Curso

SISTEMAS Y CONTROL

Clase 15

Fotogramas de los pizarrones de clases filmadas

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Clase 15 –

- Respuesta Temporal 2 (Respuesta temporal del Sistema de Parámetros Concentrados a entradas específicas)

  - Sistema de orden1 (continuación).
  - Caracterización de la respuesta a escalón.
  - Ejemplo de diseño de motor controlado

- Linealización de Sistemas no-Lineales

Esta clase hace uso de las transparencias “sistemas_1er_orden.pdf”

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + D\,u(t) \end{cases}$$

$$\begin{cases} x(t_0) = x_0 \\ u(t) \end{cases} \quad \text{conocidos}$$

$Y(s) = H(s)\,U(s)$ H: Matriz de transferencia.

con $\boxed{H(s) = C(sI - A)^{-1}B + D}$

$$H(s) = \frac{a_n \prod_{i=1}^{m_n} (s - z_i)}{\prod_{j=1}^{n_g} (s - p_j)}$$

$$U(s) = \frac{a_u \prod_{i=1}^{m_u} (s - z_i')}{\prod_{j=1}^{n_u} (s - p_j')}$$

$$Y(s) = \frac{a \prod_{i=1}^{m} (s - z_i)}{\prod_{j=1}^{n} (s - p_j)} = H(s)U(s) = A + \underbrace{\frac{A_{11}}{s - p_1} + \frac{A_{12}}{(s-p_1)^2} + \dots + \frac{A_{1\mu_1}}{(s-p_1)^{\mu_1}}}_{\mu_1} + \dots$$

$$y(t) = \sum_{i=1}^{r} \sum_{j=0}^{\mu_{i-1}} \alpha_{ij} e^{\sigma_i t} (\operatorname{sen} \omega_i t + \varphi_{ij}) + A\delta_t$$

SISTEMA de 1^er ORDEN

$$\boxed{H(s) = \frac{1}{Ts+1}}$$

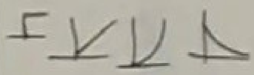
$$Y(s) = \frac{1}{Ts+1} \cdot U(s) =$$

$u(t)$	$U(s)$	$Y(s) \rightarrow y(t)$
δ	1	$\frac{1}{Ts+1}$
Y_t	$\frac{1}{s}$	$\frac{1}{s}\frac{1}{Ts+1}$
tY_t	$\frac{1}{s^2}$	$\frac{1}{s^2}\cdot\frac{1}{Ts+1}$
$t^2 Y_t$	$\frac{2!}{s^3}$	$\frac{2!}{s^3}\cdot\frac{1}{Ts+1}$

u   y

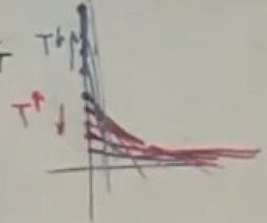
$\frac{1}{s}$ $\frac{1}{s+1/T}$

$\underline{e^{0t} Y_t}$ $\underbrace{e^{-t/T} Y_t}$

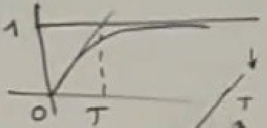


$\frac{1}{s}$ $\frac{1}{s^2}$ $\frac{1}{s^3}$ $\frac{1}{s+1/T}$

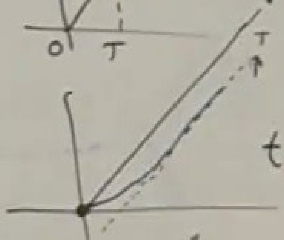
$$\frac{1}{T} e^{-t/T}$$

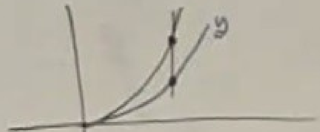


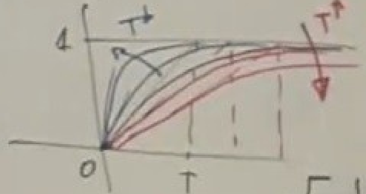
$$1 - e^{-t/T}$$

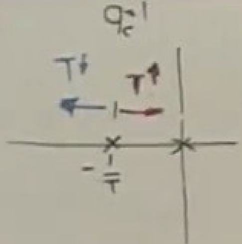


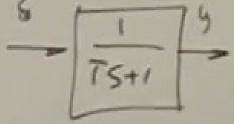
$$t + T\left(e^{-t/T} - 1\right)$$

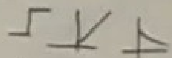




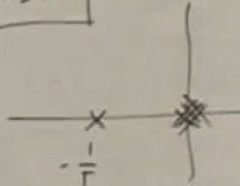




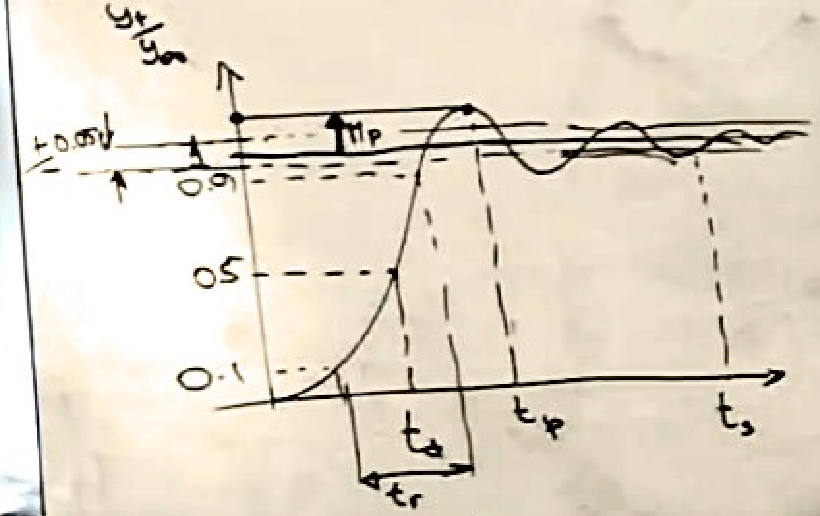




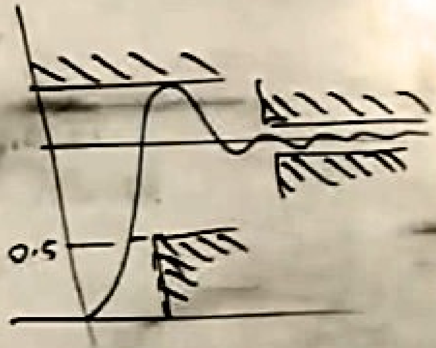
$\frac{1}{s}$ $\frac{1}{s^2}$ $\frac{1}{s+1/T}$



Caracterización de la resp. a escalón



$\frac{t}{T} \geq 2.996 \simeq 3$



t_d – retardo (delay)

t_r – t_l t. de levant. (rise-time)

t_s – t. de asent. (settling time)

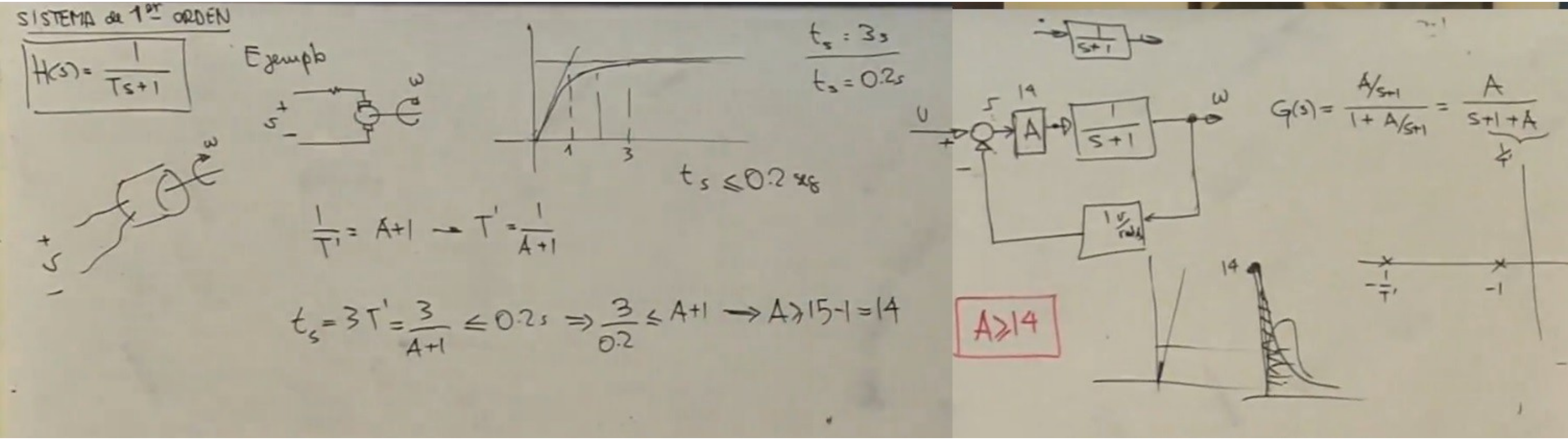
t_p – t. de pico (peak-time)

Sis 1er or. $1 - e^{-t/T} \geq 0.95$

$\Rightarrow e^{-t/T} \leq 0.05 \rightarrow \boxed{t \geq 3T}$

Ejemplo, control de un motor eléctrico de corriente contínua.

Objetivo: disminuir el tiempo de levantamiento



Linealización de Sistemas no-lineales

LINEALIZACIÓN de SIST. NO-LINEALES

Sea

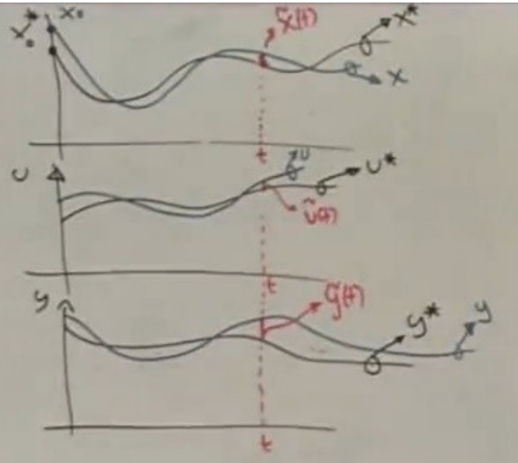
$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \end{cases} \qquad \left.\begin{array}{l} u(t) \in \mathbb{R}^r \\ y(t) \in \mathbb{R}^m \\ x(t) \in \mathbb{R}^n \end{array}\right\} \forall\, t \geq t_0$$

f, g no lineales en x, u

$$\begin{cases} x(t_0) = x_0 \\ u(t) \text{ conocida } \forall t \geq t_0 \end{cases}$$

Supongamos que conocemos una solución:

$$\text{Si } \begin{cases} x(t_0) = x_0^* \\ u^*(t) \end{cases} \Rightarrow \begin{cases} \dot{x}^*(t) = f(x^*(t), u^*(t), t) \\ y^*(t) = g(x^*(t), u^*(t), t) \end{cases}$$



aplicamos $\begin{cases} x_0 \\ u(t) \end{cases}$

$$\begin{aligned} x(t_0) &= x_0^* + \tilde{x}_0 \\ u(t) &= u^*(t) + \tilde{u}(t) \\ x(t) &= x^*(t) + \tilde{x}(t) \\ y(t) &= y^*(t) + \tilde{y}(t) \end{aligned}$$

$\to$ se obtienen $x(t), y(t)$ $\Rightarrow \begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \\ x(t_0) = x_0 \end{cases}$

$$\dot{x}(t) = f(x(t), u(t), t) = f(x(t), u(t), t)\Big|_{\substack{x = x^* \\ u = u^*}} + \frac{\partial f}{\partial x}\Big|_{\substack{x = x^* \\ u = u^*}} \cdot [x(t) - x^*(t)] + \frac{\partial f}{\partial u}\Big|_{\substack{x = x^* \\ u = u^*}} [u(t) - u^*(t)] + \ldots$$

$$y(t) = g(x(t), u(t), t) = g(x(t), u(t), t)\Big|_{\substack{x = x^* \\ u = u^*}} + \frac{\partial g}{\partial x}\Big|_{\substack{x = x^* \\ u = u^*}} \cdot [x(t) - x^*(t)] + \frac{\partial g}{\partial u}\Big|_{\substack{x = x^* \\ u = u^*}} [u(t) - u^*(t)] + \ldots$$

$$\Rightarrow \begin{cases} \dot{\tilde{x}}(t) \simeq \left.\dfrac{\partial f}{\partial x}\right|_{\substack{x=x^* \\ u=u^*}} \tilde{x}(t) + \left.\dfrac{\partial f}{\partial u}\right|_{\substack{x=x^* \\ u=u^*}} \tilde{u}(t) \\ \tilde{y}(t) = \left.\dfrac{\partial g}{\partial x}\right|_{\substack{x=x^* \\ u=u^*}} \tilde{x}(t) + \left.\dfrac{\partial g}{\partial u}\right|_{\substack{x=x^* \\ u=u^*}} \tilde{u}(t) \end{cases}$$

$\tilde{u}(t)$ conocida

$\tilde{x}_0 = x_0 - x^*_0$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$y(t) = y^*(t) + \tilde{y}(t)$

$x(t) = x^*(t) + \tilde{x}(t)$