Curso

SISTEMAS Y CONTROL

Clase 04

Fotogramas de los pizarrones de clases filmadas

Prof. Rafael Canetti

Instituto de Ingeniería Eléctrica,
Facultad de Ingeniería, Universidad de la República
Montevideo, Uruguay.
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Señales, Concatenación de Señales.

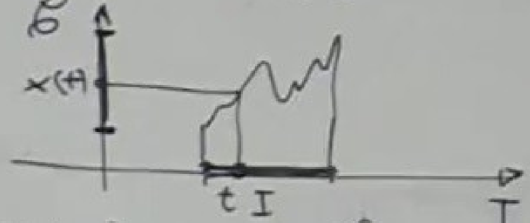
T ordenado linealmente: $\begin{cases} \mathbb{R} & \text{t. continuo} \\ \mathbb{Z}, \mathbb{N} & \text{t. discreto} \end{cases}$

$x: \mathbb{R} \to \mathbb{R}$

Señales

$x: I \to \mathcal{C}$

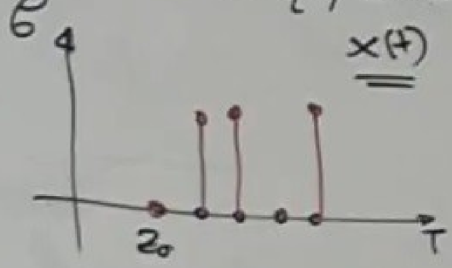
$I \subset T$



$\forall t \in I \;\; \exists \; x(t) \in \mathcal{C}$

$\mathcal{C} = \{0,1\}$

$T = \{z / z \in \mathbb{Z}, z \geq z_0\}$



x(t)

$x: \mathbb{Z} \to \{0,1\}$

Señales, Concatenación de Señales.

T ordenado linealmente: $\begin{cases} \mathbb{R} & \text{t. continuo} \\ \mathbb{Z}, \mathbb{N} & \text{t. discreto} \end{cases}$

Señales

$x : I \to \mathscr{C}$

$I \subset T$

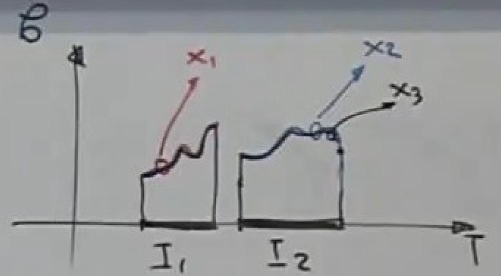
Concatenación de Señales

T ordenado linealmente $(\mathbb{R}, \mathbb{Z})$

$I_1, I_2 : \begin{cases} I_1, I_2 \subset T \\ I_1 \cap I_2 = \emptyset \end{cases}$

$\mathscr{C}$ $\quad x_1 : I_1 \to \mathscr{C}$

$x_2 : I_2 \to \mathscr{C}$



$$\boxed{x_3 = x_1 \mathring{+} x_2}$$

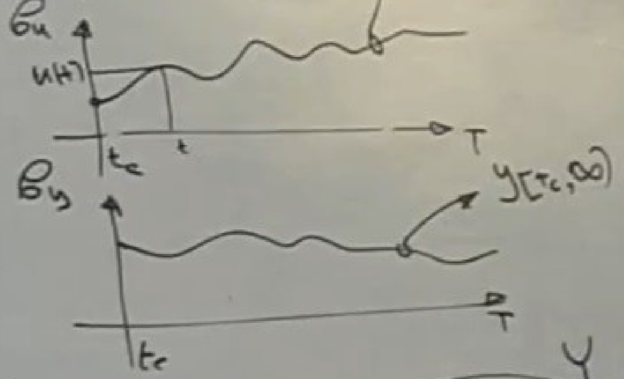
$x_3 : I \to \mathscr{C}$

$I = I_1 \cup I_2$

$$x_3(t) = \begin{cases} x_1(t) & \text{si } t \in I_1 \\ x_2(t) & \text{si } t \in I_2 \end{cases}$$

Sistema

T ordenado lin. $(\mathbb{R}, \mathbb{Z})$ $\begin{cases} \mathbb{R} & \text{tiempo continuo} \\ \mathbb{Z} & \text{tiempo discreto.} \end{cases}$



$\mathscr{C}_u, \mathscr{C}_y$

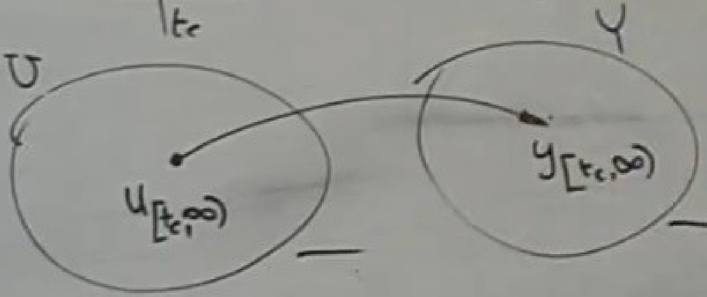
$u_{[t_c,\infty)} : [t_c, \infty) \to \mathscr{C}_u$

$[t_c, \infty) \subset T$

$y_{[t_c,\infty)} : [t_c, \infty) \to \mathscr{C}_y$

$U = \{ u_{[t_c,\infty)} \}$

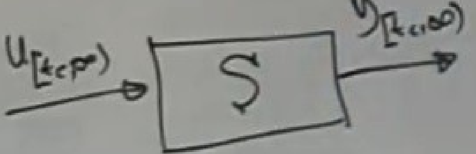
$Y = \{ y_{[t_c,\infty)} \}$



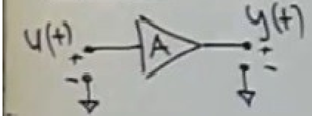
si $\exists\ S : U \to Y$

$$\boxed{y_{[t_c,\infty)} = S\left[u_{[t_c,\infty)}\right]}$$

entonces $\underline{U, Y, S}$: sistema (determinístico)

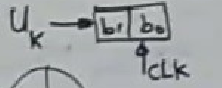


Ej. 1 Amp Ideal.

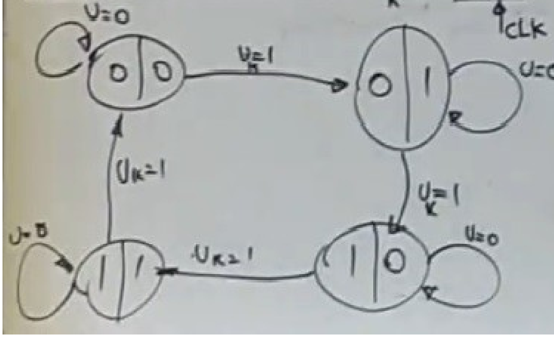


$$\boxed{y(t) = A\,u(t)}$$

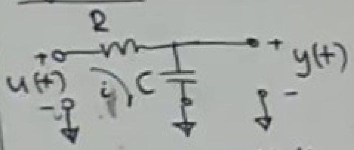
Ej 2 Contador binario



$$y_k = \begin{bmatrix} b_{1k} \\ b_{0k} \end{bmatrix}$$



Ej 3 Filtro RC

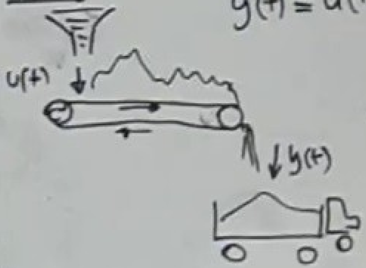


$$\left.\begin{aligned} i &= \frac{u-y}{R} \\ i &= C\frac{dy}{dt} \end{aligned}\right\} \quad \dot{y} = \frac{u-y}{RC}$$

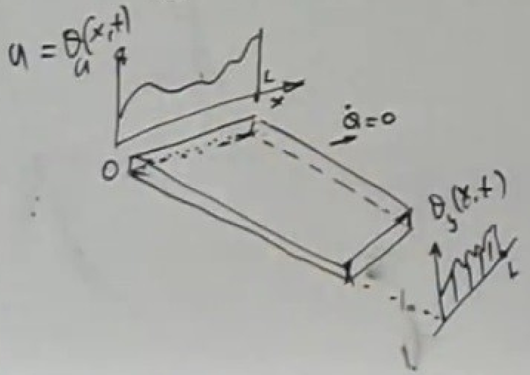
Ej. 4

4a.

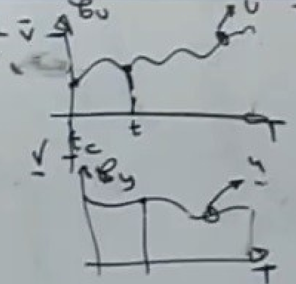
$$y(t) = u(t-\tau)$$



4b



Ej. 1



$T = \mathbb{R}, \ [t_c, \infty) \subset \mathbb{R}$

$$\mathcal{C}_u = \begin{cases} \mathbb{R} \\ [\underline{v}, \bar{v}] \subset \mathbb{R} \end{cases}$$

$$\mathcal{C}_y = \begin{cases} \mathbb{R} \\ [A\underline{v}, A\bar{v}] \subset \mathbb{R} \end{cases}$$

$u : [t_c, \infty) \to \mathbb{R}$ $\quad U = \{ u_{[t_c,\infty)} \}$ $\quad y(t) = A\,u(t)$

$y : [t_c, \infty) \to \mathbb{R}$ $\quad Y = \{ y_{[t_c,\infty)} \}$

$$\exists\, S \,/\, y_{[t_c,\infty)} = S\left[u_{[t_c,\infty)}\right]$$

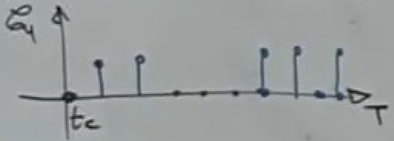
Ej. 2

$T = \{ z \,/\, z \in \mathbb{Z}, z \geq t_c \}$

$[t_c, \infty) \subset \mathbb{Z}$ $\quad \mathcal{C}_u = \{0, 1\}$

u_k

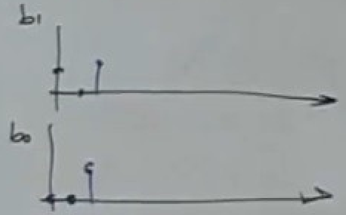
$y_k = \begin{bmatrix} b_{1k} \\ b_{0k} \end{bmatrix}$



$\mathcal{C}_y = \{ (0,0), (0,1), (1,0), (1,1) \}$

$u_{[t_c,\infty)}$

$y_{[t_c,\infty)}$



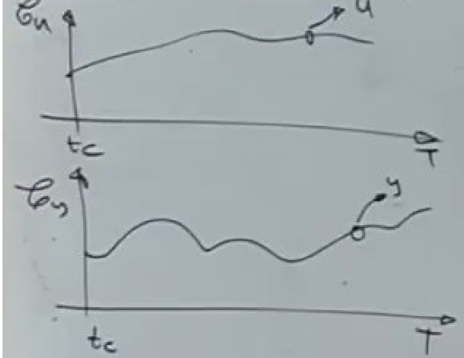
$$\begin{bmatrix} b_1 \\ b_0 \end{bmatrix}_{[t_c,\infty)} = S\left[u_{[t_c,\infty)}\right]$$

Ej 3

Filtro RC.

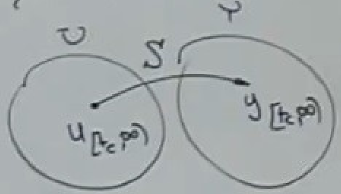
$T = \mathbb{R}$

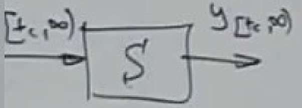
$\mathcal{C}_u = \mathbb{R}$



$\mathcal{C}_y = \mathbb{R}$

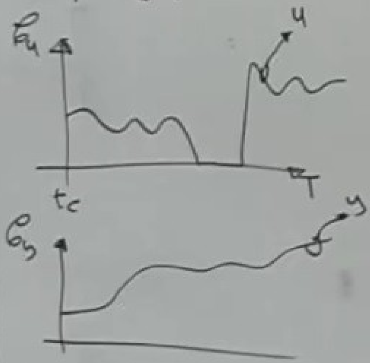
$\begin{cases} u \text{ es cont.} \\ u \text{ " secc. continua} \end{cases}$





$$\dot{y} = \frac{u}{RC} - \frac{y}{RC}$$

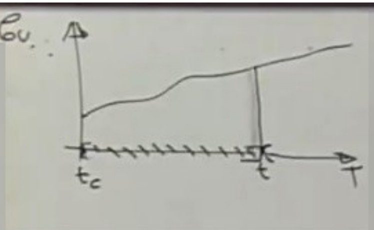
$T = \mathbb{R}$



$\mathcal{C}_u = \mathbb{R}^+$

$\mathcal{C}_y = \mathbb{R}^+$

Ej.



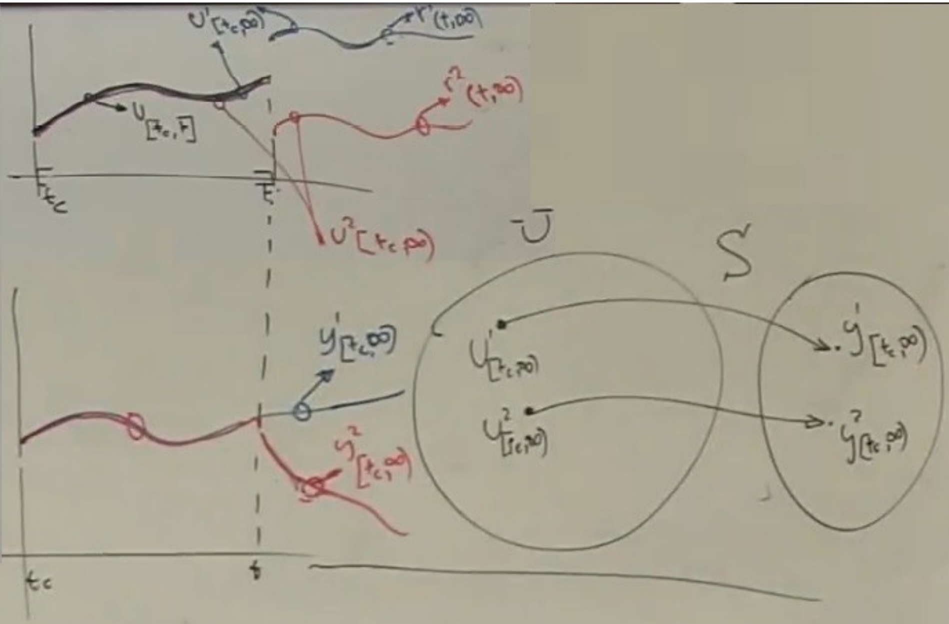
CAUSALIDAD

$$\begin{cases} u^1_{[t_c,\infty)} = u_{[t_c,t]} \dot{+} r_1(t,\infty) \\ u^2_{[t_c,\infty)} = u_{[t_c,t]} \dot{+} r_2(t,\infty) \end{cases}$$

Si $\begin{cases} \forall t > t_c \\ \forall r^1, r^2 / u^1, u^2 \in \bar{U} \end{cases} \Rightarrow \boxed{y^1_{[t_c,t]} = y^2_{[t_c,t]}}$

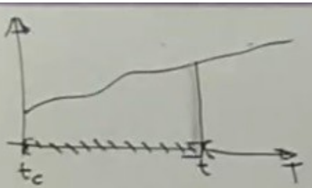
entonces S es causal

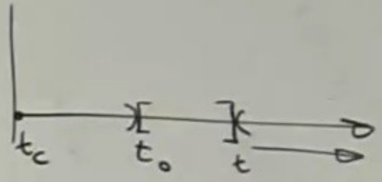
$$\begin{cases} y^1_{[t_c,\infty)} = S'[u^1_{[t_c,\infty)}] \\ y^2_{[t_c,\infty)} = S'[u^2_{[t_c,\infty)}] \end{cases}$$



Si S es causal $\Rightarrow \exists \bar{S} / \boxed{y_{[t_c,t]} = \bar{S}[u_{[t_c,t]}]}$

Gu.





Equivalencia entre entradas

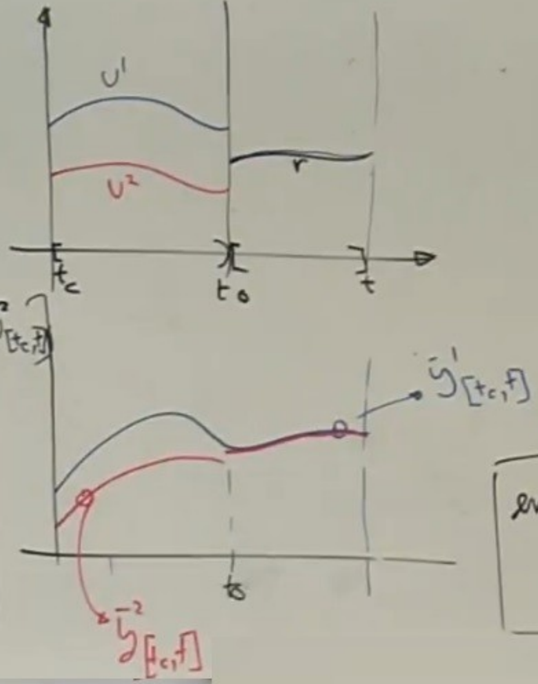
Sea S, U, Y causal $\exists\ \bar{S}$

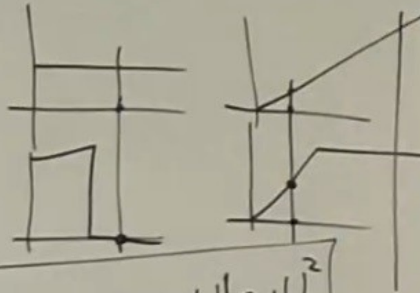
$$\begin{cases} \bar{u}^1_{[t_c,t]} = u^1_{[t_c,t_0)} + r_{[t_0,t]} \\ \bar{u}^2_{[t_c,t]} = u^2_{[t_c,t_0)} + r_{[t_0,t]} \end{cases}$$

$$\bar{y}^1_{[t_c,t]} = \bar{S}\left[\bar{u}^1_{[t_c,t]}\right] \quad / \quad \bar{y}^2_{[t_c,t]} = \bar{S}\left[\bar{u}^2_{[t_c,t]}\right]$$

Si $\forall t \geq t_0$

$\forall r \ / \ \bar{u}_1, \bar{u}_2 \in U \Rightarrow \boxed{\bar{y}^1_{[t_0,t]} = \bar{y}^2_{[t_0,t]}}$





entonces decimos que $u^1 \sim u^2$ para S en t_0