 t} + A_2 e^{p_2 t} + \dots + \underline{A_n}\, e^{p_n t}$$

$$A_k = |\alpha| \frac{\prod_{i=1}^{m}|p_k-z_i|}{\prod_{\substack{j=1\\ j\neq k}}^{n}|p_k-p_j|} \angle \sum_{i=1}^{m}\varphi_{ik} - \sum_{\substack{j=1\\ j\neq k}}^{n}\psi_{jk}$$



$D >> d \quad y(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + A_4 e^{p_4 t} + A_5 e^{p_5 t}$ 1 0.04

$\simeq e^{\sigma_2 t}(\sin \omega_2 t + \varphi_2) + \alpha_1 e^{\sigma_4 t}(\sin \omega_4 t + \varphi_4)$

$$|A_3| = \frac{d_{13}}{d'_{13}\, d'_{23}\, d'_{43}\, d'_{53}} |\alpha| \simeq |\alpha| \frac{D}{d^2\, D^2} = |\alpha| \frac{1}{D d^2}$$

$$|A_4| \simeq \cdot \; \frac{\quad}{\quad} \simeq |\alpha| \frac{d}{D D D d} = |\alpha| \frac{1}{D^3}$$

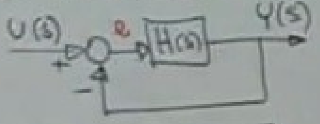
$$\frac{|A_4|}{|A_3|} = \frac{D d^2}{D^3} \simeq \left(\frac{d}{D}\right)^2$$

RESPUESTA TEMPORAL

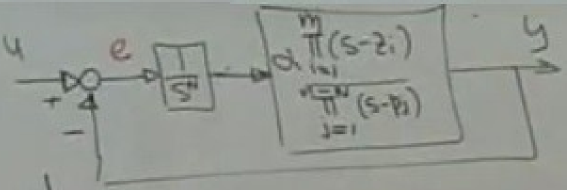
Comportamiento asintótico. - Errores permanentes, diseño de servomecanismos

Si $u(t) = t^k$ ¿ $\lim_{t\to\infty} y(t)$?

Sea $G(s) \simeq$



$G(s) = \dfrac{H(s)}{1+H(s)}$



Sistema "Tipo N"

$$\boxed{H(s) = \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)} = \frac{\alpha \prod_{i=1}^{m}(s-z_i)}{s^N \prod_{j=1}^{n-N}(s-p_j)}}$$

$$\boxed{E(s) = \frac{1}{1+H(s)} \cdot U(s)} \rightarrow E(s) = \frac{s^N}{s^N + \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n-N}(s-p_j)}}$$

$\dfrac{1}{S^r}$

u(t)	U
1_t	$\frac{1}{s}$
$t\,1_t$	$\frac{1}{s^2}$
$\frac{t^2}{2!}1_t$	$\frac{1}{s^3}$

$G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{H(s)}{1+H(s)}$ | $\underline{\underline{\text{Hip}}}$ ¡ $G(s)$ es estable!

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^{N+1}}{s^N + \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n-N}(s-p_j)}} \cdot \frac{1}{s^r} = \lim_{s\to 0} \frac{s^{N+1-r}}{s^N + \alpha \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n-N}(s-p_j)}}$$

si $\exists \lim_{t\to\infty} e(t)$ $\Updownarrow$

$$\frac{1}{1+\alpha \frac{\prod_{i=1}^{m}(-z_i)}{\prod_{j=1}^{n}(-p_j)}}$$

$$k_p : \lim_{s \to 0} H(s) \qquad t \quad t^3$$

$N=1$

$$\lim_{s \to 0} sE(s) = \frac{s^{N+1-r}}{s^1 \cdot \alpha \frac{\prod (s-z_i)}{\prod (s-p_i)}} = \frac{1}{\lim_{s \to 0} sH(s)} = \frac{1}{K_v} \qquad \lim_{s \to 0} sH(s) = K_v$$

$$sH(s)$$

$N=2$

$$\lim_{s \to 0} sE(s) = \frac{1}{\lim_{s \to 0} s^2 H(s)} = \frac{1}{K_a} \qquad \lim_{s \to 0} s^2 H(s) = K_a$$

tipo N \ u	$r=1$: Y_r	$r=2$: $t\,Y_r$	$r=3$: $\frac{t^2}{2}\,Y_r$	....	
0	$\frac{1}{1+K_p}$	∞	∞	∞	
1	0	$\frac{1}{K_i}$	∞	∞	
2	0	0	$\frac{1}{K_a}$	∞	
3	0	0	0		∞