Curso

# SISTEMAS Y CONTROL

# Clase 05

## Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
Año 2021

## CAUSALIDAD

Si $u^1_{[t_c,\infty)} = u_{[t_c,t]} \dot{+} r^1_{(t,\infty)}$

$u^2_{[t_c,\infty)} = u_{[t_c,t]} \dot{+} r^2_{(t,\infty)}$

$y^1_{[t_c,\infty)} = S\left[u^1_{[t_c,\infty)}\right]$

$y^2_{[t_c,\infty)} = S\left[u^2_{[t_c,\infty)}\right]$

Si $\forall t \geq t_c$

$\forall u, r^1, r^2 / u^1, u^2 \in \mathcal{U}$

$\Rightarrow \boxed{y^1_{[t_c,t]} = y^2_{[t_c,t]}}$

S es causal.

$\Rightarrow \exists \bar{S} / \; y_{[t_c,t]} = \bar{S}\left[u_{[t_c,t]}\right]$

## EQUIVALENCIA de ENTRADAS

Sea S causal

Si $\begin{cases} \bar{u}^1_{[t_c,t]} = u^1_{[t_c,t_0]} \dot{+} r_{(t_0,t)} \\ \bar{u}^2_{[t_c,t]} = u^2_{[t_c,t_0]} \dot{+} r_{(t_0,t)} \end{cases}$

$\bar{y}^1_{[t_c,t]} = \bar{S}\left[\bar{u}^1_{[t_c,t]}\right]$

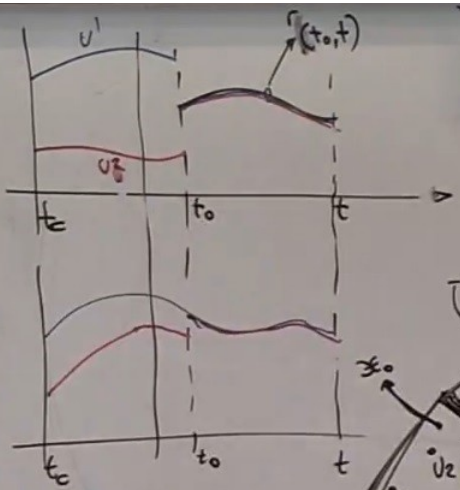
$\bar{y}^2_{[t_c,t]} = \bar{S}\left[\bar{u}^2_{[t_c,t]}\right]$

Si $\forall \begin{cases} t > t_0 \\ r / \bar{u}^1, \bar{u}^2 \in \mathcal{U}_{[t_c,t]} \end{cases}$

$\Rightarrow \bar{y}^1_{[t_0,t]} = \bar{y}^2_{[t_0,t]}$ entonces $u^1 \sim u^2$ para S en $t_0$

a) ¿es una rel. de eq?
- idéntica – $u \sim u$
- simétrica – si $u^1 \sim u^2 \Rightarrow u^2 \sim u^1$
- transitiva – si ($u^1 \sim u^2$ y $u^2 \sim u^3$) $\Rightarrow u^1 \sim u^3$

DEF. si $X \longleftrightarrow \mathcal{X}_{t_0} \Rightarrow X$: conjunto de estados de S




$\mathcal{U}_{t_0}$


$x_{t_0}$ : estado del sistema S en $t_0$

$\mathcal{X}_{t_0} = \{ x_{t_0} \}$, conjunto de estados posibles de S en $t_0$

# Sistemas

dados $T$, $\mathcal{C}_u$, $\mathcal{C}_y$

$u:[t_c,\infty)\to\mathcal{C}_u$

$y:[t_c,\infty)\to\mathcal{C}_y$

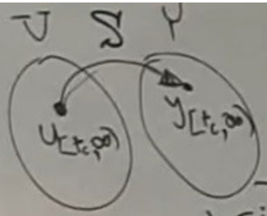
$U=\{u_{[t_c,\infty)}\}$

$Y=\{y_{[t_c,\infty)}\}$

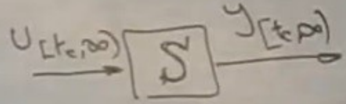
Sistema: $U, Y, S$

$$y_{[t_c,\infty)}=S\left[u_{[t_c,\infty)}\right]$$

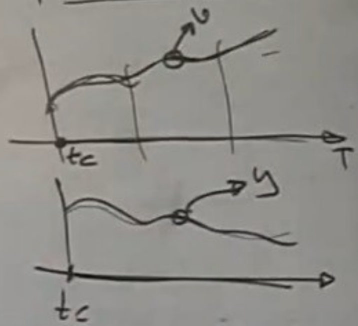
$$S: U\to Y$$

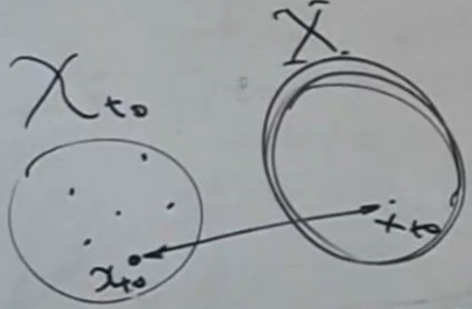



$$y_{[t_c,\infty)}=S\left[u_{[t_c,\infty)}\right]$$

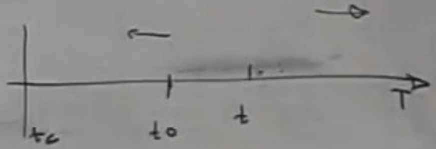



sistema determinístico.




$x_{t_0}$: estado de $S$ en $t_0$

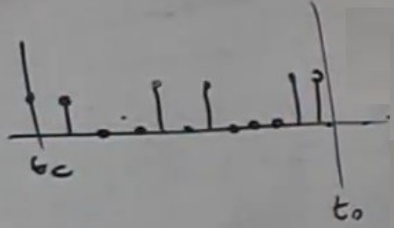



$u\in U$

$u(t)\in\mathcal{C}_u$

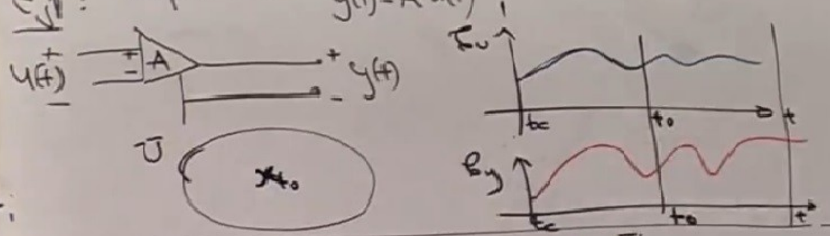
$\mathcal{C}_u=\{0,1\}$

$T=\mathbb{Z}$



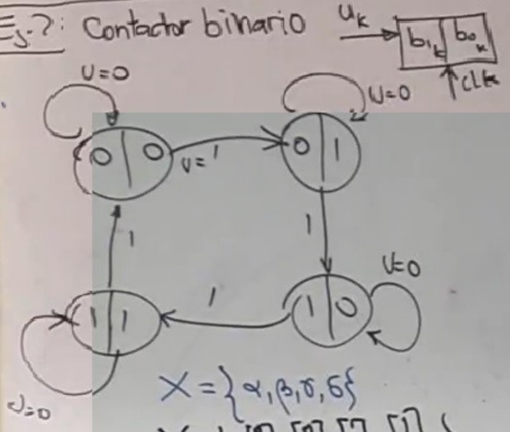
Ej. 1: Amp. Ideal

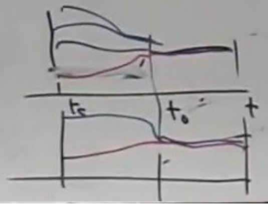
$y(t) = A\,u(t)$




Ej. 2: Contactor binario

$y_k = \begin{bmatrix} b_{1k} \\ b_{0k} \end{bmatrix}$



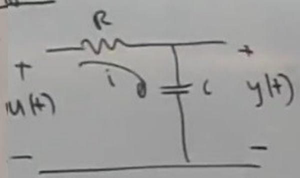




$X = \{\alpha, \beta, \gamma, \delta\}$

$X = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

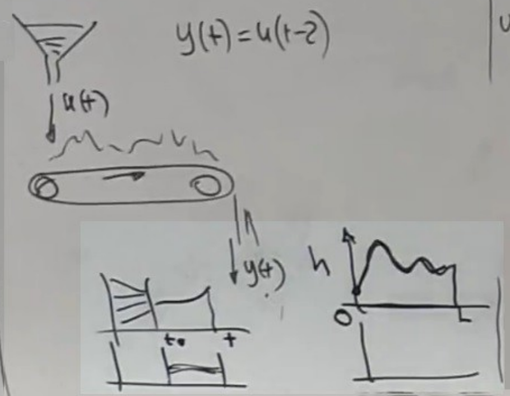
Ej. 3 Filtro RC

$\dot{y}(t) = \frac{1}{C}\left(\frac{u - y}{R}\right)$




Ej. 4 Cinta transportadora

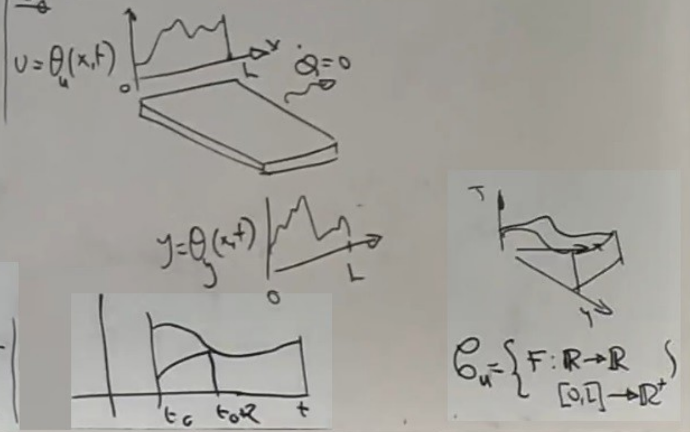
$y(t) = u(t-2)$




Ej. 5: Conducción de calor

$u = \theta_u(x,t)$ $\quad \dot{Q} = 0$

$y = \theta_y(x,t)$




$\mathcal{E}_u = \left\{ F : \mathbb{R} \to \mathbb{R},\ [0,L] \to \mathbb{R}^+ \right\}$

| Ej | | #X | |
|---|---|---|---|
| Amp | 1 | 1 | Algebraicos (sin memoria) |
| Contador | $n$ | $n$ | Autómata finito |
| Máquina de Turing | $\infty$ | $\mathbb{N}$ | Autómata infinito. |
| Filtro | $\infty$ | $\mathbb{R}^n$ | Sist. de parám. concentrados |
| Retardo | $\infty$ | espacio funcional $C$, $L_2$, $L_1$ | Sist. de parámetros distribuidos |

Sea

Si

Si t

$\Rightarrow$

a)

DEF

# Sistemas

dados $T, \mathcal{C}_u, \mathcal{C}_y$

$u:[t_c,\infty)\to\mathcal{C}_u$

$y:[t_c,\infty)\to\mathcal{C}_y$

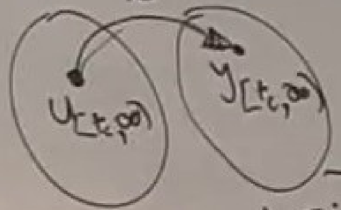
$U=\{u_{[t_c,\infty)}\}$

$Y=\{y_{[t_c,\infty)}\}$

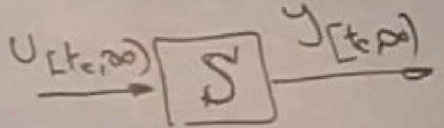
Sistema: $U, Y, S$

$y_{[t_c,\infty)} = S[u_{[t_c,\infty)}]$

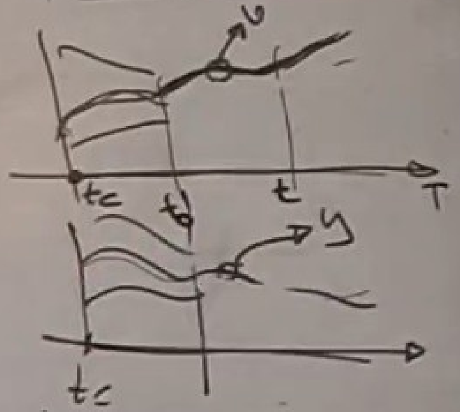
$S: U \to Y$

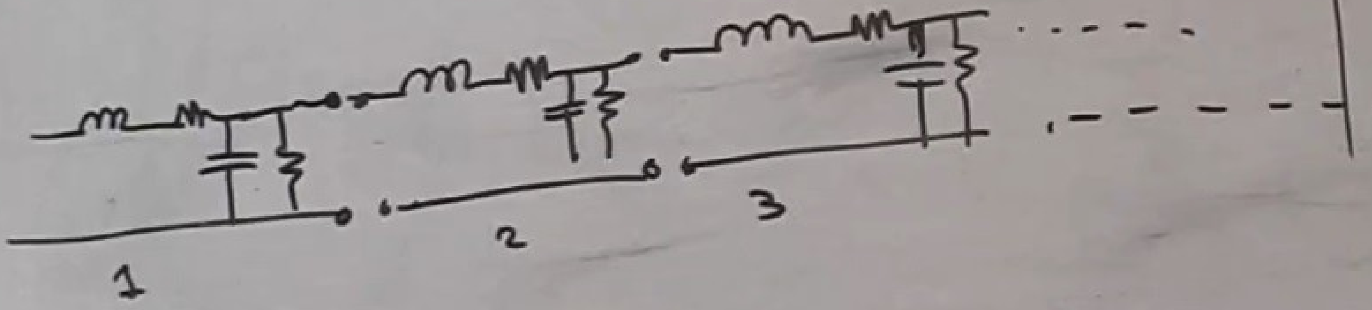



$y_{[t_c,\infty)} = S[u_{[t_c,\infty)}]$




sistema determinístico.




2n

$\mathbb{R}^{2n}$