Curso

SISTEMAS Y CONTROL

Clase 16

Fotogramas de los pizarrones de clases filmadas

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Clase 16 –

- Respuesta Temporal 3 (Respuesta temporal del Sistema de Parámetros Concentrados a entradas específicas)

  - Sistema de orden1 (continuación), valores de Mp, td, ts, tl.
  - Sistema de orden 2, respuesta a impulso
  - Sistema de orden 2, respuesta a escalón (cálculo de tp, Mp)

Esta clase hace uso de las transparencias “resp_orden2_mp.pdf” sistema 1er orden, resp. a escalón

$H(s) = \frac{1}{Ts+1}$

$e = e^{-\frac{t}{T}}$

$y(t) = 1 - e^{-t/T}$

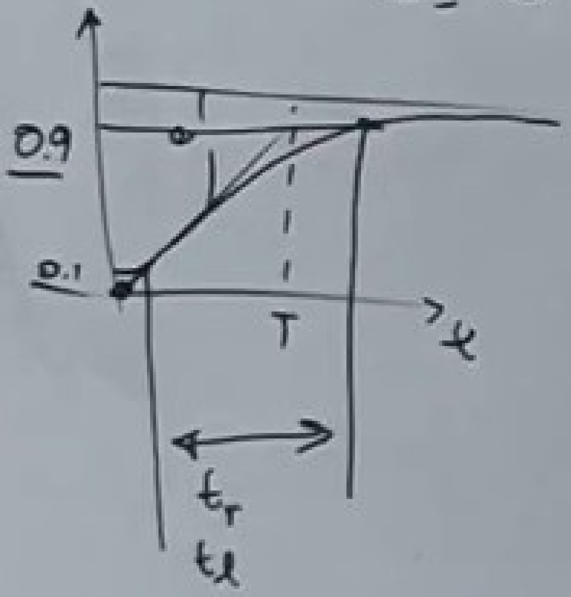
$M_p = 0$

$t_{s(95\%)} \cong 3T$

$t_s(98\%) = 3.91\,T$

$t_{r(10-90\%)} =$

$t_d = 0.693\,T$



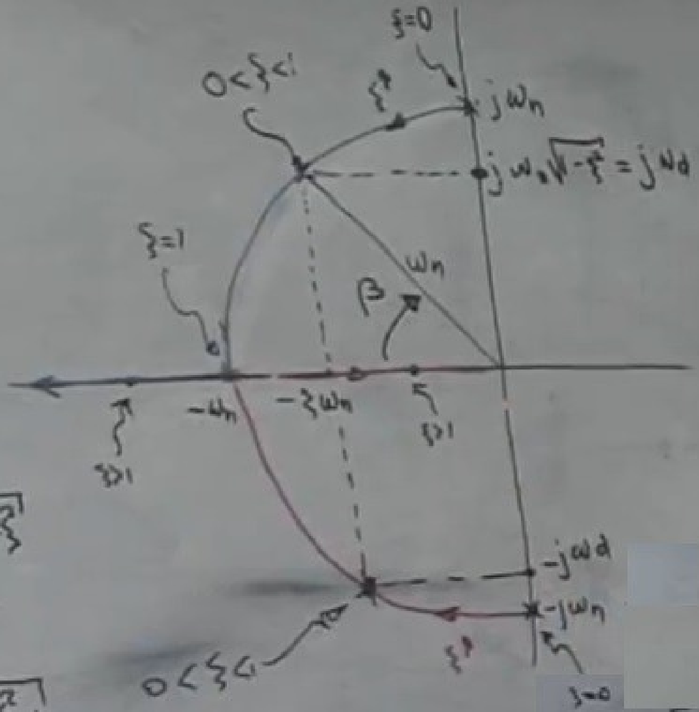
SISTEMA de 2º ORDEN – Respuesta temporal

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

polos $\quad p_1, p_2 = \dfrac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$

$$\boxed{p_1, p_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$$

	sistema	
$\xi = 0$	no amort.	$p_1, p_2 = \pm j\omega_n$
$0 < \xi < 1$	sub-amort.	$p_1, p_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$
$\xi = 1$	crític. amort.	$p_1, p_2 = -\omega_n$
$\xi > 1$	sobre amort.	$p_1, p_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$



$$\beta = \operatorname{arctg}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

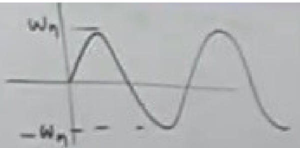
ξ : relac. de amort.

$\xi\omega_n$: factor de amort.

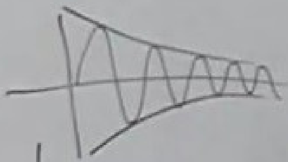
ω_n : frec. natural

$\omega_d = \omega_n\sqrt{1-\xi^2}$: frec. real o natural amortiguada

$\xi = 0$



$0 < \xi < 1$

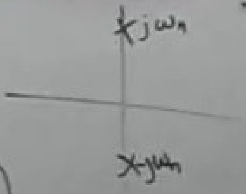


$\xi = 1$

$\xi > 1$

$$y(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

$$\propto e^{\sigma t} \operatorname{sen}(\quad + \varphi)$$



$$\underbrace{A_1 e^{pt} + A_2 t e^{pt}}_{\omega_n^2 t e^{-\omega_n t}}$$

$$\frac{\omega_n}{\sqrt{1-\xi^2}} \cdot e^{-\xi \omega_n t} \operatorname{sen}(\underbrace{\omega_n \sqrt{1-\xi^2}}_{\omega_d}\, t)$$

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

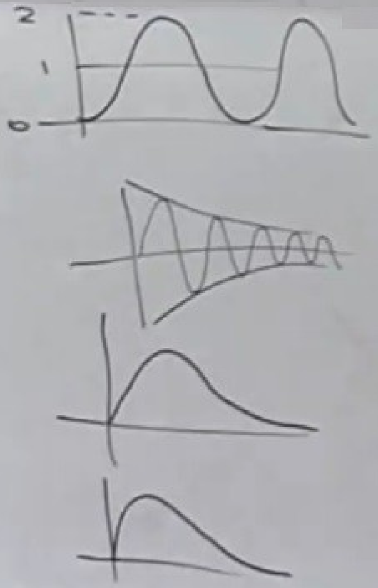
$$= A_1 Y_{1(t)} + A_2 e^{p_1 t} + A_3 e^{p_2 t}$$

$\xi = 0$

$0 < \xi < 1$

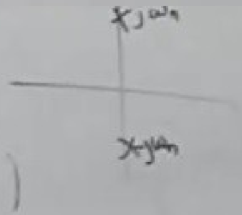
$\xi = 1$

$\xi > 1$



$$y(t) = 1 - \cos(\omega_n t)$$

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \operatorname{sen}\left(\overbrace{\omega_n\sqrt{1-\xi^2}}^{\omega d}\, t + \overbrace{\operatorname{arctg}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]}^{\beta}\right)$$



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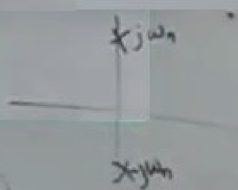
$$Y(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$= A_1 Y(t) + A_2 e^{p_1 t} + A_3 e^{p_2 t}$$

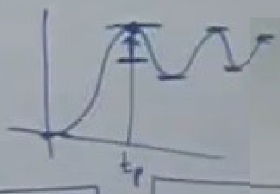
$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot sen\Big(\underbrace{\omega_n\sqrt{1-\xi^2}}_{\omega_d}\, t + \underbrace{arctg\Big[\frac{\sqrt{1-\xi^2}}{\xi}\Big]}_{\beta}\Big)$$

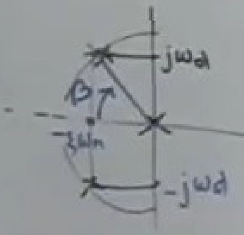


$\omega_d = \omega_n\sqrt{1-\xi^2}$

Cálculo de t_p, M_p

$y'(t^*) = 0 \qquad y'(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\, sen\big(\underbrace{\omega_n\sqrt{1-\xi^2}}_{\omega_d}\, t\big) = 0$ em t^*



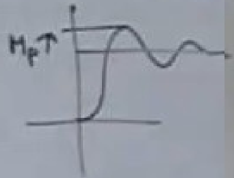


$y'(t) = 0$

$l = 0, 1, 2, \dots \Rightarrow \boxed{t^* = \frac{\pi}{\omega_d}} \qquad \boxed{t^* = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$y(t^*) = 1 + M_p = 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot sen\Big(\omega_n\sqrt{1-\xi^2}\,\frac{\pi}{\omega_n\sqrt{1-\xi^2}} + \beta\Big) =$$



$= A_1 Y_{(t)} + A_2 e^{p_1 t} + A_3 e^{p_2 t}$

$$\Rightarrow M_p = -\frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot sen(\pi + \beta) = \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot \underbrace{sen(\beta)}_{\sqrt{1-\xi^2}} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\boxed{M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$

$\beta = arctg\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \qquad \sqrt{1-\xi^2} = \frac{\omega}{\omega_n}$

$sen\,\beta = \sqrt{1-\xi^2}$

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \operatorname{sen}\Big(\underbrace{\omega_n\sqrt{1-\xi^2}}_{\omega_d}\, t + \underbrace{\operatorname{arctg}\Big[\frac{\sqrt{1-\xi^2}}{\xi}\Big]}_{\beta}\Big)$$

Cálculo de t_p, M_p

$$y'(t^*) = 0 \qquad y'(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \operatorname{sen}\big(\underbrace{\omega_n\sqrt{1-\xi^2}}_{\omega_d}\, t\big) = 0 \text{ en } t^*$$

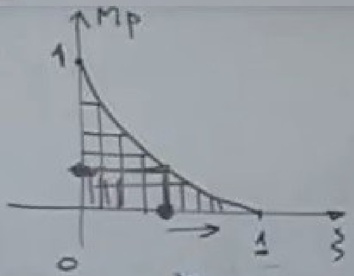
$$y'(t) = 0$$

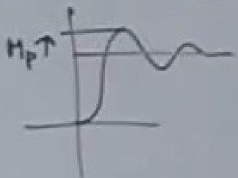
$$l = 0, 1, 2, \dots \Rightarrow \boxed{t^* = \frac{\pi}{\omega_d}} \qquad \boxed{t^* = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

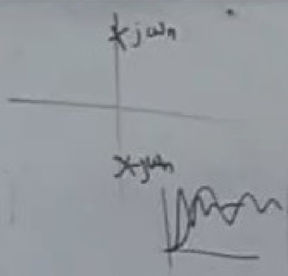
$$y(t^*) = 1 + M_p = 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot \operatorname{sen}\Big(\omega_n\sqrt{1-\xi^2}\, \frac{\pi}{\omega_n\sqrt{1-\xi^2}} + \beta\Big) =$$

$$\Rightarrow M_p = -\frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot \operatorname{sen}(\pi + \beta) = \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot \underbrace{\operatorname{sen}(\beta)}_{\sqrt{1-\xi^2}} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

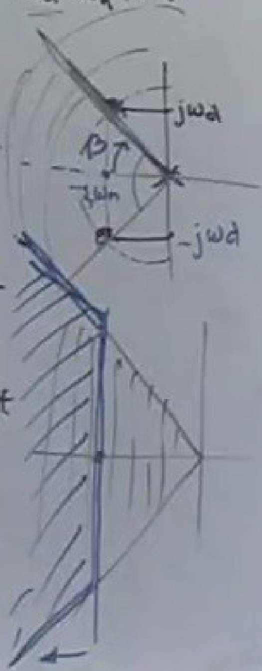
$$\boxed{M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$







$$\omega_d = \omega_n\sqrt{1-\xi^2}$$



$$Y(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$= A_1 Y_{(0)} + A_2 e^{p_1 t} + A_3 e^{p_2 t}$$

$$\beta = \operatorname{arctg}\Big(\frac{\sqrt{1-\xi^2}}{\xi}\Big)$$

$$\operatorname{sen}\beta = \sqrt{1-\xi^2}$$