3/11

14. Hallar las series de Fourier de las funciones definidas por:

a) $f(x) = |x|$, si $-2 \leqslant x \leqslant 2$, 4-periódica .

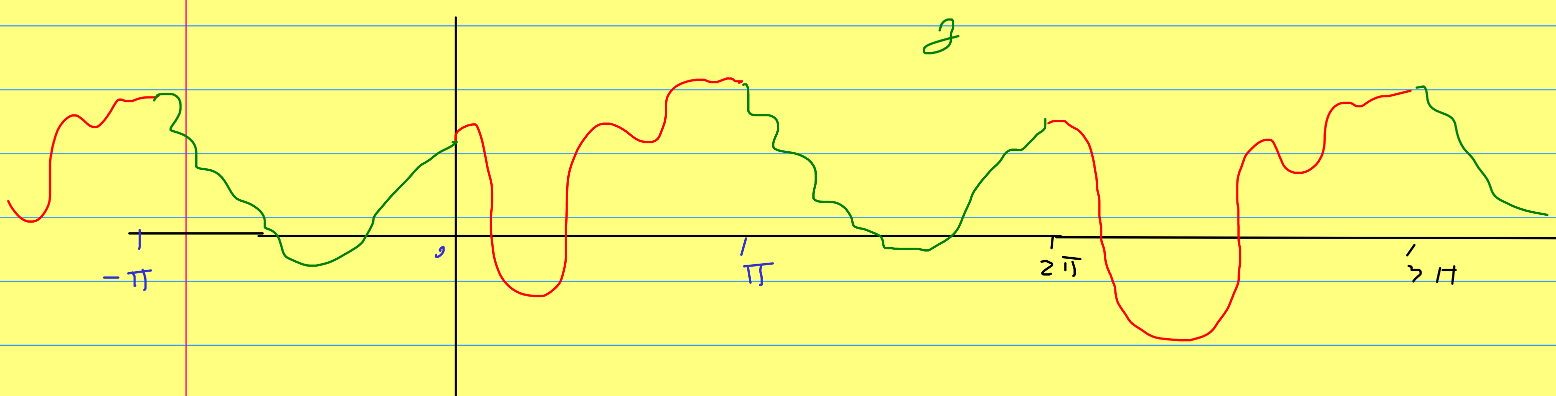
b) $f(x) = \begin{cases} 0 & \text{si} \quad -\pi \leqslant x < 0 \\ 1 & \text{si} \quad 0 \leqslant x \leqslant \frac{\pi}{2} \\ 0 & \text{si} \quad \frac{\pi}{2} < x \leqslant \pi \end{cases}$, 2π-periódica.

b) $\cos\left(\frac{K\pi}{2}\right) = 0$ si K impar

K par: $\begin{matrix} 0, & 2, & 4, & 6 \\ 1 & -1 & 1 & -1 \end{matrix}$

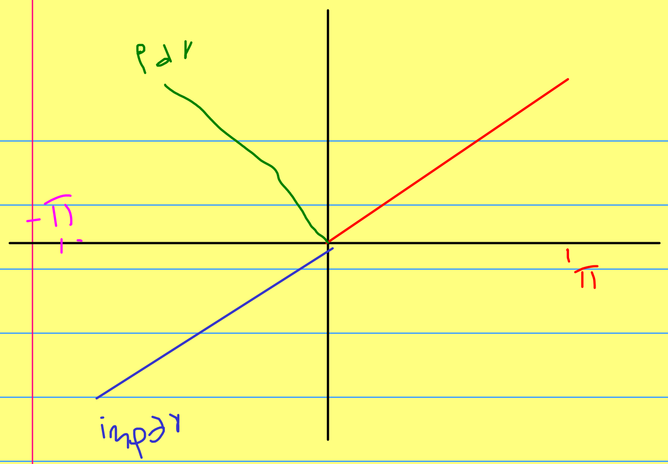
		$2\cdot 1$	$2\cdot 2$	$2\cdot 3$	$2\cdot 4$	$2\cdot 5$	$2\cdot 6$	$2\cdot 7$	
La sucesión: K	0	2	4	6	8	10	12	14	16
a_K	1	-1	1	-1	1	-1	1	-1	1

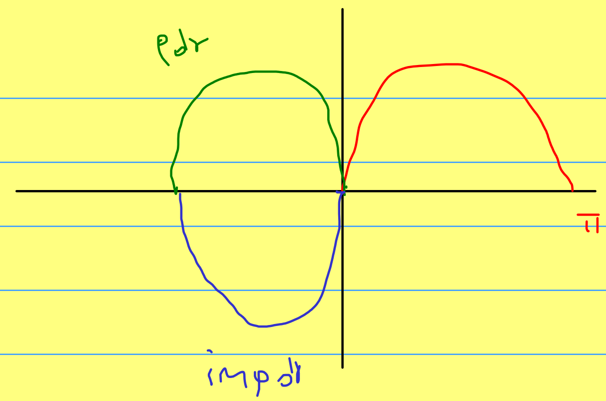
se puede escribir como $a_K = (-1)^{\frac{K}{2}}$



Extensión par $\longmapsto$ Serie de Fourier de cosenos

Extensión impar $\longmapsto$ Serie de Fourier de senos





Dada $f:[0,\pi] \to \mathbb{R}$ (integrable)
la extensión:

par: $b_k = 0$, $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$$a_k = \frac{2}{\pi}\int_0^{\pi} f(x)\cos kx\,dx$$

impar: $a_k = 0$ $\qquad b_k = \frac{2}{\pi}\int_0^{\pi} f(x)\operatorname{sen} kx\,dx$

impar

13. En los siguientes casos, escribir la serie de Fourier de senos y la serie de Fourier de cosenos de la función f definida en $(0,\pi)$ como:

par

a) $f(x) = 1$ $\qquad$ b) $\boxed{f(x) = \pi - x}$ $\qquad$ c) $f(x) = \begin{cases} 1 & \text{si } 0 < x < \frac{\pi}{2} \\ 0 & \text{si } \frac{\pi}{2} \leq x < \pi \end{cases}$

$f(x) = \pi - x.$

Fourier de cosenos: $\quad b_k = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} \pi - x\,dx$$

$$= \frac{2}{\pi}\left[\pi x\Big|_0^{\pi} - \frac{x^2}{2}\Big|_0^{\pi}\right]$$

$$= \frac{2}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \cancel{\frac{2}{\pi}} \cdot \frac{\pi^{\cancel{2}}}{\cancel{2}} = \pi \qquad \boxed{a_0 = \pi}$$

$$a_K = \frac{2}{\pi}\int_0^{\pi} f(x)\cos Kx\,dx = \frac{2}{\pi}\int_0^{\pi}(\pi - x)\cos Kx\,dx =$$

$$= \frac{2}{\pi}\left[\pi \underbrace{\cancel{\int_0^{\pi}\cos Kx\,dx}}_{\frac{\operatorname{sen} Kx}{K}\Big|_0^{\pi}} - \int_0^{\pi} x\cos Kx\,dx\right] \qquad 0$$

$$\int_0^{\pi} \overset{f}{x}\,\overbrace{\cos Kx}^{g'}\,dx \overset{\text{partes}}{=} \cancel{\frac{x\operatorname{sen} Kx}{K}\Big|_0^{\pi}} - \int_0^{\pi}\frac{\operatorname{sen} Kx}{K}\,dx \qquad 0$$

$$= -\frac{1}{K}\int_0^{\pi}\operatorname{sen} Kx\,dx$$

$$= -\frac{1}{K}\left(-\frac{\cos Kx}{K}\Big|_0^{\pi}\right)$$

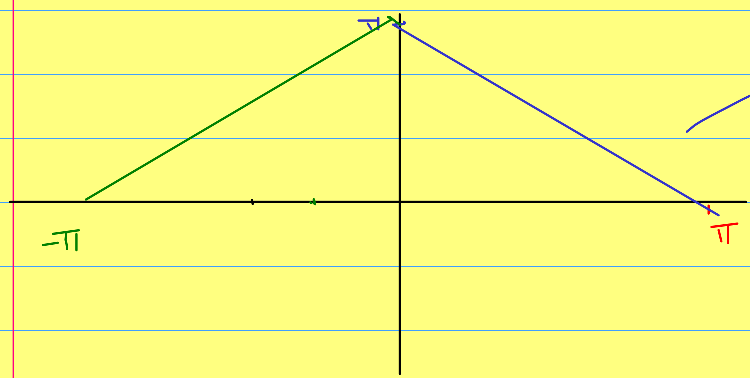
$$= \frac{\overbrace{\cos K\pi}^{(-1)^K} - \overbrace{\cos(0)}^{1}}{K^2} = \frac{(-1)^K - 1}{K^2}$$

$$a_K = -\frac{2}{\pi}\,\frac{(-1)^K - 1}{K^2} = \frac{2}{\pi}\,\frac{1 - (-1)^K}{K^2}$$

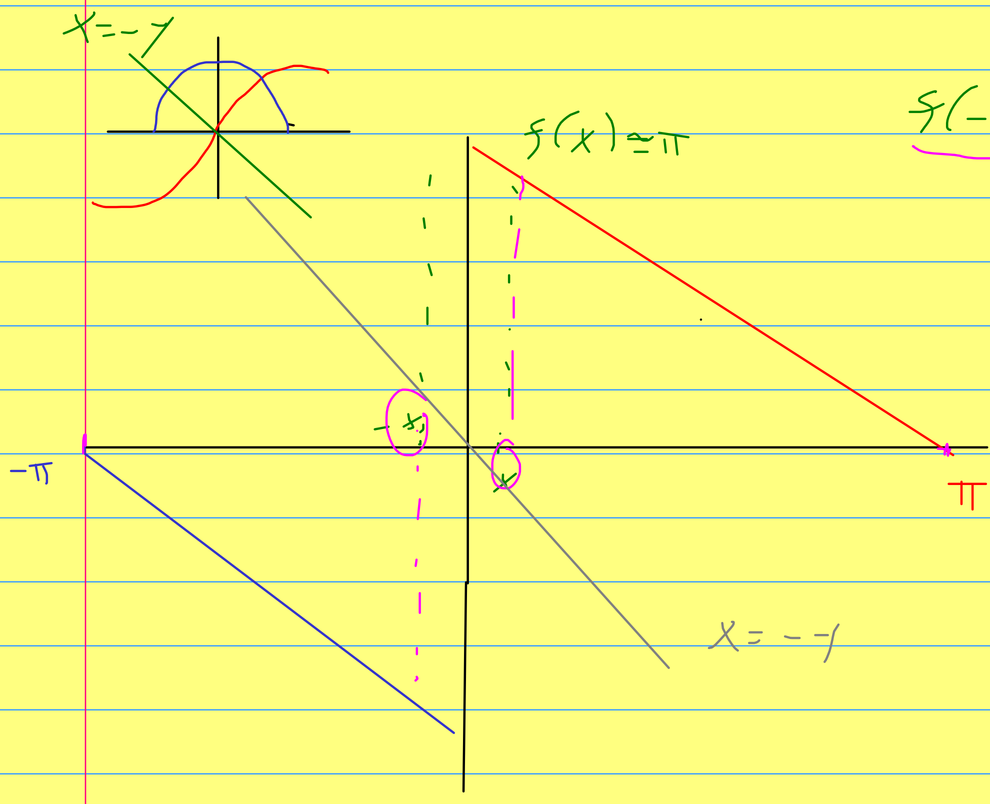
$$\boxed{a_K = \frac{2}{\pi K^2}\left[1 - (-1)^K\right]}$$

La serie de Fourier de cosenos de $f(x) = \pi - x$ es:

$$S_\infty(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{K=1}^{\infty} \frac{1-(-1)^K}{K^2} \cos Kx$$



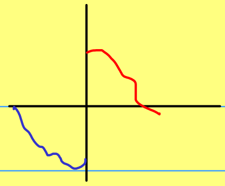
$x \in (0, \pi)$



$f(-x) = -f(x)$

$$S_n(x) = \sum_{K=1}^{n} b_K \operatorname{sen} Kx$$

- Como las S_n son continuas pero S_∞ no entonces la convergencia no puede ser uniforme.



$f(0) \neq 0$

La extensión impar no converge uniformemente

Práctico 7

1. Hallar la solución de la ecuación de ondas,

$$\boxed{u_{tt}(x,t) - c^2 u_{xx}(x,t) = 0} \quad (x,t) \in (0,L) \times (0,\infty)$$

con las condiciones de contorno:

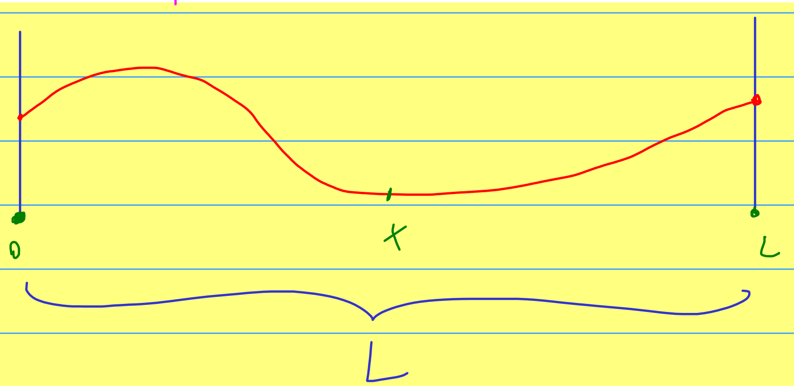
Posición inicial

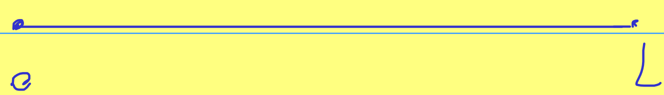
Velocidad inicial

$$\begin{cases} \boxed{u(x,0) = x(L-x)} & x \in [0,L] \\ \boxed{u_t(x,0) = 0} & x \in [0,L] \\ \boxed{u(0,t) = u(L,t) = 0} & t \in [0,\infty) \end{cases}$$

utilizando el método de separación de variables.

Extremos de la onda están fijos





$u(x,t) = 0 \quad \forall (x,t)$

Variables separables:

$$\longrightarrow \boxed{u(x,t) = X(x)\,T(t)}$$

$$u_{tt} - c^2 u_{xx} = 0$$

$$u_{tt} = X(x)\,T''(t) \quad , \quad u_{xx} = X''(x)\,T(t)$$

$$X\,T'' - c^2 X''\,T = 0$$

$$\frac{1}{c^2}\frac{T''(t)}{T} = \frac{X''(x)}{X}$$

si vale $\forall (x,t) \in (0,L) \times (0,\infty)$

$$\Rightarrow \frac{1}{c^2}\frac{T''}{T} = \underbrace{K}_{\text{constante}} = \frac{X''}{X}$$

$$\begin{cases} \dfrac{1}{c^2}\dfrac{T''(t)}{T} = K \longrightarrow T'' - Kc^2\,T = 0 & \text{I} \\[2ex] \dfrac{X''(x)}{X} = K \longrightarrow X'' - KX = 0 & \text{II} \end{cases}$$

- Veamos cuales son las condiciones iniciales de I y II a partir de las condiciones iniciales en u.

$$\begin{cases} u(x,0) = x(L-x) & x \in [0,L] \\ u_t(x,0) = 0 & x \in [0,L] \\ u(0,t) = u(L,t) = 0 & t \in [0,\infty) \end{cases}$$

(I): $\begin{cases} T'' - Kc^2 T = 0 \\ T(0) = \\ T'(0) = 0 \end{cases}$

$u(x,t) = X(x)\,T(t)$

$u(0,t) = X(0)\,T(t) = 0 \to \boxed{X(0) = 0}$

$\to T(t) = 0 \quad \forall t$

Si $T(t) = 0$

$u(x,t) = X(x)\,T(t) = 0$

es la solución trivial

$\cdot\ u(L,t) = X(L)\,\underbrace{T(t)}_{\text{hipótesis}} = 0 \to \boxed{X(L) = 0}$

$\cdot\ u_t(x,0) = 0$

Si $u = X\,T \Rightarrow u_t = X\,T'$

$X(x)\,T'(0) = 0 \to X(x) = 0 \quad \forall x$ → Genera la solución trivial

$\to \boxed{T'(0) = 0}$

$$\begin{cases} X'' - KX = 0 \\ X(0) = 0 \\ X(L) = 0 \end{cases}$$

Polinomio característico: $\lambda^2 - K = 0$

• $K = 0$:

$$\frac{X''}{X} = 0 \rightarrow X'' = 0$$

$$X(x) = ax + b$$

$X(0) = 0 \rightarrow b = 0$

$X(L) = 0 \rightarrow a \cdot L = 0 \rightarrow a = 0$

Si $K = 0 \Rightarrow X(x) = 0 \ \forall x$. No queremos esa solución.

• $K > 0$:

$$X(t) = C_1 e^{\sqrt{K}x} + C_2 e^{-\sqrt{K}x}$$

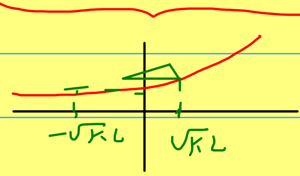
$0 = X(0) = C_1 + C_2$

$0 = X(L) = C_1 e^{\sqrt{K}L} + C_2 e^{-\sqrt{K}L}$

$$\begin{cases} C_1 + C_2 = 0 \rightarrow C_2 = -C_1 \\ \underbrace{C_1 e^{\sqrt{K}L} + C_2 e^{-\sqrt{K}L}}_{} = 0 \end{cases}$$

$$C_1 e^{\sqrt{K}L} - C_1 e^{-\sqrt{K}L} = 0$$

$$C_1 \left[\underbrace{e^{\sqrt{K}L} - e^{-\sqrt{K}L}}_{>0} \right] = 0$$



$\rightarrow \boxed{C_1 = 0} \rightarrow \boxed{C_2 = 0}$

Nos queda la solución nula.

- $K < 0$:

$$\lambda^2 = -|K| \qquad K = -|K|$$

$$\lambda = \pm\sqrt{|K|}\, i \qquad +\ 0$$

$$X(x) = C_1 e^{\overset{1}{Re(\lambda)x}} \cos(Im(\lambda)x) + C_2 e^{\overset{1}{Re(\lambda)x}} sen(Im(\lambda)x)$$

$$X(x) = C_1 \cos(\sqrt{|K|}\,x) + C_2\, sen(\sqrt{|K|}\,x)$$