Curso

SISTEMAS Y CONTROL

Clase 14

Fotogramas de los pizarrones de clases filmadas

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Clase 14 –

- Respuesta del Sistema de Parámetros Concentrados (– resumen).

- Respuesta Temporal 1 (Respuesta temporal del Sistema de Parámetros Concentrados a entradas específicas)
  - Expresión general
  - Sistema de orden1

Esta clase hace uso de las transparencias “sistemas_1er_orden.pdf” [redacted]SIST. DIN. de PARÁMETROS CONCENTRADOS. LINEAL — $Y_\sigma(t)$ — $Y_\sigma(s)$

[redacted]SIST. — SOL GRAL.

$$\begin{cases}\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)\\ y(t) = C(t)\,x(t) + D(t)\,u(t)\end{cases}$$

$$x(t) = \phi(t,t_0)x_0 + \int_{t_0}^{t} \phi(t,\sigma)B(\sigma)u(\sigma)\,d\sigma$$

$$y(t) = C(t)\phi(t,t_0)x_0 + \int_{t_0}^{t} \left[C(t)\phi(t,\sigma)B(\sigma) + \delta_{t-\sigma}D(\sigma)\right]u(\sigma)\,d\sigma$$

$$y_\sigma(t) = \int_{t_0}^{t} \mathcal{H}(t,\sigma)\,u(\sigma)\,d\sigma$$

con $\mathcal{H}(t,\sigma) = \left[C(t)\phi(t,\sigma)B(\sigma) + \delta_{t-\sigma}D(\sigma)\right]$

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

$$x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-\sigma)}B\,u(\sigma)\,d\sigma$$

$$y(t) = Ce^{A(t-t_0)}x_0 + \underbrace{\int_{t_0}^{t}\left[Ce^{A(t-\sigma)}B + \delta_{t-\sigma}D\right]u(\sigma)\,d\sigma}_{y_\sigma(t)}$$

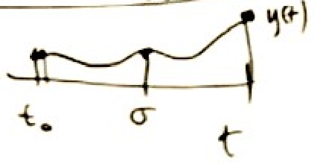
$$\boxed{y_\sigma(t) = \int_{t_0}^{t} \mathcal{H}(t-\sigma)\,u(\sigma)\,d\sigma}$$

con $\underset{2}{\mathcal{H}(t-\sigma)} = \left[Ce^{A(t-\sigma)}B + \delta_{t-\sigma}D\right]$

$$\boxed{Y_\sigma(s) = H(s)\,U(s)}$$

con $H(s) = \left[C(sI-A)^{-1}B + D\right]$

$$H(s) = \mathcal{L}\{\mathcal{H}(t)\}$$

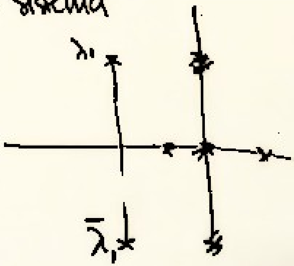


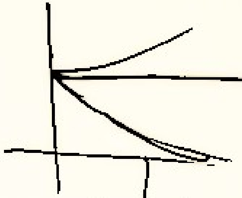
(1) $\mathcal{L}\{e^{At}\} = (sI - A)^{-1}$ $\quad d(s) = \det(sI - A)$

$$\mathcal{L}\{e^{At}\} = \left\{ \frac{n_{kl}(s)}{d(s)} \right\} \Rightarrow e^{At} = \left\{ \sum_{i=1}^{r} \sum_{j=0}^{\mu_i - 1} \alpha_{ij}^{kl}\, t^{j} e^{\sigma_i t} \left(\operatorname{sen} \omega_i t + \varphi_{ij}^{kl}\right) \right\}$$

$d(\lambda_i) = 0$ $\quad \lambda_i$, $i = 1..n$ autovalores de A, polos del sistema

$\lambda_i = \sigma_i + j\omega_i$



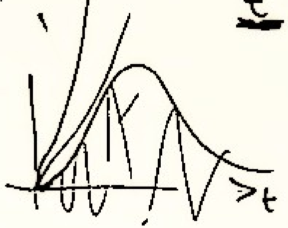


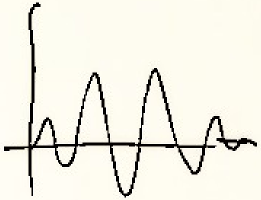
$e^{\sigma_i t}$

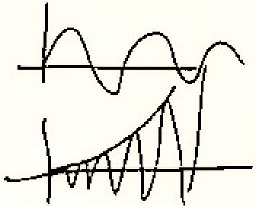
$t e^{\sigma_i t}$

$t^2 e^{\sigma_i t}$

$\underline{t^{\mu_i - 1}}\ \underline{e^{\sigma_i t}}$



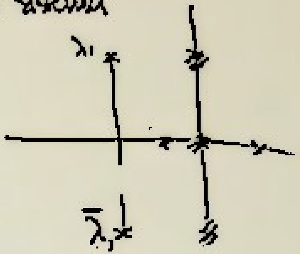




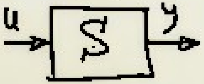
① $\mathcal{L}\{e^{At}\} = (sI-A)^{-1}$ $\quad d(s) = \det(sI-A)$ $\quad d(\lambda_i) = 0$ $\quad \lambda_i$, $i=1..n$ autovalores de A, polos del sistema

$\lambda_i = \sigma_i + j\omega_i$

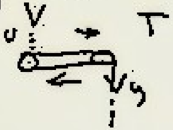
$$\mathcal{L}\{e^{At}\} = \left\{ \frac{n_{ij}(s)}{d(s)} \right\} \Rightarrow e^{At} = \left\{ \sum_{i=1}^{r} \sum_{j=0}^{\mu_i - 1} \alpha_{ij}^{kl} t^j e^{\sigma_i t} (\operatorname{sen} \omega_i t + \varphi_{ij}^{kl}) \right\}$$



② $y_\sigma = \int_{t_0}^{t} \mathcal{H}(t-\sigma)\, u(\sigma)\, d\sigma$



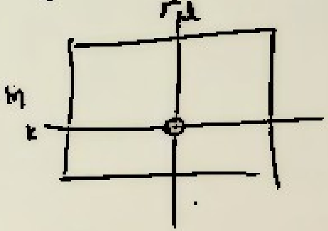
p.ej.: Retardo $\quad y(t) = u(t-T)$



$h(t) = \delta_{t-T}$

$$\boxed{H(s) = e^{sT}}$$

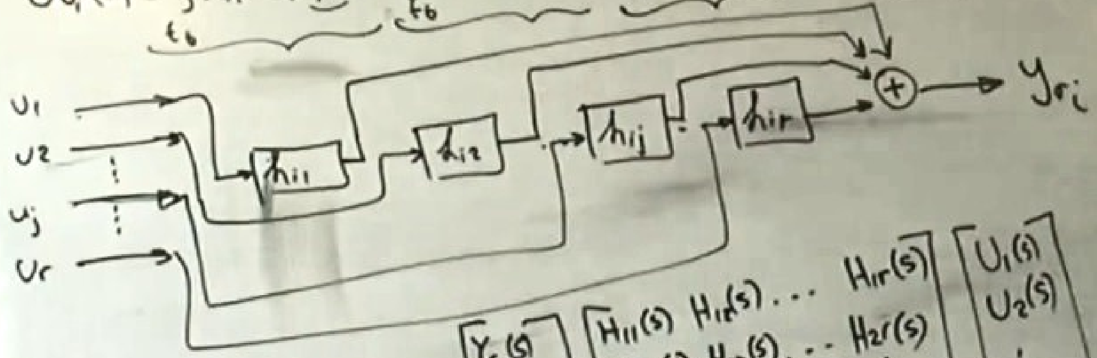
$$H(s) = \left\{ \frac{n'_{kl}(s)}{d(s)} \right\}$$



③ ¿ $y_\sigma(t)$?

$$y_\sigma(t)=\begin{bmatrix} y_{\sigma_1}(t)\\ y_{\sigma_2}(t)\\ \vdots\\ y_{\sigma_i}(t)\\ \vdots\\ y_{\sigma_m}(t)\end{bmatrix}=\int_{t_0}^{t}\overbrace{\begin{array}{c|c} & \begin{array}{cccccc} & & & j & & \end{array}\\ \begin{array}{c} \\ \\ i \\ \\ \end{array} & \begin{bmatrix} h_{11}(t-\sigma) & h_{12}(t-\sigma) & \cdots & \vdots & \cdots & h_{1r}(t-\sigma)\\ h_{21}(t-\sigma) & h_{22}(t-\sigma) & \cdots & \vdots & \cdots & h_{2r}(t-\sigma)\\ \boxed{h_{i1}(t-\sigma)} & h_{i2}(t-\sigma) & \cdots & h_{ij}(t-\sigma) & \cdots & h_{ir}(t-\sigma)\\ \vdots & & & \vdots & & h_{mr}(t-\sigma)\\ h_{m1}(t-\sigma) & h_{m2}(t-\sigma) & \cdots & \vdots & \cdots & \end{bmatrix}\end{array}}^{H(t-\sigma)}\begin{bmatrix}u_1(\sigma)\\ u_2(\sigma)\\ \vdots\\ u_j(\sigma)\\ \vdots\\ u_r(\sigma)\end{bmatrix}d\sigma$$

$$y_{\sigma_i}(t)=\underbrace{\int_{t_0}^{t} h_{i1}(t-\sigma)u_1(\sigma)\,d\sigma}+\underbrace{\int_{t_0}^{t} h_{i2}(t-\sigma)u_2(\sigma)\,d\sigma}+\cdots+\underbrace{\int_{t_0}^{t} h_{ij}(t-\sigma)u_j(\sigma)\,d\sigma}+\cdots+\underbrace{\int_{t_0}^{t} h_{ir}(t-\sigma)u_r(\sigma)\,d\sigma}$$



$$Y_\sigma(s)=H(s)U(s)$$

$$\underbrace{\begin{bmatrix} Y_{\sigma_1}(s)\\ Y_{\sigma_2}(s)\\ \vdots\\ \vdots\\ Y_{\sigma_m}(s)\end{bmatrix}}_{Y_\sigma(s)}=\begin{bmatrix} H_{11}(s) & H_{12}(s) & \cdots & H_{1r}(s)\\ H_{21}(s) & H_{22}(s) & \cdots & H_{2r}(s)\\ \vdots & & & \vdots\\ \vdots & & & H_{mr}(s)\\ H_{m1}(s) & H_{m2}(s) & \cdots & \end{bmatrix}\underbrace{\begin{bmatrix}U_1(s)\\ U_2(s)\\ \vdots\\ \vdots\\ U_r(s)\end{bmatrix}}_{U(s)}$$