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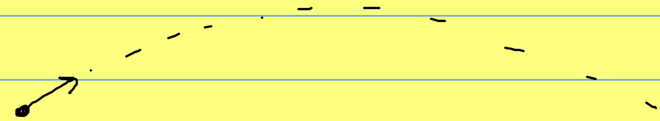
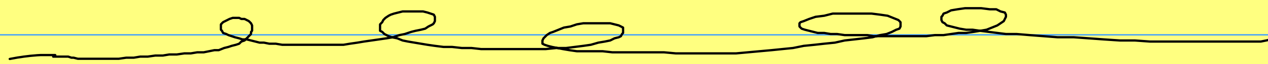
Ej: $x' = x$ tiene solución $x(t) = Ke^{t}$

$$\begin{cases} F = m x'' \\ x'(0) = v_0 \\ x(0) = x_0 \end{cases}$$





5. a) $y' - 3y = e^{2x}$ con $y(0) = 0$.

$$y' - 3y = e^{2x} \longmapsto y' + f(x)y = g(x)$$

- $y(x) = Ke^{-\int f(x)dx} + e^{-\int f(x)dx}\int g(x)e^{\int f(x)dx}dx$

- $y(x) = y_H(x) + y_p(x)$

Es solución de $\underbrace{y' + f(x)y = 0}_{\text{Es de variables separables}}$

Una solución particular de $y' + f(x)y = g(x)$

En nuestro caso: $y' - 3y = e^{2x}$

La homogenea es $y' - 3y = 0$

y_H:

$$y' - 3y = 0$$
$$y' = 3y$$
$$\frac{y'}{y} = 3 \qquad \text{Si } y \neq 0$$

$$\int \frac{y'}{y}\,dx = 3\int dx$$

$y = y(x) \rightarrow dy = y'\,dx$

$$\underbrace{\int \frac{1}{y}\,dy}_{\ln(|y_H(x)|)} = 3\underbrace{\int dx}_{3x+K}$$

$$\ln(|y_H(x)|) = 3x + K$$

- $y_H(x) = e^{3x+K} = e^{3x}\,\underbrace{e^{K}}_{A} = Ae^{3x}$

$$\boxed{y_H(x) = Ae^{3x}}$$

Verificación: • $y_H' - 3y_H = 0$

$y_H' = 3\underbrace{Ae^{3x}}_{y_H} = 3y_H$ $\Big\}$ $y_H' - 3y_H = 0$ ✓

- Solución particular:

$$y' - 3y = e^{2x}$$

Proponemos $y(x) = Ke^{2x}$. $\rightarrow y' = 2Ke^{2x}$

$y' - 3y = 2Ke^{2x} - 3Ke^{2x}$
$= -Ke^{2x} \underset{?}{=} e^{2x}$ $\Big\}$ $\boxed{K = -1}$

Queremos

Una solución particular es $y_p(x) = -e^{2x}$

La solución general de $y' - 3y = e^{2x}$ es

$$y(x) = y_H(x) + y_p(x)$$ es decir

$$\boxed{y(x) = A e^{3x} - e^{2x}}$$

Se halla usando la condición inicial.

$$\underset{\underset{0}{\shortparallel}}{y(0)} = A e^0 - e^0 = A - 1$$

$$A - 1 = 0 \rightarrow \boxed{A = 1}$$

La solución a $\begin{cases} y' - 3y = e^{2x} \\ y(0) = 0 \end{cases}$ es

$$\boxed{y(x) = e^{3x} - e^{2x}}$$

1. **Variables separables.**

Resolver las siguientes ecuaciones:

a) $y' = y^2 - 1$.

$$y' = y^2 - 1$$

$$\int \frac{dy}{y^2 - 1} = \underbrace{\int 1\, dx}_{x + K}$$

- $\displaystyle\int \frac{1}{y^2-1}\, dy$

- Las raíces de $y^2 - 1$ son ± 1

$$\frac{1}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1} = \frac{A(y+1) + B(y-1)}{\underbrace{(y-1)(y+1)}_{y^2-1}}$$

$$\frac{1}{y^2-1} = \frac{(A+B)y + A - B}{y^2-1}$$

$$\begin{cases} A + B = 0 & (i) \\ A - B = 1 & (ii) \end{cases}$$

$(i) + (ii)$: $2A = 1 \rightarrow \boxed{A = 1/2}$

$A + B = 0 \rightarrow \boxed{B = -A = -\frac{1}{2}}$

$$\frac{1}{y^2-1} = \frac{1/2}{y-1} - \frac{1/2}{y+1}$$

$$\int \frac{1}{y^2-1}\,dx = \frac{1}{2}\int \frac{1}{y-1}\,dx - \frac{1}{2}\overbrace{\int \frac{1}{y+1}\,dx}^{\ln(|y+1|)}$$

$\int \frac{1}{y-1}\,dy$: $\quad u = y-1 \rightarrow du = dy$

$$\rightarrow \int \frac{1}{u}\,du = \ln(|u|) = \ln(|y-1|)$$

$$\int \frac{1}{y^2-1}\,dx = \frac{1}{2}\left[\ln(|y-1|) + \ln(|y+1|)\right]$$

$$\boxed{\frac{1}{2}\left[\ln(|y-1|) + \ln(|y+1|)\right] = x + K}$$

$$\ln(|y-1|) + \ln(|y+1|) = 2x + 2K$$

$$e^{\ln(|y-1|)}\, e^{\ln(|y+1|)} = Ae^{2x}$$

$$|y-1||y+1| = Ae^{2x}$$

$$|(y-1)(y+1)| = Ae^{2x}$$

$$|y^2-1| = Ae^{2x}$$

- Si $|y| \geq 1 \Rightarrow y^2(x) - 1 = Ae^{2x}$

$$y(x) = \pm\sqrt{Ae^{2x}+1}$$

• Si $|y| < 1$ $\quad -(y^2-1) = Ae^{2x}$

$$y^2 - 1 = -Ae^{2x}$$

$$y(x) = \pm\sqrt{1 - Ae^{2x}}$$

Si $y(0) = 1 \Rightarrow \underset{\underset{1}{\parallel}}{y(0)} = \sqrt{Ae^{0}+1} = \sqrt{A+1}$

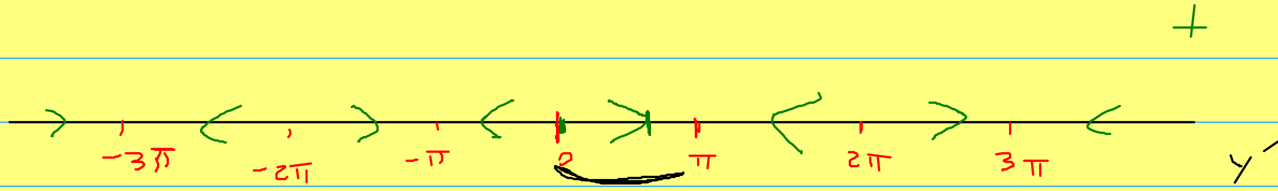
$$\sqrt{A+1} = 1$$

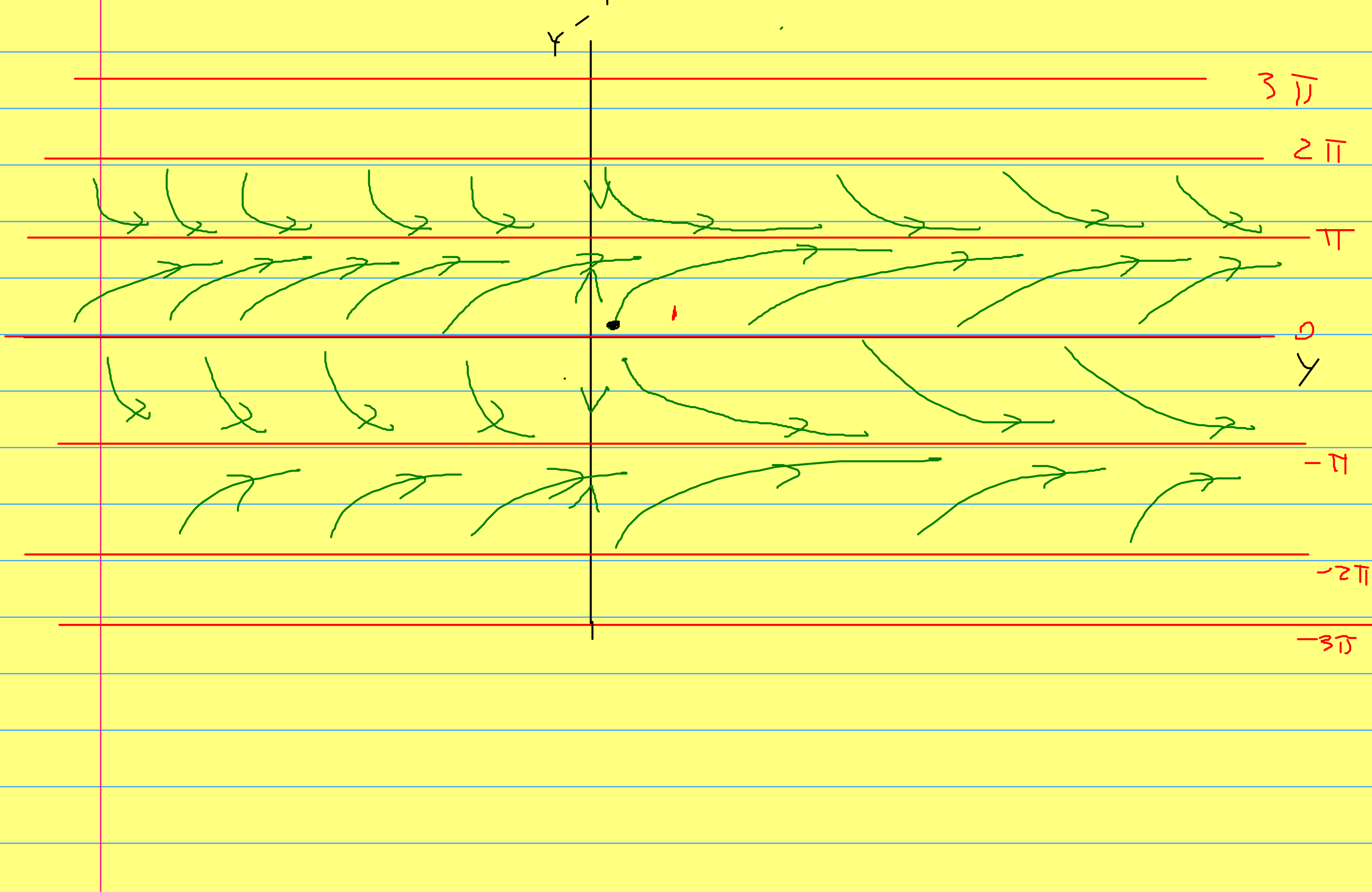
$$A+1 = 1$$

$$\boxed{A = 0}$$

3. d) $y' = \operatorname{sen}(y)$.

1º) Las raíces: $\operatorname{sen}(y) = 0 \iff y = k\pi \quad k \in \mathbb{Z}$





Ecuaciones diferenciales de orden 2 con coeficientes constantes:

$$\underbrace{a}\,y'' + \underbrace{b}\,y' + \underbrace{c}\,y = f(x)$$

$a, b, c \in \mathbb{R}$

Def: El polinomio característico es

$$a\lambda^2 + b\lambda + c =: p(\lambda)$$

Ej:
$$3y'' - 5y' + 20\pi y = \cos(x)$$

$$3\lambda^2 - 5\lambda + 20\pi = p(\lambda)$$

1) Hay dos raíces reales distintas α y β

2) Una raíz doble real α

3) Hay raíces complejas conjugadas $\alpha \pm i\beta$

1) $y_H(x) = C_1 e^{\alpha x} + C_2 e^{\beta x}$

2) $y_H(x) = C_1 e^{\alpha x} + C_2 x e^{\alpha x}$

3) $y_H(x) = C_1 e^{\alpha x} \cos(\beta x) + C_2 e^{\alpha x} \operatorname{sen}(\beta x)$

11. a) $y'' + 3y' - 10y = 6e^{4x}$.

y_H: $p(\lambda) = \lambda^2 + 3\lambda - 10$

$$\lambda = \frac{-3 \pm \sqrt{9 + 40}}{2} = \frac{-3 \pm 7}{2} \begin{cases} \alpha = 2 \\ \beta = -5 \end{cases}$$

$$\boxed{y_H(x) = C_1 e^{2x} + C_2 e^{-5x}}$$

y_p: $y(x) = Ae^{4x} \rightarrow y' = 4Ae^{4x} \rightarrow y'' = 16Ae^{4x}$

$$y'' + 3y' - 10y = 6e^{4x}$$

$$16A\cancel{e^{4x}} + 12A\cancel{e^{4x}} - 10A\cancel{e^{4x}} = 6\cancel{e^{4x}}$$

$$18A = 6 \rightarrow A = \frac{6}{18} = \frac{1}{3} \rightarrow \boxed{A = \frac{1}{3}}$$

$$\Rightarrow y_p(x) = \frac{1}{3}e^{4x}$$

Entonces, la solución a $y'' + 3y' - 10y = 6e^{4x}$ es

$$\boxed{y(x) = C_1 e^{2x} + C_2 e^{-5x} + \frac{1}{3}e^{4x}}$$