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(k) $A = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix}$

- Valores propios: $p(\lambda) = (-3-\lambda)(-1-\lambda) + 2$

$$= \lambda^2 + 4\lambda + 3 + 2$$
$$= \lambda^2 + 4\lambda + 5$$

$$\lambda = \frac{-4 \pm \sqrt{16-20}}{2} = \underbrace{-2}_{Re(\lambda)} \pm \underbrace{1}_{Im(\lambda)} i$$

$\boxed{\lambda = -2 + i}$ $\quad B = \begin{pmatrix} Re(\lambda) & Im(\lambda) \\ -Im(\lambda) & Re(\lambda) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$

$$A = P B P^{-1}$$

- Vectores propios:

$v \in Ker(A - \lambda Id)$

(k) $A = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix}$

$\lambda = -2 + i$

$$A - \lambda I = \begin{pmatrix} -3+2-i & 2 \\ -1 & -1+2-i \end{pmatrix} = \begin{pmatrix} -1-i & 2 \\ -1 & 1-i \end{pmatrix}$$

$$\begin{cases} (-1-i)x + 2y = 0 \\ -x + (1-i)y = 0 \end{cases}$$

$-x + (1-i)y = 0 \to x = (1-i)y$

$(-1-i)(1-i)y + 2y = 0$

$[(-1-i)(1-i) + 2]y = 0 \to y$ libre en $\mathbb{C}$

$-1^2 + i^2 = -2$

$y = 1 \rightarrow x = 1 - i$

$V = (1-i,\ 1) \in \mathbb{C} \times \mathbb{C}$

Diferencia: $V = (z, w)$ donde $z, w \in \mathbb{C}$

$$P = \begin{pmatrix} a = Re(z) & b = Im(z) \\ c = Re(w) & d = Im(w) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{B = \begin{pmatrix} Re(\lambda) & Im(\lambda) \\ -Im(\lambda) & Re(\lambda) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}}$$

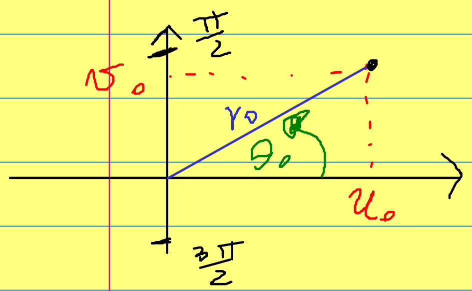
$\lambda = -2 + i$

$$\begin{cases} u' = -2u + v \\ v' = -u - 2v \end{cases}$$

$$Y(t) = \left(r_0 e^{Re(\lambda)t} \cos(Im(\lambda)t + \theta_0),\ r_0 e^{Re(\lambda)t} sen(Im(\lambda)t + \theta_0)\right)$$

$$= \left(r_0 e^{-2t} \cos(t + \theta_0),\ r_0 e^{-2t} sen(t + \theta_0)\right)$$

$r_0 = \|(u_0, v_0)\| = \sqrt{u_0^2 + v_0^2}$

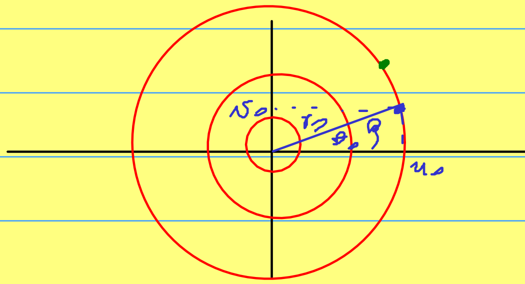


$$\theta_0 = \begin{cases} arctg\left(\frac{v_0}{u_0}\right) & \text{si } u_0 \neq 0 \\ \frac{\pi}{2} & \text{si } u_0 = 0,\ v_0 > 0 \\ \frac{3\pi}{2} & \text{si } u_0 = 0,\ v_0 < 0 \end{cases}$$

$X(t) = P\,Y(t)$

- $\lambda = 0 + b i$

$$\gamma(t) = (r_0 \cos(bt + \theta_0), r_0 \operatorname{sen}(bt + \theta_0))$$
$$= r_0 (\cos(bt + \theta_0), \operatorname{sen}(bt + \theta_0))$$



$b > 0 \to$ Gira en sentido antihorario

$b < 0 \to$ Gira en sentido horario

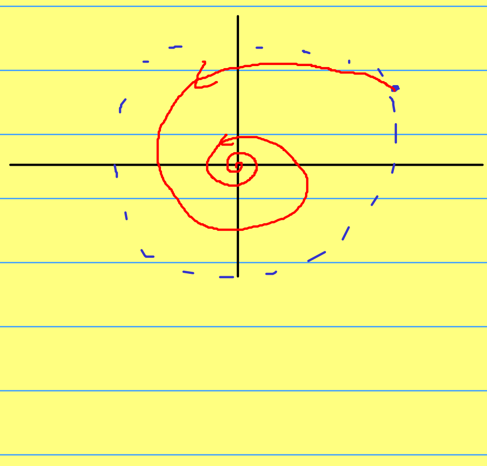
$\lambda = a + bi$

$$\gamma(t) = (r_0 e^{at} \cos(bt + \theta_0), r_0 e^{at} \operatorname{sen}(bt + \theta_0))$$
$$= r_0 e^{at} \underbrace{(\cos(bt + \theta_0), \operatorname{sen}(bt + \theta_0))}_{\text{Circunferencia}}$$

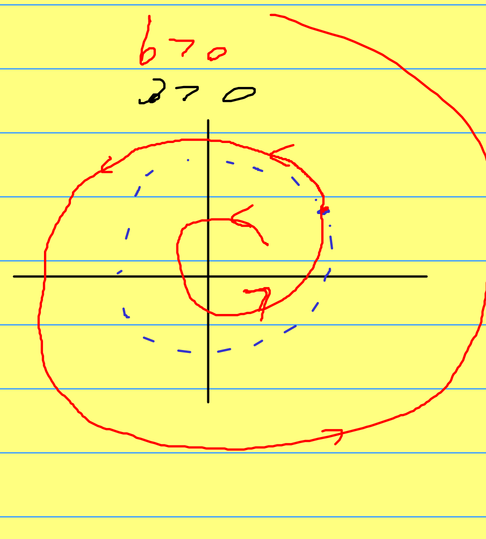
$$\|\gamma(t)\| = r_0 e^{at} \qquad \xrightarrow[a<0]{} 0$$

$$\xrightarrow[a>0]{} +\infty$$

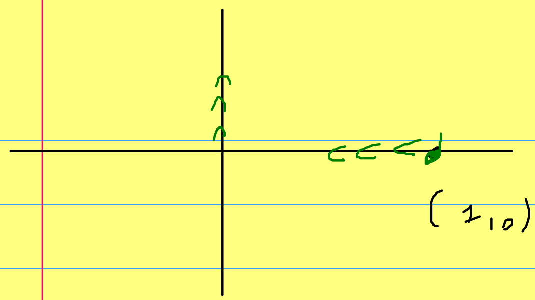
$b > 0$
$a < 0$

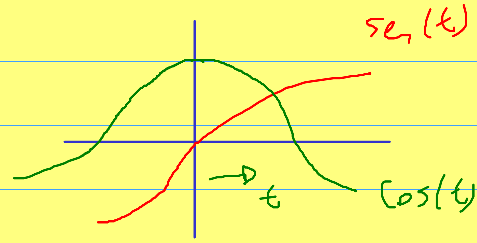


$b > 0$
$a > 0$

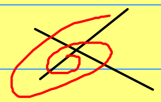


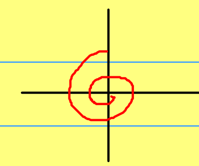
$((\cos(t), \operatorname{sen}(t))$





$$\dot{x} = A\,x \xrightarrow{\;y = P^{-1}x\;} \dot{y} = B\,y$$





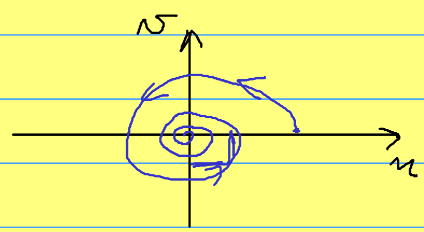
$$x = P\,y$$

En nuestro ejercicio. $\lambda = -2 + i$

$$y(t) = (r_0 e^{-2t} \cos(t+\theta_0),\ r_0 e^{-2t} \operatorname{sen}(t+\theta_0))$$

$x = Py.$

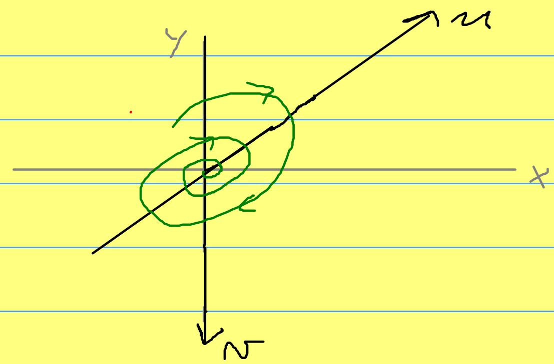
¿Cómo cambia el diagrama de fase?



$a = \operatorname{Re}(\lambda) = -2 < 0$

$b = \operatorname{Im}(\lambda) = 1 > 0$

$$P = \begin{array}{cc} u & v \\ \left(\begin{matrix} 1 & -1 \\ 1 & 0 \end{matrix}\right) & \end{array}$$



$\dot{x} = A\,x$

$\dot{x} = Ax \rightsquigarrow \dot{y} = By$ con $B = \begin{pmatrix}\mu & 0\\ 0 & \lambda\end{pmatrix}$

(e) $\mu < 0 < \lambda$

$\begin{cases} u' = \mu u \\ v' = \lambda v\end{cases}$ $\to u(t) = u_0 e^{\mu t}$

$\to v(t) = v_0 e^{\lambda t}$

• $u(t) = u_0 e^{\mu t} \to t = \frac{1}{\mu} \ln\left(\frac{u}{u_0}\right)$

$\to v(t) = v_0 \, e^{\lambda\left(\frac{1}{\mu}\ln\left(\frac{u}{u_0}\right)\right)} = v_0 \, \underline{e^{\overset{b}{\frac{\lambda}{\mu}} \ln\left(\overset{a}{\frac{u}{u_0}}\right)}}$

$a^b = e^{b\ln(a)}$

$= v_0 \left(\frac{u}{u_0}\right)^{\frac{\lambda}{\mu}}$

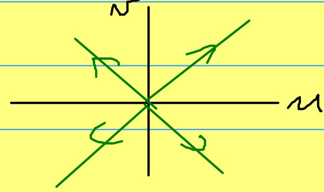
$= \frac{v_0}{u_0^{\frac{\lambda}{\mu}}} \cdot u^{\frac{\lambda}{\mu}}$

$v = \overset{K}{\boxed{\frac{v_0}{u_0^{\frac{\lambda}{\mu}}}}} u^{\frac{\lambda}{\mu}} = K u^{\frac{\lambda}{\mu}}$

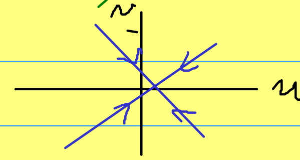
$S_g(\lambda) = S_g(\mu)$

• $\underline{\lambda = \mu}$: $\to v = Ku \to$ Rectas

$\lambda, \mu > 0$



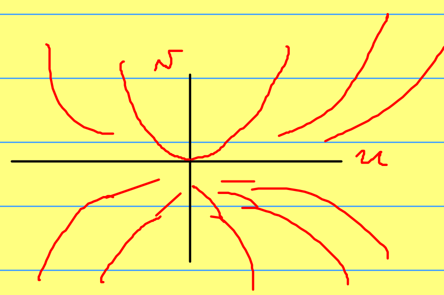
$\lambda, \mu < 0$



• $\underline{\lambda > \mu}$:

$v = K u^{\frac{\lambda}{\mu}}$ $\frac{\lambda}{\mu} > 2$

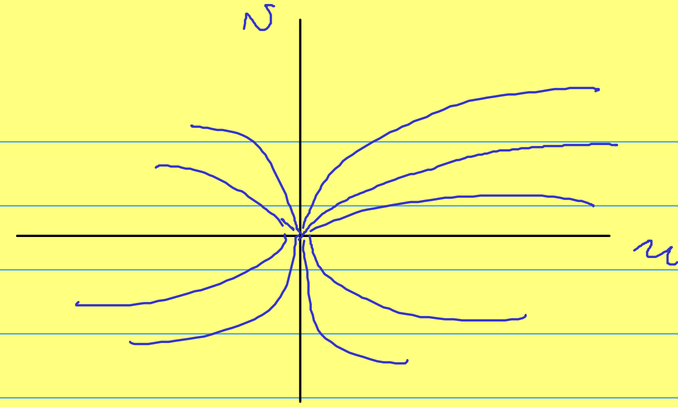
$v = u^2$

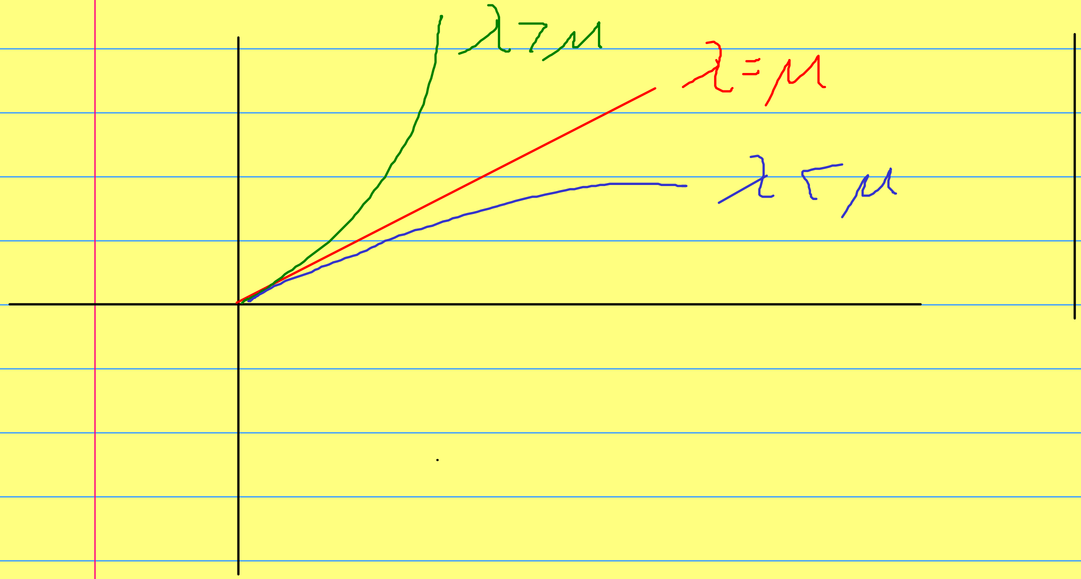


• $\underline{\lambda < \mu}$

$v = K u^{\frac{\lambda}{\mu}}$, $\frac{\lambda}{\mu} < 1$

Ej: $v = u^{1/2}$



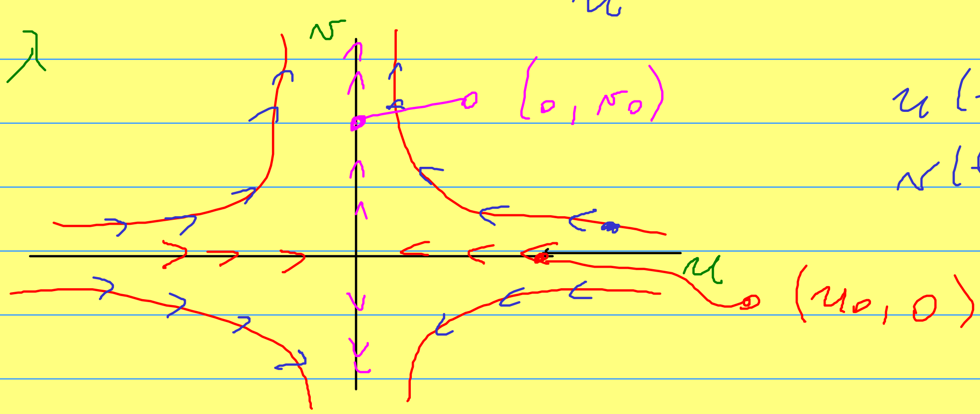


$s_g(\lambda) = s_g(\mu)$

$s_g(\lambda) \neq s_g(\mu) \rightarrow \frac{\lambda}{\mu} < 0$

$\rightarrow v = K u^{\frac{\lambda}{\mu}} = K \frac{1}{u^{|\frac{\lambda}{\mu}|}} \Rightarrow \boxed{v u^{|\frac{\lambda}{\mu}|} = K}$ → Hipérbolas

$\mu < 0 < \lambda$

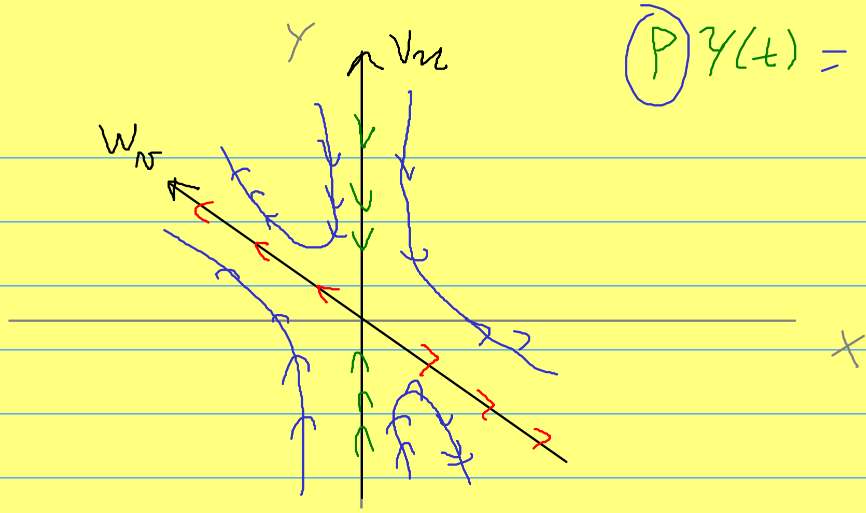


$u(t) = u_0 e^{\mu t} \rightarrow 0$

$v(t) = v_0 e^{\lambda t} \rightarrow \pm\infty$

$V_u = (0, 1)$

$W_v = (-1, 1)$



$(P)\, Y(t) = X(t)$

$$P = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \to X(t) = \left(a u_0 e^{\mu t} + b v_0 e^{\lambda t},\ c u_0 e^{\mu t} + d v_0 e^{\lambda t} \right)$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = X(0)$$

$$= \lambda^2 + 4\lambda + 3 + 2$$

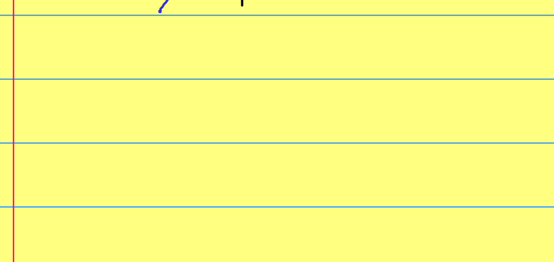
$$p(\lambda)\ (\lambda^2 + 4\lambda + 5)(-1-\lambda) \qquad A = \begin{pmatrix} \boxed{\begin{matrix} -3 & 2 \\ -1 & -1 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & -1 \end{pmatrix}$$

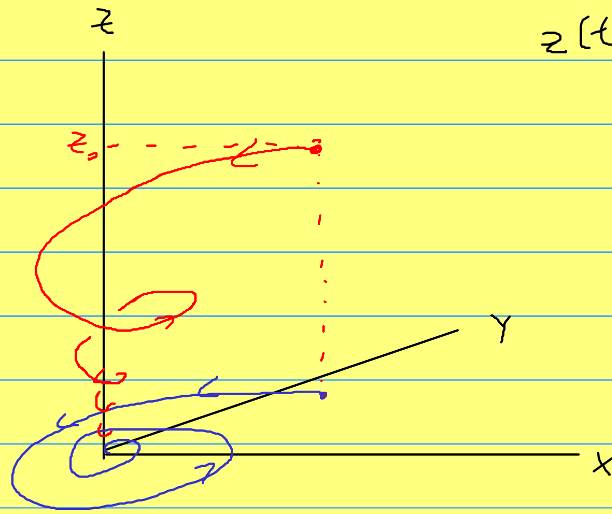
$$z' = -z$$

$$\lambda = -2 \pm i \to X(t) = \left(\underline{r_0 e^{-2t} \cos(t+\theta_0),\ r_0 e^{-2t} \sin(t+\theta_0)},\ z_0 e^{-1t} \right)$$

$$\lambda = -1$$

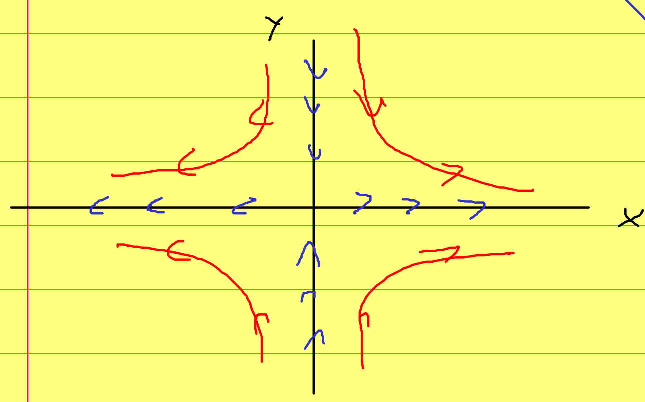
plano xy





$z(t) = z_0 e^{-t}$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \longrightarrow p(\lambda) = \underbrace{(1-\lambda)(-1-\lambda)}_{\substack{\lambda = 1 \\ \lambda = -1}} \underbrace{(-2-\lambda)}_{\lambda = -2}$$



$x(t) = x_0 e^{t}$

$y(t) = y_0 e^{-t}$

$z(t) = z_0 e^{-2t}$

$\longrightarrow z(t) = z_0 e^{-2t}$