Curso

# SISTEMAS Y CONTROL

# Clase 08

## Fotogramas de los pizarrones de clases filmadas

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# MODELADO

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) \\ y(t) = g(x(t), u(t), t) \end{cases}$$

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$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

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$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

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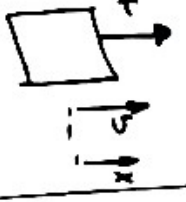
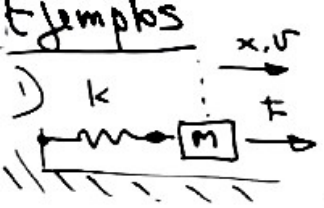
# FEN. MECÁNICOS de TRASLACIÓN

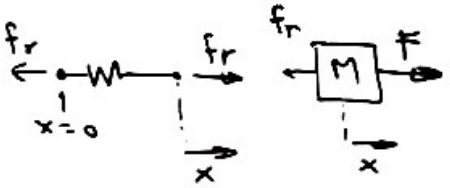
F, v
F, x

| Nombre | Parám. | Símbolo | Leg |
|---|---|---|---|
| INERCIA (Masa) | M (masa) |  | $F = M\frac{dv}{dt}$ <br> $F = M\dot{x}$ |
| ELASTICIDAD (Resorte) | K |  | $\frac{dF}{dt} = K(\underbrace{v_2 - v_1}_{v})$ <br> $F = K(\underbrace{x_2 - x_1}_{x})$ |
| FRICCIÓN | B |  | $F = B(\underbrace{v_2 - v_1}_{v})$ <br> $F = B(\underbrace{\dot{x}_2 - \dot{x}_1}_{\dot{x}})$ |

# Ejemplos

1)



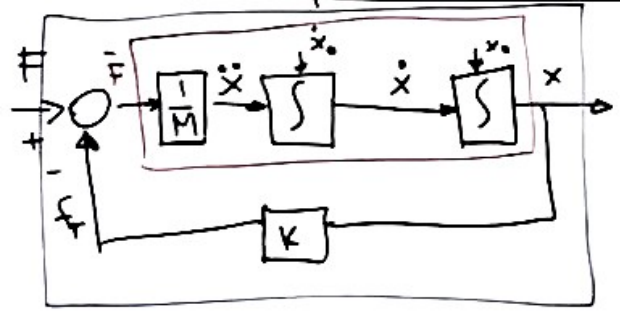





$$f_r = kx$$

$$M\ddot{x} = (F - f_r) = F - kx$$

$$\ddot{x} = -\frac{k}{M}x + \frac{1}{M}F$$

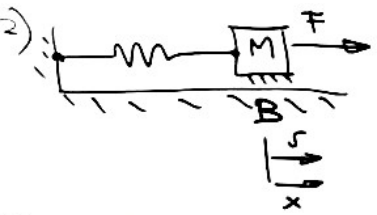



$$z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

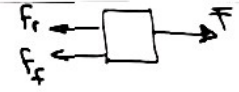
$$\dot{z}(t) = \begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}}_{B} \underbrace{F(t)}_{u} \qquad \dot{z} = Az + Bu$$

$\dot{z}$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} \underbrace{F}_{u}$$



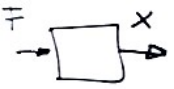

$F - f_r - f_f$

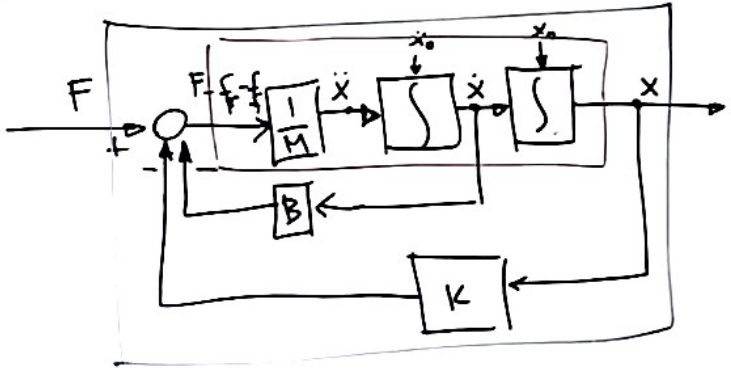



$$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$

$$M\ddot{x} = F - f_r - f_f = F - kx - B\dot{x}$$

$$\ddot{x} = \frac{1}{M}F - \frac{k}{M}x - \frac{B}{M}\dot{x}$$

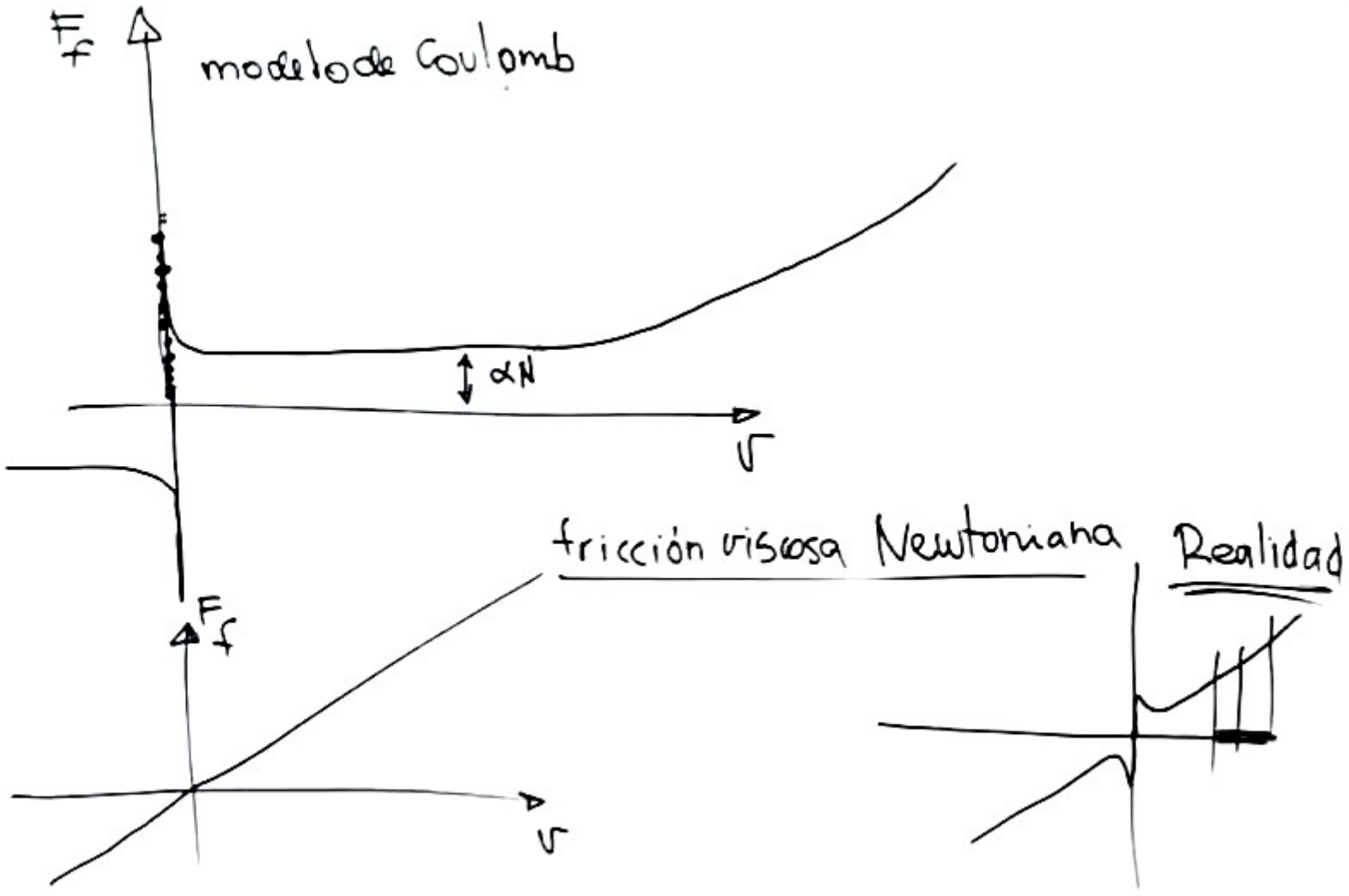



$$\dot{z}(t) = \begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{M} & -\frac{B}{M} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}}_{B} \underbrace{F(t)}_{u}$$

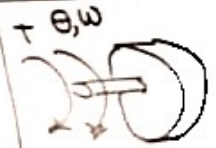
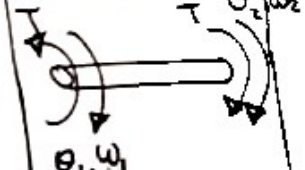
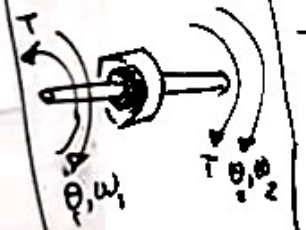
$$\begin{cases} \dot{z} = Az + Bu \\ y = Cz + Du \end{cases}$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} \underbrace{F(t)}_{u}$$


$F_f$
modelo de Coulomb
$\alpha N$
v
$F_f$
v
fricción viscosa Newtoniana
Realidad

# FEN. MECÁNICOS de ROTACIÓN

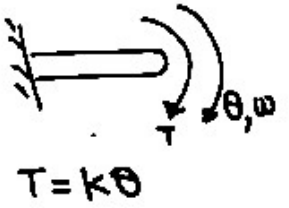
T, ω
T, θ

| Nombre | Parám. | Símbolo | Ley |
|---|---|---|---|
| INERCIA (momento de inercia) | J |  | $J\frac{d\omega}{dt} = T$ <br> $J\ddot{\theta} = T$ |
| ELASTICIDAD (Resorte de Torsión) | K |  | $\frac{dT}{dt} = k(\omega_2 - \omega_1)$ <br> $T = k(\theta_2 - \theta_1)$ |
| FRICCIÓN | B |   | $T = B(\omega_2 - \omega_1)$ <br> $T = B(\dot{\theta}_2 - \dot{\theta}_1)$ |

Leyes de conjunto:

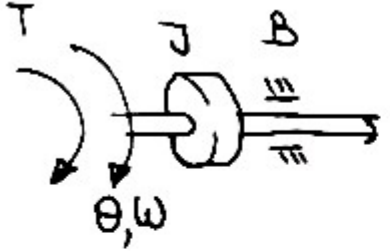
$$\sum_{nodo} F_i = 0$$

$$\sum_{malla} x_{ij} = 0$$



$T = k\theta$

$\boxed{T = B\omega}$




$$J\ddot{\theta} = (T - B\dot{\theta})$$

$$\ddot{\theta} = \frac{1}{J}T - \frac{B}{J}\dot{\theta}$$

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} T$$