PROBLEMA 1

Examen de diciembre de 2022
Solución del problema 1

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1)
- Primera ecuación: $M\ddot{x} = f - Kx - B\dot{x}$
- Vínculo entre ambas: $x = r\cdot\theta$
- Segunda ecuación a la polea: $J\ddot{\theta} = T_m - f\cdot r$; $T_m = A\Phi\cdot i$
- Malla eléctrica del motor CC: $v = R\cdot i + A\Phi\cdot\dot{\theta}$; $e = A\Phi\cdot\dot{\theta}$

$$J\ddot{\theta} = A\Phi\left[\frac{v - A\Phi\cdot\dot{\theta}}{R}\right] - f\cdot r \Rightarrow f = \frac{A\Phi}{R\cdot r}\cdot v - \frac{(A\Phi)^2}{R\cdot r}\cdot\dot{\theta} - \frac{J}{r}\ddot{\theta}$$

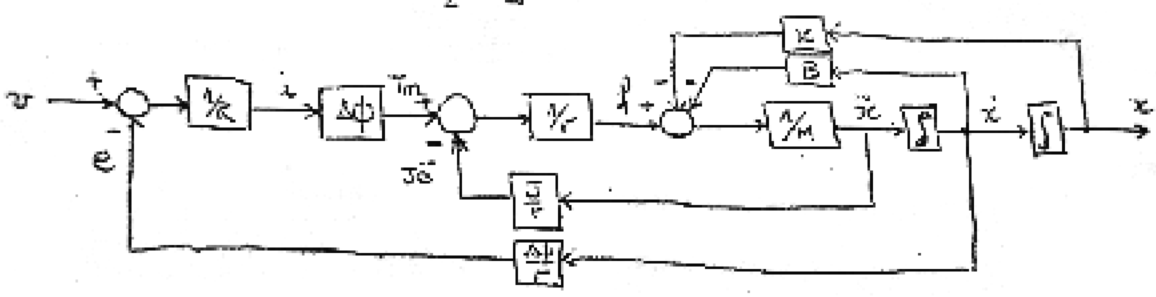
$$M\ddot{x} = \frac{A\Phi}{R\cdot r}\cdot v - \frac{(A\Phi)^2}{R\cdot r}\cdot\frac{\dot{x}}{r} - \frac{J}{r}\cdot\frac{\ddot{x}}{r} - K\cdot x - B\dot{x}$$

$$\boxed{\left(M + \frac{J}{r^2}\right)\ddot{x} = \frac{A\Phi}{R\cdot r}\cdot v - K\cdot x - \left(B + \frac{(A\Phi)^2}{R\cdot r^2}\right)\dot{x}}$$

Sean $\begin{cases} M' = M + \dfrac{J}{r^2} \\ B' = B + \dfrac{(A\Phi)^2}{R r^2} \end{cases}$

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix} -\dfrac{B'}{M'} & -\dfrac{K}{M'} \\ 1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ x\end{bmatrix} + \begin{bmatrix}\dfrac{A\Phi}{R\cdot r\cdot M'}\\ 0\end{bmatrix}[v]$$

$$[x] = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\dot{x}\\ x\end{bmatrix} + [0][v]$$



2) Transferencia $\dfrac{X}{V}(s) = \dfrac{A\Phi/R\cdot r}{M's^2 + B's + K} = \dfrac{G\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Del régimen de la resp. escalón: $G = 0{,}05$

Del sobretiro: $\zeta = \sqrt{\dfrac{\ln\left[\frac{0.00581}{0.005} - 1\right]^2}{\pi^2 + \ln\left[\frac{0.00581}{0.05} - 1\right]^2}} \rightarrow \zeta = 0.5$

Del t_R: $\omega_n = \dfrac{0.366e^{2.5} + 0.6536}{t_R} \rightarrow \omega_n = 20$

$$\Rightarrow \frac{X}{V}(s) = \frac{20}{s^2 + 20s + 400}$$

PROBLEMA 1

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3) a) $v(t) = \begin{cases} 1 & -\infty < t < 0 \\ 2 & 0 \le t < \infty \end{cases}$ Es 1 escalón unitario desde $-\infty$ más 1 escalón unitario que comienza en $t=0$

→ La salida será la superposición del régimen de la 1ª + el transitorio de la 2ª ($t>0$)

$x_1(t) = 0{,}05$

$x_2(t) = 0{,}05 \cdot \left[1 - e^{-10t}\left(\cos\sqrt{3}\,10t + \frac{1}{\sqrt{3}} \operatorname{sen}\sqrt{3}\,10t\right)\right]$ $(\xi\omega_n = 10)$ $(\sqrt{1-\xi^2}\,\omega_n = \sqrt{3}\,10)$

$\Rightarrow \boxed{x(t) = 0{,}05 \cdot \left[2 - e^{-\omega_n t}\left(\cos\sqrt{3}\,10t + \frac{1}{\sqrt{3}} \operatorname{sen}\sqrt{3}\,10t\right)\right]}$ $t>0$

b) $v(t) = 4 \cdot \operatorname{sen}(20t)$ Salida en régimen: $\boxed{x(t) = 4 \cdot \frac{0{,}05}{1} \cdot \operatorname{sen}\left(20t - \frac{\pi}{2}\right)}$

4) $H(z) = \frac{z-1}{z} \cdot \mathcal{Z}\left\{ \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] \Big|_{t=kT} \right\}$ donde $H(s) = 0{,}05 \cdot \frac{400}{s^2 + 20s + 400}$ y $T = 0{,}1$ seg

Tablas: $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightsquigarrow \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$

$\omega_n = 20$
$\xi = 0{,}5$

$b_1 = 1 - \alpha\left(\beta + \frac{\xi\omega_n}{\omega}\cdot\gamma\right)$
$b_2 = \alpha^2 + \alpha\left(\frac{\xi\omega_n}{\omega}\cdot\gamma - \beta\right)$
$a_1 = -2\alpha\beta$
$a_2 = \alpha^2$

$\omega = \omega_n\sqrt{1-\xi^2} = \sqrt{3}\,10$
$\alpha = e^{-\xi\omega_n T} = e^{-10T} = e^{-1{,}1}$
$\beta = \cos(\omega T) = \cos(1{,}1\sqrt{3})$
$\gamma = \operatorname{sen}(\omega T) = \operatorname{sen}(1{,}1\sqrt{3})$

Así, $\boxed{H(z) = 0{,}05 \cdot \frac{0{,}9277 z + 0{,}4046}{z^2 + 0{,}2185 z + 0{,}1108}}$

$G(z) = \frac{K_p H(z)}{1 + K_p H(z)} = \frac{0{,}05 K_p (0{,}9277 z + 0{,}4046)}{z^2 + \underbrace{(0{,}05 \cdot K_p \cdot 0{,}9277 + 0{,}2185)}_{a} z + \underbrace{(0{,}05 K_p \cdot 0{,}4046 + 0{,}1108)}_{b}}$

Estable $\Leftrightarrow \begin{cases} b < 1 \\ b > a - 1 \\ b > -a - 1 \end{cases} \Leftrightarrow$

$0{,}0204\, K_p < 0{,}8892 \Leftarrow \boxed{K_p < 44{,}23}$

$0{,}0204\, K_p + 0{,}1108 > 0{,}2185 - 1 + 0{,}0464 \cdot K_p$

$0{,}0263\, K_p < 0{,}8923 \Leftarrow \boxed{K_p < 33{,}92}$

$0{,}0204\, K_p + 0{,}1108 > -0{,}2185 - 1 - 0{,}0464 \cdot K_p$

$0{,}0665\, K_p > -1{,}3293 \Leftarrow \boxed{K_p > -19{,}99}$

$\boxed{-20 < K_p < 33{,}9}$

4) (Cont.)

Los polos de $G(z)$ deben ser reales para que la respuesta al pulso no tenga componentes sinusoidales.

Los polos de $G(z)$ son las soluciones de $1+K_p H(z)=0$: $1+K_p\cdot\frac{1}{20}\cdot\frac{b_1z+b_2}{z^2+a_1z+a_2}=0$

$K_p=0$ (polos de $H(z)$) : $z^2+a_1z+a_2=0 \longrightarrow z_0=-0{,}1073 \pm j\,0{,}3144$

$K_p=\infty$ (ceros de $H(z)$) : $b_1z+b_2=0 \longrightarrow z_\infty=-0{,}4329$

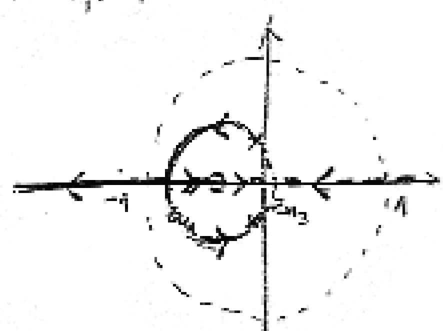
Ptos múltiples : $\frac{d}{dz}\frac{b_1z+b_2}{z^2+a_1z+a_2}=0 \Rightarrow b_1(z^2+a_1z+a_2)-(b_1z+b_2)(2z+a_1)=0$

$$b_1z^2+2b_2z+a_1b_2-a_2b_1=0$$

$z_{m_1}=-0{,}8841$ y $K_{m_1}=-20\left.\frac{(z^2+a_1z+a_2)}{b_1z+b_2}\right|_{z=z_{m_1}}=33{,}41$

$z_{m_2}=0{,}0183$ y $K_{m_2}=\quad "\quad \Big|_{z=z_{m_2}}=-5{,}501$

$\Rightarrow$ Rango de soluciones : $\begin{cases}-20<K_p\leq-5{,}5\\ 33{,}4<K_p<33{,}9\end{cases}$



- Si $K_p=-10$ $\quad G(z)=-\dfrac{0{,}4839z+0{,}2095}{z^2-0{,}2454z-0{,}0900}$ $\quad\begin{matrix}\lambda_1=0{,}4468\\ \lambda_2=-0{,}2014\end{matrix}$

$$G(z)=\frac{0{,}05K_p(b_1z+b_2)}{(z-\lambda_1)(z-\lambda_2)}=0{,}05K_p\cdot\frac{1}{\lambda_1-\lambda_2}\cdot\left[\frac{b_1\lambda_1+b_2}{z-\lambda_1}-\frac{b_1\lambda_2+b_2}{z-\lambda_2}\right]\cdot(z\cdot z^{-1})$$

$$\Rightarrow g_1(k)=\mathcal{Z}^{-1}\{G(z)\}=\frac{K_p}{20(\lambda_1-\lambda_2)}\left[(b_1\lambda_1+b_2)\lambda_1^{k-1}-(b_1\lambda_2+b_2)\lambda_2^{k-1}\right]$$

- si $K_p=K_{m_1}$ (rango ajustado y al límite de amortiguamiento)

$$G(z)=0{,}05K_p\frac{(b_1z+b_2)}{(z-z_{m_1})^2}=\frac{K_p}{20}\left[\frac{b_1z_{m_1}+b_2}{(z-z_{m_1})^2}+\frac{b_1}{z-z_{m_1}}\right]\cdot(z\cdot z^{-1})$$

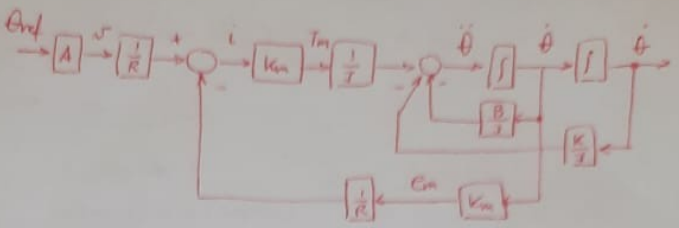
$$\Rightarrow g_2(k)=\frac{K_p}{20}\cdot\left[\frac{b_1z_{m_1}+b_2}{z_{m_1}}(k-1)\,z_{m_1}^{k-1}+b_1\cdot z_{m_1}^{k-1}\right]$$

Examen de diciembre de 2022
Solución del problema 2

uaciones:

$$\begin{cases} v = A\,\theta_{ref} \\ v = Ri + e_m \\ e_m = K_m\,\dot{\theta} \\ T_m = K_m\,i \end{cases}$$

$$J\ddot{\theta} = T_m - B\dot{\theta} - k\theta$$



MVE:

$$\ddot{\theta} = \frac{1}{J}\frac{K_m}{R}\left(A\,\theta_{ref} - K_m\dot{\theta}\right) - \frac{B}{J}\dot{\theta} - \frac{K}{J}\theta$$

$$\ddot{\theta} = -\frac{1}{J}\left(\frac{K_m^2}{R} + B\right)\dot{\theta} - \frac{K}{J}\theta + \frac{K_m A}{JR}\theta_{ref}$$

$$\dot{\begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix}} = \begin{bmatrix} -\frac{1}{J}(K_m^2/R + B) & -K/J \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} + \begin{bmatrix} K_m A/JR \\ 0 \end{bmatrix}\theta_{ref}$$

$$\theta = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix}$$

TF:

$$\boxed{H(s) = \frac{\theta(s)}{\theta_{ref}(s)} = \frac{K_m A/JR}{s^2 + \frac{1}{J}(K_m^2/R + B)\,s + K/J}}$$

Valores numéricos: $J = 5$, $K = 20$, $B = 1$, $A = 20$, $R = 1$, $K_m = 10$

TF:

$$H(s) = \frac{40}{s^2 + 20.2s + 4} = \frac{40}{(s+20)(s+0.2)} \approx \frac{2}{s+0.2} = \tilde{H}(s)$$

$$H(0) = 10 = \tilde{H}(0)$$

INSTITU

$e_{ss\ rampa} < 10\%$

$$k_v = \lim_{s\to 0} s\, G_{OL}(s) = \lim_{s\to 0} s\left(k_p + k_I/s\right)\frac{2}{s+0.2} = 10\, k_I$$

$$e_{ss} = \frac{1}{k_v} < 0.1 \iff \boxed{k_I > 1} \ (1)$$

estabilidad: $G_{cl}(s) = \dfrac{2(sk_p + k_I)}{s(s+0.2) + 2(sk_p + k_I)} = \dfrac{2(sk_p + k_I)}{s^2 + (0.2 + 2k_p)s + 2k_I}$

estable $\iff \boxed{k_I > 0}$, $\boxed{k_p > -0.1}$ (2)

ii)

$e_{ss\ rampa} = 5\% \Rightarrow \boxed{k_I = 2}$

$\zeta = 0.8 \Rightarrow \zeta = \dfrac{0.2 + 2k_p}{2\sqrt{4}} \Rightarrow \boxed{k_p = 1{,}5}$

Tiempo discreto:

$C(z)$: $\text{area}(k) = \text{area}(k-1) + T \cdot e_{k-1} \Rightarrow \dfrac{\text{area}(z)}{e(z)} = \dfrac{T}{z-1}$

$\Rightarrow C(z) = k_p + k_I \dfrac{T}{z-1}$

$G_{cl}(z) = \dfrac{C(z)\cdot \tilde{H}(z)}{1 + C(z)\cdot\tilde{H}(z)}$; $\tilde{H}(z) = 10\,\dfrac{1 - e^{-0.2T}}{z - e^{-0.2T}}$

$C(z) = \dfrac{k_p(z-1) + k_I T}{z-1}$

$$G_{cl}(z) = \frac{(k_p(z-1) + k_I T)\, 10\,(1 - e^{-0.2T})}{(z-1)(z - e^{-0.2T}) + 10(k_p(z-1) + k_I T)(1 - e^{-0.2T})}$$

$$d(z) = z^2 + \underbrace{\left(10\, k_p(1 - e^{-0.2T}) - (1 + e^{-0.2T})\right)}_{a(T)} z + \underbrace{e^{-0.2T} + 10(k_I T - k_p)(1 - e^{-0.2T})}_{b(T)}$$

Jury: estable $\iff \begin{cases} -1 < b(T) < 1 \\ -(b(T)+1) < a(T) < b(T)+1 \end{cases}$